Notes by-

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Kinematics of fluids

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*1) State the basic principles of mechanics which are applicable to fluids, in motion. ...... 7 *

2) W Distinguish between stream function and velocity potential in a fluid flow. The stream function and velocity potential for a flow are given by $\psi = 2xy$, $\phi = (x^2 - y^2)$. Show that the conditions of continuity and irrotational flow are satisfied. 10

3) The velocity components of a 2D flow are $u = \frac{y^3}{3} + 2x - x^2 y$
$v = xy^2 - 2y - x^3/3$. Show that it satisfies the continuity equation for an irrotational flow. Also find the ϕ and ψ values. 10

4) Determine whether following velocity components satisfy continuity equation (a) $u = (3x - y)$ $v = (2x + 3y)$

5) Calculate the unknown velocity components so that they satisfy continuity equation $u = 2x^2$ $v = xyz$ $w = ?$

6) Determine which of the following velocity fields represent possible example of irrotational flow.

(i) $u = Cx$ $v = -Cy$ (ii) $u = Ax^2 - Bxy$ $v = -2Axy + \frac{1}{2}By^2$

7) Check, $\phi = x^2 - y^2 + y$ represents the velocity potential for 2-dimensional irrotational flow. If it does, then determine the stream function ψ.
$(\psi = -2xy + x)$

8) If $\psi = y^2 - x^2$ determine whether the flow is rotational or irrotational. Then determine the velocity potential ϕ.
$(\phi = -2xy + c)$

9) If $\phi = A(x^2 - y^2)$ is the velocity potential, determine ψ

10) If the velocity field in three dimensional fluid flow is given by $V = 12xy\hat{i} + 6x^3\hat{j} + (xt^2x + z)\hat{k}$ find the local acceleration at the point (3, 4, 4) when t = 3 s and also the total acceleration at point (1, 1, 1) when t = 3 s.

[54 m/s² in z dirⁿ, 479.804 m/s²]

11) Check whether following functions represent the possible irrotational flow phenomenon. (i) $\phi = x + y + z$ (Y)
(ii) $\phi = Zx^2 - y^2 - z^2$ (N)
(iii) $\psi = 2Axy$ (Y)
(iv) $\psi = 2xy + y$. (Y)

Kolhe P.S.

Kinematics of Fluid Flow

Fluid Kinematics:- Branch of FM, in which geometry of motion ~~is~~ of fluid is taken into account without considering force causing the motion.

As fluid mass is composed of individual particles, vel. & acceln of a particle varies w.r.t. space & time (as in case of solid, vel. & acceln of particles is same as body)

Langrangian Mtd:- In this method, single fluid particle is selected & study is made at that single fluid particle.

Eulerian Mtd:- In this mtd, point occupied by fluid is selected & study is made at that point, occupied by fluid mass.

Velocity of Fluid Particle:- $V = \lim_{dt \to 0} \frac{ds}{dt}$

u, v, w are components of velocity in x, y & z directions.

$\therefore \bar{V} = u\hat{i} + v\hat{j} + w\hat{k}$

$V = \sqrt{u^2 + v^2 + w^2}$ (m/s)

Path line:- The actual path traced by a fluid particle over a period of time.

Stream line:- It is an imaginary line drawn through flowing mass of fluid such that tangent at any point represents direction of velocity at that point.

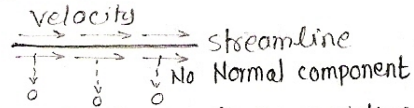
$\tan\theta = \frac{v}{u} = \frac{dy}{dx}$

$\Rightarrow \frac{dy}{v} = \frac{dx}{u}$

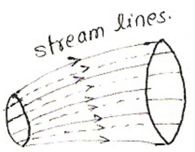
$\therefore$ For 3D flow

$\boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}} \longrightarrow$ Eqn of stream line.

$\therefore$ As velocity is tangential to streamline, there is no component of velocity normal to stream line. Hence flow is along streamline only.



Stream tube:- It is the tubular space bounded by surface consisting of stream line. Flow can takes place only through end faces of tube, similar to flow in open conduits.



Streak line:- It is the line traced by series of fluid particle from a fixed point in a flowing fluid mass.

For steady flow, path line, stream line & streak lines coinsides with each other.

