Notes by-

Pravin S Kolhe,

BE(Civil), Gold Medal, MTech (IIT-K)

Assistant Executive Engineer,

Water Resources Department,

www.pravinkolhe.com

Dynamics of Fluid Flow

Er. Pravin Kolhe

Def^n:- Study of motion of fluid including the study of forces causing the motion & corresponding energy changes that takes place, is covered under Dynamics of fluid flow.

Dynamics of fluid flow is also governed by Newtons IInd law of motion i.e. rate of change of momentum is proportional to the external force in the direct^n of force.

i.e: $\Sigma F = m \cdot a$ & if F_x, F_y & F_z are components of forces in x, y & z direct^n & a_x, a_y, a_z are components of accel^n in x, y, z direct^n resp. then,

$\Sigma F_x = m \cdot a_x$, $\Sigma F_y = m \cdot a_y$; $\Sigma F_z = m \cdot a_z$.

ΣForce = Gravity force + Pressure force + Viscous force + Turbulent force + Surface ten. force + elastic force

i.e. $\Sigma F = M \cdot a = F_g + F_p + F_v + F_t + F_s + F_e$

If fluid is incompressible ($F_e = 0$); depth of flow is large ($F_s = 0$)

$\therefore M \cdot a = F_g + F_p + F_v + F_t \Rightarrow$
$M a_x = F_{gx} + F_{px} + F_{vx} + F_{tx}$
$M a_y = F_{gy} + F_{py} + F_{vy} + F_{ty}$
$M a_z = F_{gz} + F_{pz} + F_{vz} + F_{tz}$
} Reynold's Eq^n (Applicable to turbulent flow)

If flow is laminar, $F_t = 0$

$\therefore M a = F_g + F_p + F_v \Rightarrow$
$M \cdot a_x = F_{gx} + F_{px} + F_{vx}$
$M \cdot a_y = F_{gy} + F_{py} + F_{vy}$
$M \cdot a_z = F_{gz} + F_{pz} + F_{vz}$
} Navier-Stokes eq^n. (Applicable to viscous force)

If fluid is ideal ; $F_v = 0$

$\therefore M a = F_g + F_p \Rightarrow$
$M \cdot a_x = F_{gx} + F_{px}$
$M \cdot a_y = F_{gy} + F_{py}$
$M \cdot a_z = F_{gz} + F_{pz}$
} Eulers Eq^n [Applicable to steady, incompressible, non viscous (ideal fluid)]

Eulers Eq^n of motion along a stream line for steady, non viscous, compre/Incomp. fluids:-

$$g \cdot dz + \frac{dP}{\rho} + V \cdot dV = 0$$

Eulers eq^n of motion for 3D eq^n for gravity & pre. forces.

$$X = \frac{1}{\rho} \cdot \frac{\partial P}{\partial x} + \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$$

$$Y = \frac{1}{\rho} \frac{\partial P}{\partial y} + \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z}$$

$$Z = \frac{1}{\rho} \cdot \frac{\partial P}{\partial z} + \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z}$$

} Applicable for steady or unsteady, compressible or incompressible, non viscous ~~flo~~ fluids.

Bernoullies Eq^n as a integration of Eulers eq^n:-

Assumptions:-
1. Flow is steady, incompressible, non viscous, irrotational.
2. Eq^n is applicable along a stream line.
3. Only pressure force & gravity forces are predominant.
4. Velocity is uniform over the c/s.

Bernoullies eq^n:- $\frac{P}{\gamma} + \frac{V^2}{2g} + Z = \text{Constant}$ ... for ideal fluids.

Statement:- In a steady flow of an ideal fluid (compressible & non viscous) the total head (i.e. pre. head + vel. head + Datum head) remains constant along stream line.

Bernoullies eq^n for real fluids (Head loss):-

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_L$$

b) For addition of head : (Pump) $\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 + h_P = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$

c) For ~~sub~~ extraction of energy : (Turbine) $\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 - h_T = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$

* Hydraulic Grade line : (HGL): It is the line joining piezometric head along a pipe. If HGL is above centre line of pipe, fluid flows under pre, If HGL is ~~below~~ coincides ℄ of pipe fluid is under atmospheric pressure, if HGL is below ℄ of pipe, pressure is -ve (vacuum)

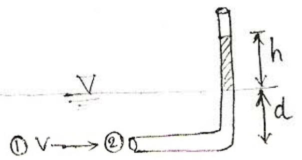
HG at a pt = $\frac{P}{\gamma} + Z \rightarrow$ Datum head.

* **Total Energy Line (TEL)**: The line joining the points representing total head ($\cancel{z}\ \frac{p}{\gamma}+\frac{V^2}{2g}+Z$) along a pipe is known as TEL

In case of ideal fluids, as there is no head loss, TEL is horizontal but in case of real fluids TEL is always slopping down towards direction of flow, since head is lost due to losses.

Applications of Bernoullies Eq^n:-

① Pitot tube: If velocity of flow = 0, at a pt. known as stagnat^n pt, the KE of fluid becomes zero. But as energy can not be created nor destroyed. ∴ KE or vel. head is converted in to pressure head. Thus vel. at a pt. can be determined.



