Notes by-

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Laminar Flow

Defⁿ:- The flow in which particles of fluid behaves in orderly manner without intermixing with other & the flow takes place in number of sheets, layers or laminae, each sliding over the other is called as laminar flow.

eg. flow of oil in lubricating mechanism, flow of oil in measuring instruments, flow of fluid in capillary tubes, flow of blood through veins, flow of sap in tree.

Shear stress gradient (τ): There is no relative motion betⁿ the boundary & the ~~H~~ real fluid. This condⁿ is known as no slip condⁿ. Vel. is zero at the boundary & goes on increasing away from the boundary. Thus diff. layers of fluid moves with diff. velocities. The relative motion betⁿ the layers gives rise to shear stress. as $\tau = \mu \cdot \frac{du}{dy}$, as vel. is higher at free surface than boundary, the shear stress gradient ~~increases~~ decreases as we go towards boundary.

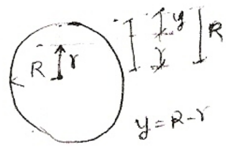
Relation betⁿ shear & pressure gradient in 2D, steady, uniform, laminar flow.

$$\boxed{\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y}}$$

Thus, pressure gradient in the directⁿ of flow is equal to the shear stress gradient in the normal direction.

$$\therefore \frac{\partial p}{\partial x} = \mu \cdot \frac{\partial^2 u}{\partial y^2}$$

Steady laminar flow through a circular pipe :-



1) $\tau = \left(-\frac{\partial p}{\partial x}\right) \cdot \frac{r}{2}$ .. ① where τ = shear stress
p = intensity of pressure
r = radius of concentric cylinder.

2) $\tau_{max} = \left(-\frac{\partial p}{\partial x}\right) \cdot \frac{R}{2}$ $(r = R)$

3) Velocity :- $\tau = \mu \cdot \frac{\partial u}{\partial y}$; $(y = R - r)$ $\Rightarrow$ $\frac{dy}{dr} = -1 \Rightarrow dy = -dr$

$\therefore \tau = -\mu \frac{du}{dr}$ ... equating with ① $\Rightarrow$ $-\frac{\partial p}{\partial x} \cdot \frac{r}{2} = -\mu \cdot \frac{\partial u}{\partial r}$

$\therefore \frac{\partial u}{\partial r} = \frac{1}{2\mu} \cdot \frac{\partial p}{\partial x} \cdot r$ $\therefore$ Integrating w.r.t. 'r' $\Rightarrow$ $u = \frac{1}{2\mu} \frac{\partial p}{\partial x} \cdot \frac{r^2}{2} + c = \frac{1}{4\mu} \frac{\partial p}{\partial x} \cdot r^2 + c$

To find c, at $r = R$, $u = 0$ $\Rightarrow$ $0 = \frac{1}{4\mu} \cdot \frac{\partial p}{\partial x} \cdot R^2 + c \Rightarrow c = -\frac{1}{4\mu} \cdot \frac{\partial p}{\partial x} \cdot R^2$

$\therefore u = \frac{1}{4\mu} \cdot \frac{\partial p}{\partial x} \cdot r^2 - \frac{1}{4\mu} \cdot \frac{\partial p}{\partial x} \cdot R^2 = \frac{1}{4\mu} \cdot \frac{\partial p}{\partial x} (r^2 - R^2)$ $\Big|$ $u = -\frac{1}{4\mu} \cdot \frac{\partial p}{\partial x} (R^2 - r^2)$ $\Big|$

$\therefore$ Velocity distⁿ is parabolic in nature.

i.e. $u \propto r^2$

for vel. to be max, $r = 0$

$$\therefore \boxed{u_{max} = \frac{1}{4\mu} \cdot \frac{\partial p}{\partial x} \cdot R^2}$$

$$\& \boxed{u = u_{max}\left[\frac{R^2 - r^2}{R^2}\right]} \quad \text{— (a)}$$

4) Discharge : $\boxed{Q = \frac{\pi R^4}{8\mu}\left(-\frac{\partial p}{\partial x}\right)}$