Classification of Flow:-

Flow

- Based on Time
  - steady: $\frac{\partial}{\partial t}(\ ) = 0$
  - Unsteady: $\frac{\partial}{\partial t}(\ ) \neq 0$
- Based on space
  - Uniform: $\frac{\partial}{\partial s}(\ ) = 0$
  - Non Uniform: $\frac{\partial}{\partial s}(\cdot) \neq 0$
- Based on Layers
  - Laminar: $Re < 2000$
  - Turbulent: $Re > 4000$
  - Transition: Re betn 2000 - 4000
- ① Rotational (Forced vortex) ② Irrotational (free vortex)
- ① 1D ② 2D ③ 3D
- ① Critical $F=1$ ② Sub critical $F<1$ ③ Supercritical $F>1$

Basic principle of Flow analysis:-

① Conservation of Mass ⇒ Continuity eqn:- $\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$

$Q = A_1V_1 = A_2V_2$ (1D continuity eqn)

② Conservation of Energy ⇒ Energy eqn:-

③ Conservation of momentum ⇒ Momentum eqn:-

$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial u}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt} + \frac{\partial u}{\partial t}\cdot\frac{\partial t}{\partial t}$

$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$

Diff. w.r.t. x, y, z & t

Acceleration of Fluid Particle:-

$a = \lim_{dt \to 0} \frac{dV}{dt}$

$\bar{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$

Space ~~Time~~ dependent convective acceln

$\therefore a_x = u\frac{du}{dx} + v\frac{du}{dy} + w\frac{du}{dz} + \frac{du}{dt}$

$a_y = u\frac{dv}{dx} + v\frac{dv}{dy} + w\frac{dv}{dz} + \frac{dv}{dt}$

$a_z = u\frac{dw}{dx} + v\frac{dw}{dy} + w\frac{dw}{dz} + \frac{dw}{dt}$

Local or Temporary acceln Time dependent

Tangential & Normal Component of acceleration:-

Velocity is always tangential to stream line & accel^n have normal & tangential component. Let a_s & a_n are tangential & normal components.

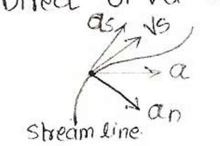
$$a_s = V_s \cdot \frac{dv_s}{ds} + \frac{dv_s}{dt}$$

$$a_t = V_s \cdot \frac{dv_n}{ds} + \frac{dv_n}{dt}$$

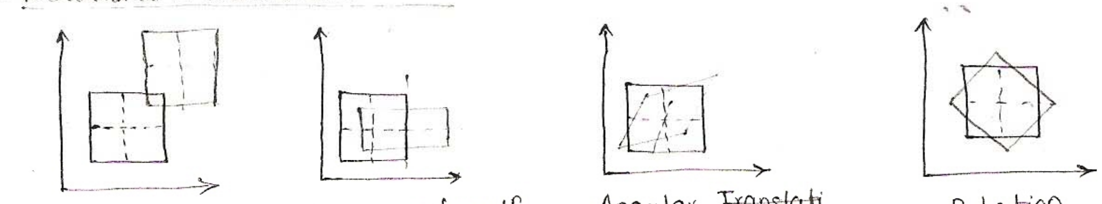
convective — Local or Temporary

As magnitude of vel. changes ⇒ Tangential accel^n

Direct^n of vel. changes ⇒ Normal accel^n.



* Rotational & Irrotational Motion:-



Pure Translation — Linear Deformat^n — Angular ~~Translati~~ deformation — Rotation

* Component of Rotation: The rotation component about any axis may be defined as the average of the sum of angular velocities of any two infinitesimal linear elements which are ⊥ar to each other & also ⊥ar to the axis of rotation.

$$\omega_x = \frac{1}{2}\left[\frac{\partial w^*}{\partial y} - \frac{\partial v^*}{\partial z}\right] \qquad \omega_y = \frac{1}{2}\left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right] \qquad \omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

How to Remember. $x \longrightarrow y \xrightarrow{(-)} z$, w^*, v^*

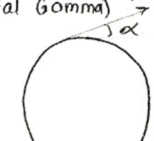
$$\bar{\omega} = \bar{\omega}_x \hat{i} + \bar{\omega}_y \hat{j} + \bar{\omega}_z \hat{k}$$

$$\omega = \sqrt{\omega_x^2 + \omega_y^2 + \omega_z^2}$$

Irrotational Flow:- If at every point in the flowing fluid, the rotation component $\omega_x, \omega_y, \omega_z$ are zero, the flow is known as irrotional.

i.e. $\frac{\partial v^*}{\partial x} = \frac{\partial u}{\partial y^*}$ $\quad \frac{\partial w}{\partial y} = \frac{\partial v}{\partial z}$; $\frac{\partial u}{\partial z} = \frac{\partial w}{\partial x}$

Circulation (Γ) (Capital Gomma) Circulation is defined as flow around closed curve. Mathematically circulation is defined as the line integral of the tangential component of velocity vector taken around a closed curve.



$$\Gamma = \oint_c V_s \cos\alpha \cdot ds$$

circulation around a rectangulare element : $\Gamma = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \cdot dx\, dy$

Vorticity: It is the ratio of circulation around infinitesimal close curve at a point to the area of curve.

$$\zeta \text{ (zeta)} = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 2\omega_z$$

* Velocity Potential / Potential function (ϕ) It is defined as the scalar function ~~whose~~ of space & time such that its -ve derivative w.r.t. any direction gives the fluid velocity in that direction.