To measure vel. of flow in open channel, pitot tube is immersed in water. Due to vel. of flow water level rises by 'h' in pitot tube

∴ Applying Bernoullies eq^n at ~~sect^n~~ pt. ① & ②

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+Z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+Z_2 \quad \text{... Neglecting losses.}$$

~~As pre. head is same~~ at ① & ②, & $Z_1=Z_2$ & Velocity of stagnat^n pt. ② is = 0

$$\therefore d+\frac{V^2}{2g}+\cancel{0}=\cancel{\frac{p_2}{\gamma}}(h+d)+0+0$$

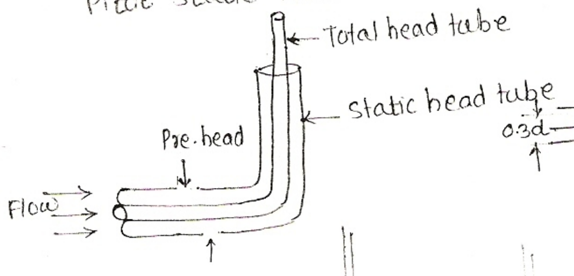
$$\therefore \frac{V^2}{2g}=h$$

$$\therefore V=\sqrt{2gh}$$

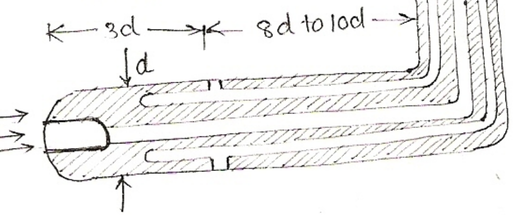
$$\boxed{V=C\sqrt{2gh}}$$

Where C = Coeff. of pitot tube = 0.97 − 1.

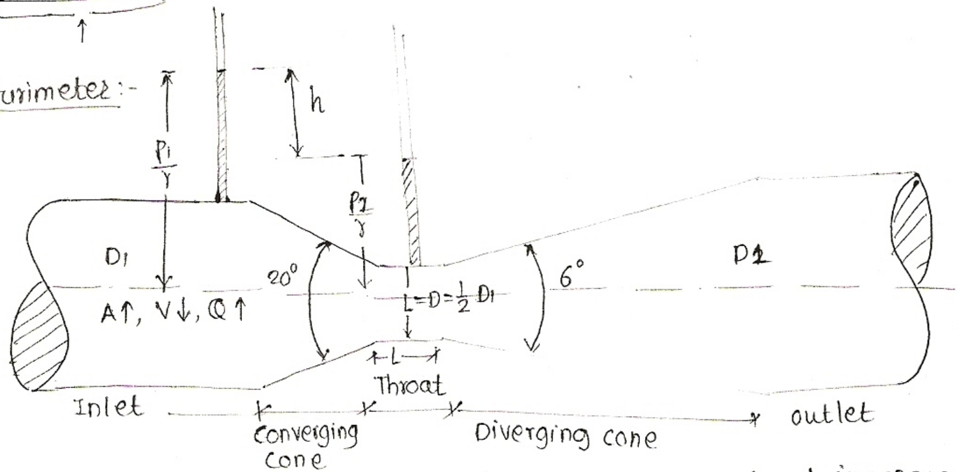
Pitot Static tube:



Prandtl Pitot tube



② Venturimeter:-



By decreasing area of throat, velocity & hence velocity head increases. This increased vel. head reduces pressure head (P/γ) at throat section.

Applying Bernoullies eq^n from Inlet to throat section.

$$\therefore \frac{P_1}{\gamma}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+Z_2 \quad \text{Neglect losses.}$$

as $Z_1=Z_2$

$$\therefore \frac{V_2^2-V_1^2}{2g}=\frac{P_1}{\gamma}-\frac{P_2}{\gamma}=h \quad \text{—(i)}$$

From continuity eq^n ⇒ $Q=A_1V_1=A_2V_2$ ⇒ $V_2=\frac{A_1V_1}{A_2}$ ⇒ Eq^n (i) becomes,

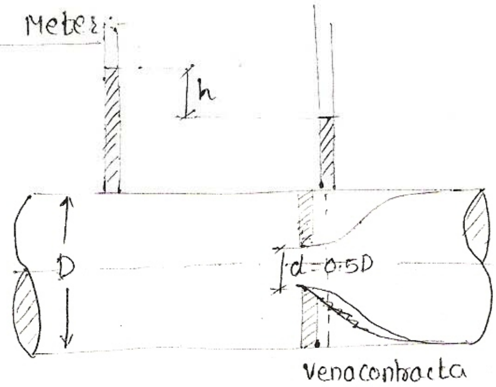
$$\therefore -V_1^2 + \frac{A_1^2 V_1^2}{A_2^2} = 2gh \Rightarrow V_1^2\left(-1 + \frac{A_1^2}{A_2^2}\right) = 2gh \Rightarrow V_1^2 = 2gh\left(\frac{A_2^2}{A_1^2 - A_2^2}\right)$$

$$V_1^2\left(\frac{-A_2^2 + A_1^2}{A_2^2}\right) = 2gh \Rightarrow V_1 = \frac{A_2}{\sqrt{A_1^2 - A_2^2}} \cdot \sqrt{2gh}$$

$$Q = A_1 V_1 = \frac{A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}} = C\sqrt{h}$$ where C = const. of Venturimeter $= \frac{A_1 A_2 \sqrt{2g}}{\sqrt{A_1^2 - A_2^2}}$