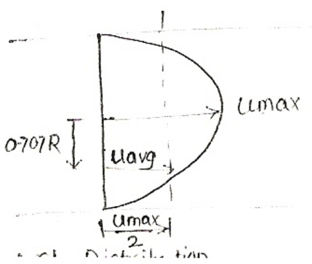
$$\boxed{u_{avg} = \frac{R^2}{8\mu}\left(-\frac{\partial p}{\partial x}\right)}$$

$$\boxed{u_{avg} = \frac{u_{max}}{2}} \quad \text{— (b)}$$

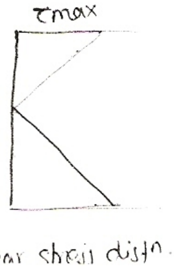
Equating (a) & (b)

$u_{max}\left(\frac{R^2 - r^2}{R^2}\right) = \frac{u_{max}}{2}$ $\Rightarrow$ $R^2 - r^2 = \frac{R^2}{2}$ or $r^2 = \frac{R^2}{2}$ $\Rightarrow$ $\left(\frac{r}{R}\right)^2 = \frac{1}{2}$

$\boxed{r = 0.707R}$ Thus avg. vel. occurs at radial dist. 0.707R from centre of pipe.



Distribution



Shear stress distⁿ.

⑤ Pressure drop over a given pipe length.

$$P_1 - P_2 = \frac{32\mu \cdot u_{avg} \cdot L}{D^2} = \frac{128\,\mu \cdot Q L}{\pi D^4}$$ } Hagen - Poiseuille eqn

⑥ Loss of head:-

$$h_f = \frac{P_1 - P_2}{\gamma} = \frac{32\mu \cdot u_{avg} \cdot L}{\gamma \cdot D^2} = \frac{128 \cdot \mu \cdot Q L}{\gamma \cdot \pi D^4}$$

⑦ friction factor :-

$$f = \frac{64}{Re}$$

⑧ Power = $P = (P_1 - P_2) \cdot Q = \gamma \cdot Q \cdot h_f$

* Laminar flow through inclind pipes :-

① velocity : $u = -\frac{1}{4\mu} \cdot \gamma \cdot \left(-\frac{\partial h}{\partial x}\right) \cdot (R^2 - r^2)$

② Power : $P = \gamma Q (h_1 - h_2)$

* Laminar flow through parallel plates (both plates fixed)

① shear Relatn betn shear stress & pressure gradient : $\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y} = \mu \cdot \frac{\partial^2 u}{\partial y^2}$

② velocity :- $u = \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)(By - y^2)$

③ $u_{max} = \frac{B^2}{8\mu}\left(-\frac{\partial p}{\partial x}\right)$

④ Discharge $q = \frac{B^3}{12\mu}\left(-\frac{\partial p}{\partial x}\right)$

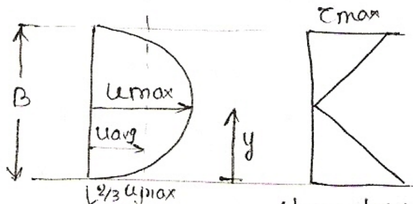
⑤ Avg. vel. = $u_{avg} = \frac{B^3}{12\mu}\left(-\frac{\partial p}{\partial x}\right) = \frac{2}{3} u_{max}$.

⑥ Pressure drop over given length of plates = $P_1 - P_2 = \frac{12\mu \cdot u_{avg} \cdot L}{B^2}$

⑦ Head loss = $\frac{P_1 - P_2}{\gamma} = h_f = \frac{12 \cdot \mu \cdot u_{avg} \cdot L}{\gamma \cdot B^2}$

⑧ shear stress = $\tau = -\left(\frac{\partial p}{\partial x}\right)\left(\frac{B}{2} - y\right)$

⑨ Max. shear stress = $\tau_{max} = \left(-\frac{\partial p}{\partial x}\right) \cdot \frac{B}{2}$



vel. Distn.   Shear stress distn.