i.e. $u = -\frac{\partial \phi}{\partial x}$, $v = -\frac{\partial \phi}{\partial y}$, $w = -\frac{\partial \phi}{\partial z}$

significance of -ve sign:-
as $s \uparrow$, ϕ decreases
(same as electric potential)

Let 3D continuity eq^n is, $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \Rightarrow \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z} = 0 \Rightarrow \nabla^2 \phi = 0 \Rightarrow$ Laplace Eq^n

condition for irrotational flow:-

$$\omega_x = \omega_y = \omega_z = 0$$

$$\frac{\partial^2 \phi}{\partial y \partial z} = \frac{\partial^2 \phi}{\partial z \partial y} \quad ; \quad \frac{\partial^2 \phi}{\partial z \partial x} = \frac{\partial^2 \phi}{\partial x \partial z} \quad ; \quad \frac{\partial^2 \phi}{\partial x \partial y} = \frac{\partial^2 \phi}{\partial y \partial x}$$

Thus existance of ϕ which satisfies Laplase eq^n is a possible case of steady, incompressible, irrotational flow. Such a case is called as potential flow.

* Stream function (ψ) ψ

It is defined as a scalar function of space & time such that its derivative w.r.t. any direct^n gives vel. component at right angle (in anticlockwise direct^n) to that direct^n.

i.e. $u = -\frac{\partial \psi}{\partial x}$, $v = \frac{\partial \psi}{\partial y}$

$$\left.\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} \; ; \; -\frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x}\right\}$$ Cauchy- Reiman conditions.

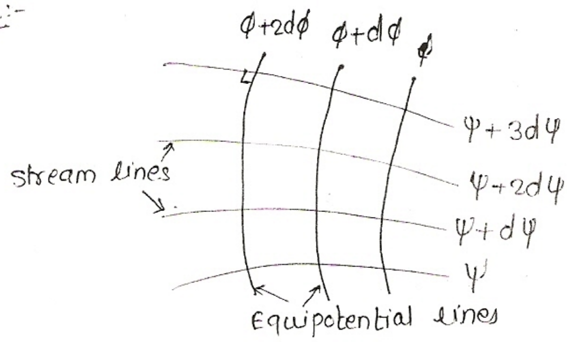
* ~~Equipotential line: The line along~~
condition for possible case of fluid flow: $\frac{\partial^2 \psi}{\partial x \cdot \partial y} = \frac{\partial^2 \psi}{\partial y \partial x}$
Condition for irrotational flow: $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$ — Poissons eq^n.

Equipotential line: This is the line along which the velocity potential ϕ is constant.
i.e ϕ = constant, $d\phi = 0$, $= \frac{\partial \phi}{\partial x} \cdot dx + \frac{\partial \phi}{\partial y} \cdot dy = 0 \Rightarrow \boxed{u dx + v dy = 0}$ - Eq^n of equipotential line.

Stream line: The line for which stream function ψ is constant is called as streamline.
i.e. ψ = constant, $\partial\psi = 0 = \frac{\partial \psi}{\partial x} \cdot dx + \frac{\partial \psi}{\partial y} \cdot dy = 0 \Rightarrow \boxed{v dx - u dy = 0}$
or $\boxed{\frac{dy}{v} = \frac{dx}{u}}$ Eq^n of stream line.

Relation bet^n ϕ line & ψ line:-
The product of slopes of ϕ line & ψ line is -1. i.e. they are orthogonal to each other.

Flow net:-



There is no flow possible tangential to equipotential line or across the streamline. Flow always takes place perpendicular to the equipotential line. Space bet^n two adjacent stream lines may be considered as a flow channel & discharge flowing through it is proportional to $\psi_2 - \psi_1$

Flow net is graphical representation of irrotational flow in 2D. it is the net formed by intersection of stream lines & equipotential lines.

* conditions ~~of~~ for drawing flow net :- Flow should be steady, irrotational & without governed by gravitational force.
* Methods :- a) Analytical
  b) Graphical
  c) Experimental Analogy → Electrical analogy, → Membrane analogy, → Viscous flow analogy.
  d) Relaxation mtd.
* Uses ① For given set of boundaries there is only one flow net, the same flow net can be used for geometrically similar boundaries of any size.
  ② Vel. at one pt. is known, vel. at other pt. can be found out by continuity eqn.
  ③ Efficient boundary profiles can be developed.
  ④ Boundaries can be modified to avoid or reduce seperation of flow.
  ⑤ seepage loss in hydraulic str. can be calculated.
  ⑥ To determine uplift pre.
* Limitations:-
  ① Flow net indicates that there is always some vel. near boundary but in real fluid, vel. is zero at boundary. so flow net theory is not applicable to flow near the boundary.
  ② It is not used for unsteady, rotational & gravitational flow.
  ③ The flow on u/s side of body is not disturbed while it is disturbed at d/s side. ∴ Flow net can be used for u/s side but not for d/s side.

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