$\therefore \boxed{Q = K \cdot C \cdot \sqrt{h}}$ where $K = f\left(Re, \frac{D_2}{D_1}\right) = 0.98$

③ Orifice Meter:-



Applying Bernoullies eq^n

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$ Neglect losses.

$$\therefore \frac{V_2^2 - V_1^2}{2g} = \frac{P_1}{\gamma} - \frac{P_2}{\gamma} = h \quad (Z_1 = Z_2)$$

$$\therefore V_2^2 - V_1^2 = 2gh \quad \text{—— (i)}$$

$Q = AV_1 = a_c V_2$ & $a_c = C_c \cdot a$

A = Area of pipe

a_c = Area of jet at venacontract.

a = Area of orifice.

$$\therefore Q = AV_1 = C_c \cdot a \cdot V_2 \Rightarrow V_2 = \frac{AV_1}{C_c \cdot a} \text{ or } V_1 = C_c \cdot \frac{a}{A} V_2$$

$\therefore$ Eq^n (i) becomes,

$$V_2^2 - C_c^2\left(\frac{a}{A}\right)^2 V_2^2 = 2gh \Rightarrow V_2^2\left[1 - \left(\frac{C_c \cdot a}{A}\right)^2\right] = 2gh \Rightarrow V_2 = \sqrt{\frac{2gh}{1 - C_c^2 (a/A)^2}}$$

$\therefore$ considering losses,

$$V_2 = C_v \sqrt{\frac{2gh}{1 - (a/A)^2 \cdot C_c^2}}$$

$$\therefore Q = C_c \cdot a \cdot V_2 = C_c \cdot a \cdot C_v \sqrt{\frac{2gh}{1 - C_c^2 (a/A)^2}}$$

But $\frac{a}{A} = \left(\frac{d}{D}\right)^2$; $C_c = 1$, $C_c \cdot C_v = C$

$$\boxed{Q = C \cdot \frac{a}{\sqrt{1 - (d/D)^4}} \cdot \sqrt{2gh}}$$

$$\therefore Q = K \cdot a\sqrt{2gh}$$

Venturimeter	Orifice meter.
① $Q = KC\sqrt{h} = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \cdot \sqrt{2gh}$	① $Q = K \cdot a\sqrt{2gh} = C \cdot \frac{a}{\sqrt{1 - (\frac{d}{D})^4}} \cdot \sqrt{2gh}$
② Losses are very low	② Losses are more
③ Very long & requires more space	③ Compact
④ Suitable for measuring flow in large pipe	④ smaller pipes.
⑤ costlier	⑤ Cheaper

Flow through orifices:- classification

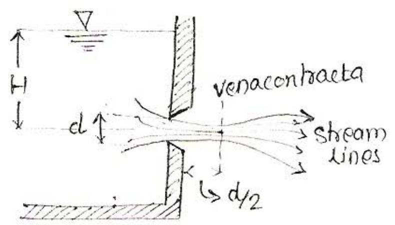
Based on size
1) Small (H>2d)
2) Large (H<2d)

Based on shape
① Circular
② Rectangular
③ Square
④ Triangular

Based on shape of u/s edge
① sharp edged
② Bell mouthed with round corner
③ Square Edge

Based on Discharg cond^n
① free
② Drowned or submerged
a) Partially
b) Fully

Venacontracta: ① Circular, ~~ed~~ sharp edged orifice discharging freely.



Venacontracta is a section of jet where stream lines are parallel to each other & ⊥ar to the plane of orifice. The velocity of each partical is in ~~ø~~ the direction ⊥ar to the plane of orifice.

Venacontracta occurs at a distance d/2 from the plane orifice where 'd' is dia. of orifice.

∴ Applying Bernoulies eqⁿ betⁿ liquid surface & centre of jet at venacontracta.

$$\therefore \frac{P_1}{\gamma}+\frac{V_1^2}{2g}+Z_1 = \frac{P_2}{\gamma}+\frac{V_2^2}{2g}+Z_2 \quad \text{... Neglecting losses}$$

But $Z_1 = Z_2$

$\frac{P_1}{\gamma} = H$, $\frac{P_2}{\gamma} = 0$, $V_1 = 0$

$$\therefore H+0+0 = 0+\frac{V_2^2}{2g}+0$$

$$\therefore \boxed{V=\sqrt{2gH}} \rightsquigarrow \text{Torricelli Theorem}$$

Hydraulic coeff. of an orifice:-

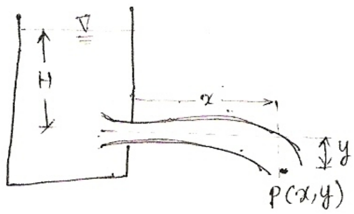
① Coeff. of Contraction $= C_c = \frac{\text{Area of jet at Venacontracta}}{\text{Area of orifice}} = \frac{a_c}{a} = 0.61 \text{ to } 0.69$

② Coeff. of Velocity $= C_v = \frac{\text{Actual vel. of jet at Venacontracta}}{\text{Theoretical vel. of jet}} = \frac{V_a}{V} = \frac{V_a}{\sqrt{2gH}} = 0.97 \text{ to } 0.98$

③ Coeff. of discharge $= C_d = \frac{\text{Actual discharge}}{\text{Theoretical Dis.}} = \frac{Q_a}{Q} = \frac{a_a \cdot V_a}{a \cdot V} = \frac{a_a \cdot V_a}{a \cdot \sqrt{2gH}} = C_c \cdot C_v = 0.61 - 0.6$

④ Coeff. of resistance $= C_r = \frac{\text{Loss of KE in orifice}}{\text{Actual KE of fluid}} = \left(\frac{1}{C_v^2}-1\right)$

Determination of coeff. of an orifice:-



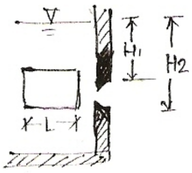
① Coeff. of vel $= C_v = \sqrt{\frac{x^2}{4Hy}}$

② Coeff. of Discharge $= C_d = \frac{Q_{act}}{a\sqrt{2gH}}$

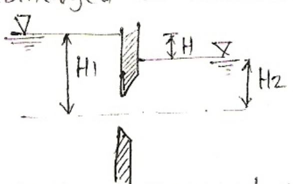
③ Coeff. of contractⁿ $= C_c = \frac{C_d}{C_v}$

④ Head loss $= h_L = H(1-C_v^2)$

② Large Rectangular orifice: $Q = \frac{2}{3} C_d \cdot \sqrt{2g} \cdot L \cdot (H_2^{3/2} - H_1^{3/2})$

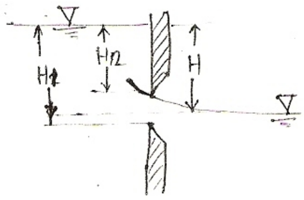


③ Submerged or Drowned orifice:



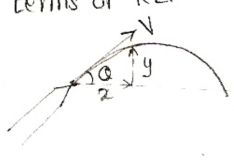
$Q = C_d \times a \times \sqrt{2gH}$

④ Partially submerged orifice:-



$Q = \frac{2}{3} C_{d1} \cdot b\sqrt{2g}\,[H^{3/2} - H_2^{3/2}] + C_{d2} \cdot b \cdot (H_1 - H)\sqrt{2gH}$

Nozzle:- It is a short piece of converging tube used to convert pressure energy in terms of KE.



1) Max. ht. reached ⇒ $y_{max} = \frac{V^2 \sin^2\theta}{2g}$

2) Time reqd. to reach $y_{max} = \frac{V\sin\theta}{g}$

3) Time of complete flight $= \frac{2V\sin\theta}{g}$

4) Horizontal range of flight $= \frac{V^2 \sin 2\theta}{g}$

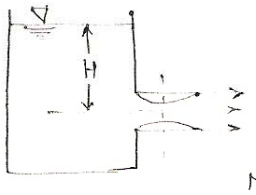
5) Max. Range (at $\theta = 45°$) $= \frac{V^2}{g}$

* Mouth Pieces:- It is a short piece of tube having dia. same as orifice & length about 2.5 to 3 times the dia. of orifice attached either internally or externally to the orifice in order to increase the discharge through the orifice. (3)

Mouthes Pieces

- According to shape
  1) cylindrical
  2) convergent
  3) convergent-divergent
- According to ~~mode of~~ Discharge condition
  1) Running full
  2) Running free
- According to position of mouthpiece
  1) External
  2) Internal.

① External cylindrical mouth piece:-



$C_c = \frac{a_c}{a} = 1$

$C_v = 0.855$

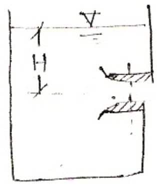
$C_d = 0.855$

~~A - Surface area~~

$\boxed{Q = 0.82 A \sqrt{2gH}}$

Max. head of water to avoid cavitat^n = 11.61 m

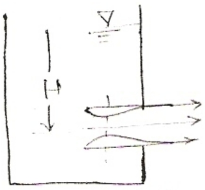
② Internal Mouth piece or Borda's mouth piece



Running free

$C_c = 0.5, \ C_v = C_d = 1$

$\boxed{Q = 0.5 \cdot A \cdot \sqrt{2gH}}$



Running full

$C_v = 0.707, \quad C_c = 1, \ C_d = 0.707$

$\boxed{Q = 0.707 \cdot A \cdot \sqrt{2gH}}$

* Notches & Weirs:-

Notch is an opening in the side of tank such that liquid level in the tank is below the top edge of the opening.

Weir is a notch on large scale. It is a obstruction built across a river or channel to raise liquid level on u/s side to allow excess liquid to flow over it. otches are used to measure flow on small scale as in lab while weirs are used to measure flow on large scale as in river. Notches are thin structure made from metallic plates & have sharp edge while weirs are large structures & may have sharp edge & /or width.

classificat^n of Notches

- According to shape
  - Rectangular
  - Triangular (V)
  - Trapezoidal
  - Stepped/compound
- According to effect of sides.
  - Notch with end contraction
  - Notch without end contraction or suppressed notch.

classification of Weir.

- Based on shape
  1) Rectangular
  2) Triangular
  3) Trapezoidal
- Based on shape of crest
  1) Sharp crested
  2) Narrow crested
  3) Broad crested (B > 0.5H)
  4) Ogee shaped
- Based on effect of sides
  1) Weir with end contraction
  2) Weir without end contract^n
- Based on discharge cond^n
  1) Free weir
  2) Submerged or drowned weir.

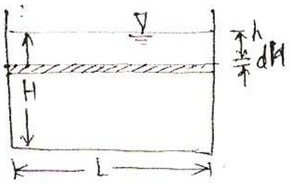
Nappe :- The sheet of liquid flowing over notch or weir

Crest (Sill) - The bottom edge of notch or top edge of weir over which fluid flows.

Head over weir :- Ht. of free liquid surface above crest or sill of notch or weir.

Height of crest :- Ht. of crest from bottom of tank or channel.

① Discharge over sharp crested rectangular Notch :-



$$dQ_t = (L\cdot dh)\cdot\sqrt{2gH}$$

$$dQa = Cd\cdot L\cdot dh\cdot\sqrt{2gh}$$

$$\therefore Qa = \int_0^H Cd\cdot L\,dh\cdot\sqrt{2gh}$$

$$\boxed{\therefore Qa = \frac{2}{3}Cd\cdot\sqrt{2g}\cdot L\cdot H^{3/2}}$$ ... The liquid is assumed as steady. (1)

If liquid has some initial velocity when it approaches to weir, (Va) then additional head is considered as $Va^2/2g = ha$ then,

$$\boxed{Q = \frac{2}{3}Cd\cdot\sqrt{2g}\cdot L\left[(H+ha)^{3/2} - (ha)^{3/2}\right]}$$ H + ha = Still water head. ... (2)

$$\text{Velocity of approach} = Va = \frac{Q}{\text{Area of approach}} \quad \ldots (3)$$

Thus as Va & Q both are unknown at initial stage, a series of successive approximation are used as follows -

① calculate Q without considering vel. of approach Va ... eqⁿ ①

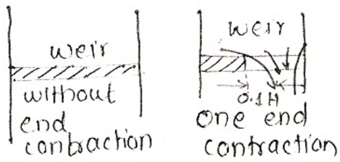
② Calculate Va from eqⁿ (3)

③ Calculate Q from eqⁿ (2) by considering vel. of approach calculated in step ②

④ Repeat procedure ② & ③ till there is very little diff. betⁿ two consecutive discharge.

* Empirical Formulae for discharge over rectangular weir :-

① Francis Formula :- After conducting experiments Francis found that due to ~~no~~ one end contraction, width of nappe reduces by 0.1H. This reduces effective length of weir from L to (L - 0.1H).



If a weir has 'n' number of end contractions, eff. length is (L - 0.1n·H)

$$\boxed{\therefore Q = \frac{2}{3}Cd\cdot\sqrt{2g}\,(L - 0.1\,n\,H)\cdot H^{3/2}}$$ .. Without considering vel. of approach.

$$\boxed{Q = \frac{2}{3}Cd\cdot[L - 0.1n(H+ha)]\sqrt{2g}\,[(H+ha)^{3/2} - (ha)^{3/2}]}$$ → After considering Va

Where $Cd = 0.623$.

② Bazins Formula :- As Cd is not constant for rectangular weir, & depends on height of weir above sill & head.

$$\therefore \boxed{Q = \left(0.405 + \frac{0.003}{H}\right)\sqrt{2g}\cdot L\cdot H^{3/2}}$$ .. without considering Va.

Where $Cd = 0.405 + \frac{0.003}{H}$

If liquid has velocity of approach Va, then $H_1 = H + \alpha\cdot\frac{V_a^2}{2g}$ where $\alpha = 1.6$

$$\therefore \boxed{Q = \left(0.405 + \frac{0.003}{H}\right)\cdot\sqrt{2g}\cdot L\cdot H_1^{3/2}}$$ .... With 'Va'.

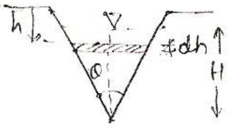
③ Rehbock Formula :-

$$\boxed{Q = \left(1.782 + 0.24\frac{He}{P}\right)\cdot L\cdot He^{3/2}}$$ ... without considering Va

where $He = (H + 0.0011)\,m$

P = Ht. of weir crest above bottom of channel.

② Discharge over triangular or V-notch / weir :-



Area of strip = Width x Depth

$= 2(H-h) \cdot \tan\frac{\theta}{2} \cdot dh$

$\therefore dQ = Cd \cdot [2(H-h) \cdot \tan\frac{\theta}{2} \cdot dh] \cdot \sqrt{2gh}$

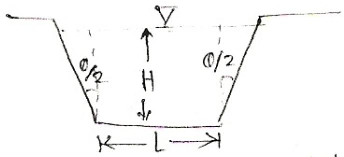
$\therefore \boxed{Q = \frac{8}{15} Cd \cdot \sqrt{2g} \cdot \tan\frac{\theta}{2} \cdot H^{5/2}}$ ... without considering va

If $\theta = 90°$, & $Cd = 0.6$, $\boxed{Q = 1.417 H^{5/2}}$

$\boxed{Q = \frac{8}{15} Cd \cdot \sqrt{2g} \tan\frac{\theta}{2} [(H+ha)^{5/2} - Ha^{5/2}]}$ ... considering va

Rectangular Notch	Triangular Notch
① Cd varies with head	① Cd is constant.
② Low discharge can not be measured accurately, · Used for large discharge.	② As low heads are sufficiently high, it can be measured accurately.
③ Ventilation is reqd.	③ Ventilat^n is not necessary.
④ For low head effect of surface tension is high.	④ Effect of surface tension is less.
⑤ Formula can not be simplified.	⑤ For 90° notch formula can be simplified & gives discharge directly by plotting calibration curves Q vs H

③ Discharge over trapezoidal Notch:-



$\boxed{Q = \frac{2}{3} Cd_1 \sqrt{2g}\, H^{3/2} \cdot L + \frac{8}{15} \cdot Cd_2 \cdot \sqrt{2g} \cdot \tan\frac{\theta}{2} \cdot H^{5/2}}$

i.e. discharge = Through V notch + through rectangular notch

Cippoletti Notch :- cippoletti notch is a trapezoidal notch which gives discharge equal to discharge over rectangular not without end contractions.

As, due to end contraction, discharge reduces. Thus discharge which is reduced by end contract^n in rectangular notch is increased by adding triangular notch on either side.

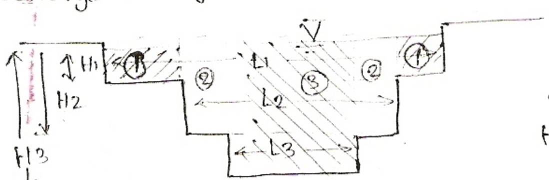
$Q =$ Discharge through Cippoletti notch $= \frac{2}{3} Cd \cdot \sqrt{2g}\; L \cdot H^{3/2}$ ... (i)

$Q =$ Discharge through trapezoidal notch $= \frac{2}{3} Cd \cdot \sqrt{2g}\; (L - 0.1nH) \cdot H^{3/2} + \frac{8}{15} Cd \cdot \sqrt{2g} \cdot \tan\frac{\theta}{2} \cdot H^{5/2}$ ... (ii)

$n =$ (End contraction = 2)

$\therefore$ Equating ① & ⓘⓘ, $\boxed{\frac{\theta}{2} = 14°}$

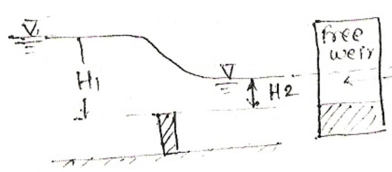
④ Discharge through stepped Notch / Compound Notch :-



Stepped notch is a combination of rectangular notches. This notch is useful for measuring accurate discharge at low & High head.

$\therefore Q = \frac{2}{3} Cd \cdot \sqrt{2g} (L_1 - L_2) \cdot H_1^{3/2} + \frac{2}{3} \sqrt{2g} \cdot Cd \cdot (L_2 - L_3) \cdot H_2^{3/2} + \frac{2}{3} Cd \cdot \sqrt{2g}\, L_3 \cdot H_3^{5/2}$

⑤ Discharge through submerged weir / Drowned weir :-



$Q_{\text{free weir}} = \frac{2}{3} \cdot Cd \cdot \sqrt{2g} \cdot L \cdot (H_1 - H_2)^{3/5}$

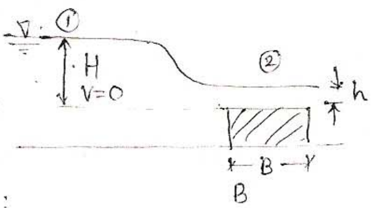
$Q_{\text{Drowned orifice}} = Cd \cdot L \cdot H_2 \cdot \sqrt{2g(H_1 - H_2)}$

$\therefore Q = Q_{\text{free weir}} + Q_{\text{Drowned orifice}}$.

Time of reqd. to empty a reservoir with rectangular weir :-

$t = \frac{2A}{\frac{2}{3} Cd \sqrt{2g} \cdot L} \left[\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}} \right]$

* ⑥ Discharge through broad crested weir :- $(B > 0.5)$



If width of weir is greater than half the head weir is known as broad crested.

∴ Applying Bernoullies eqⁿ at ① & ②

$\therefore H + 0 + 0 = h + \frac{V^2}{2g} + 0$

$\Rightarrow V = \sqrt{2g(H-h)}$

$\therefore Q = Cd \times L \times h \times V$

$\boxed{\therefore Q = Cd \times L \times \sqrt{2g(Hh^2 - h^3)}}$

For $Q \to max$, $\frac{dQ}{dh} = 0 \rightarrow \frac{dQ}{dh} = Cd \cdot L \cdot \sqrt{2g} \cdot \frac{d}{dh}(\sqrt{H \cdot h^2 - h^3}) = 0 \Rightarrow \frac{d}{dh}(H \cdot h^2 - h^3) = 0$

∴ $\Rightarrow 2H \cdot h - 3h^2 = 0 \Rightarrow 2H = 3h \Rightarrow \boxed{h = 2/3 H}$

$\boxed{\therefore Q_{max} = 1.705\, Cd \cdot L \cdot H^{3/2}}$

as $V = \sqrt{2g(H-h)} = \sqrt{2g(\frac{3}{2}h - h)} = \sqrt{2g(h/2)} = \sqrt{gh}$ or $\frac{V}{\sqrt{gh}} = 1 \Rightarrow Fr = 1$.

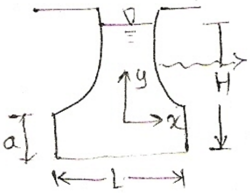
as critical depth at which Q is max.

∴ For $H = \frac{3}{2}h$, flow is critical & h is known as critical depth at which Q is max.

⑦ Discharge through Proportionate weir or Sutro weir :-

As $Q \propto V^n$, where $n = 2/3$ for rectangular weir
$= 5/2$ for triangular weir.

∴ A sutro weir or proportionate weir is a weir in which shape of weir is designed such that discharge varies directly with the head i.e. $Q \propto H$.



$\therefore Q = K(H - \frac{a}{3})$ where $K = Cd \cdot L \cdot \sqrt{2ga}$

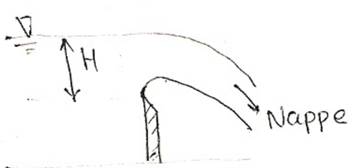
Eqⁿ of boundary $\frac{2x}{L} = [1 - \frac{2}{\pi}\tan^{-1}(\frac{y}{a})]$

$Cd = 0.6$ to 0.65.

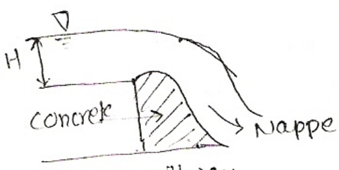
This weir is used as control device in case of dosing & sampling.

⑧ Discharge through Ogee weir or Ogee Spillway :-

Spillway is a section provided in dam which allows to flow excess water



sharp Crested weir



ogee spillway

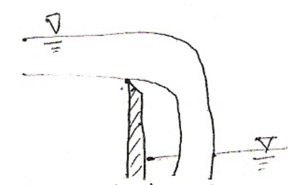
$\boxed{Q = \frac{2}{3} Cd \cdot \sqrt{2g} \cdot L \cdot H^{3/2}}$

$Q = C \cdot L \cdot H^{3/2}$

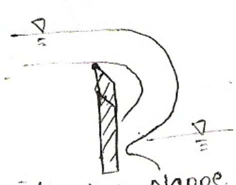
where C = Coeff. of spillway

* Aeration of weir or ventilation of weir :-

In case of suppresed weir, width of weir = width of channel it self.



Decreased Nappe
$Q_1 = 1.06Q$



Clinging Nappe
$Q_2 = 1.3Q$

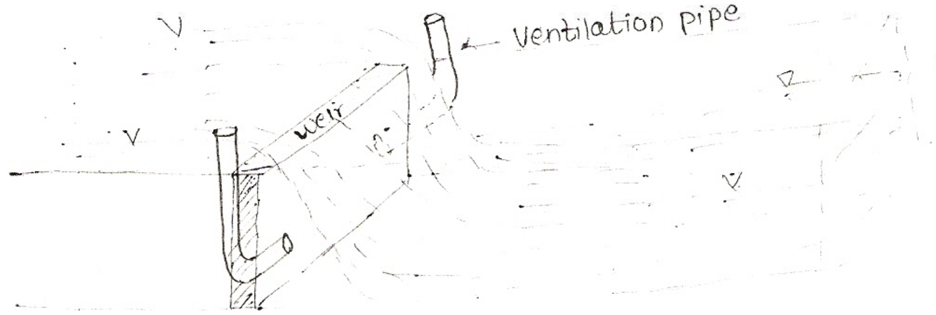
If channel sides are continued as shown in the coloured portion betⁿ d/s face of weir & lower nappe air is entrapped when flow starts from weir. This entrapped air is gradually removed by the flow of fast moving fluid. As the air has no way to go below nappe, the Subatmospheri (vaccume) pressure exist betⁿ the nappe which gives rise to depressed nappe. It this time, discharge is increased by 6%.

If further air is removed, nappe goes towards weir & discharge further increases. A stage will reach when nappe touches to weir. as shown, such nappe is known as clinging nappe & discharge increases by 28 to 30% that of free dis.

Eventhough clinging nappe increases discharge by 28% to 30% that of free discharge & depends on head, height of crest & water level on d/s side, but as we derived various formulae based on free nappe, so we must provide aeration or ventilation for avoiding clinging nappe. This can be achieved by providing a pipe from both ends.

Aeration of Nappe has following advantages:-

1. Stable condition of discharge is obtained, so we can use std. formulae.
2. As Q is constant, Cd is constant.
3. Slapping of weir by nappe can be avoided.



* Linear Momentum Eqⁿ:-

Basic eqⁿ of fluid flows : Torricelli's eqⁿ ⇒ $V = \sqrt{2gH}$

: Bernoullies Eqⁿ ⇒ $\frac{P}{\gamma} + \frac{V^2}{2g} + z = \text{const.}$

: Continuity Eqⁿ ⇒ $Q = A_1V_1 = A_2V_2$.

& eqⁿ of lineal momentum is also basic eqⁿ.

As Newtons IInd law of motion is, $\Sigma F = m \cdot a$

$$\Sigma F = m \cdot \frac{dv}{dt}$$

$$\boxed{\Sigma F = \frac{d}{dt}(m \cdot v)}$$ ⟿ Linear Momentum eqⁿ.

i.e. rate of change of momentum in any direction is equal to the vector sum of all external forces in that direction.

$$\therefore \Sigma F_x = \frac{d}{dt}(m \cdot V) = m \cdot \frac{dV}{dt} = \rho \cdot Q \cdot \frac{dV}{dt}$$

$$\Sigma F_x = \rho \cdot Q \cdot \Sigma \bar{V}_x$$
$$\Sigma F_y = \rho \cdot Q \cdot \Sigma \bar{V}_y$$
$$\Sigma F_z = \rho \cdot Q \cdot \Sigma V_z$$

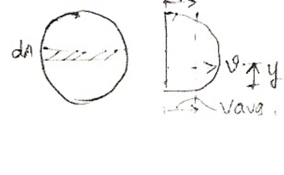
Application of momentum eqⁿ :-
1. Forces on bends, elbows etc in a pipe line
2. Jet propulsion
3. Propellers.
4. Stationary & moving plates or vanes in hydraulic machines.
5. Loss of head due to sudden expansion.
6. Loss of energy in hydraulic jump in open channel.

* Kinetic Energy Correction factor:-

While calculating velocity $V^2/2g$, in Bernoullies eqⁿ, Vel. is assumed to be uniform over a c/s. However, vel distribution in case of real fluid is parabolic. ∴ KE correctⁿ factor is added to get actual KE due to variation in vel.

$$\therefore \alpha = \frac{1}{A}\int_A \left(\frac{v}{V}\right)^3 dA$$

where v = Velocity at dist y.
A = Area bound by curve
V = Avg. vel.



∴ According to ~~Be~~ Energy correction factor, Bernoullies eqⁿ is modified as,

$$\frac{P_1}{\gamma} + \alpha_1 \cdot \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \alpha_2 \cdot \frac{V_2^2}{2g} + Z_2$$

* Momentum Correction factor (β)

Momentum theorem is based on assumption that velocity is uniform across the c/s, however vel. is not uniform in practice. Therefore to account for the variation of vel. across the channel in momentum theorem, Momentum Correction factor (β) is introduced.

$$\boxed{\beta = \frac{1}{A} \int_A \left(\frac{v}{V}\right)^2 dA}$$

For laminar Flow through circular pipe, β = 1·33.
For Turbulent Flow, β = 1·01 to 1·05 ∴ Neglected.

* Cavitation:-

According to Bernoullies theorem, Total head at a point = Constant.

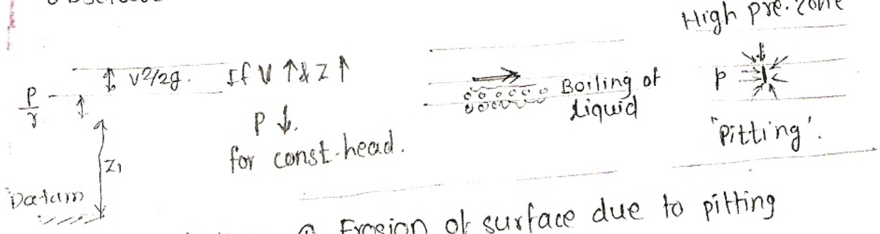
i.e. $\frac{P}{\gamma} + \frac{V^2}{2g} + Z = \text{constant}$. If V & Z are increased, P will decreased. A stage at which P at pt becomes vapour pressure, of liquid reaches, liquid starts boiling & vapour bubbles are formed. These bubbles or cavities travels with fluid flow. When these bubbles reaches to high pressure, they collapse & liquid near the vicinity of the bubble rushes into cavities which creates high local pressure.

The process of formation of cavity or bubble due to increase vel &/or datum/elevation, their travel to high pressure region & their subsequent collapse is known as 'Cavitation. The force created due to momentum of fluid rushes to collapsed e cavity is acts on very small area. & as P = F/A, Pressure increases considerably. This phenomenon generally occurs at boundaries. Therefore boundaries are subjected to higher pressure within small time, which erodes the matl. of boundary & called as 'pitting.'

Eg: of ~~cavit~~ possibilities of cavitation: Syphons, pumps, turbines, ship propellers, water missiles, stilling basin, nozzle of firehouse, venturimeter etc.

To check the possibility of cavitatⁿ: $\sigma = \frac{P - P_v}{\frac{1}{2}\rho V^2} = \frac{h - h_v}{V^2/2g}$

where, P & Pv are local & vapour pre. at a pt under consideratⁿ & h, hv are corresponding heads. ~~hence σ=0~~, when σ = 0, P = Pv & h = hv & boiling starts. For safe design cavitation is assumed to occur at a pre. head of 2·4m of water absolute.



Effects of cavitation:- ① Erosion of surface due to pitting
② Creatⁿ of noise
③ Creatⁿ of vibration
④ Change in flow pattern where erosion is prevented.
⑤ Loss of efficiency.

Mtds. to avoid cavitation:- ① Proper design of str.
② Proper installatⁿ of machines. (Hydraulic)
③ Proper choice of matl. of boundary.
④ Proper machining .. i.e. Highly finished surfaces.

—— END ——