## Notes by-

# Pravin S Kolhe,

BE(Civil), Gold Medal, MTech (IIT-K)

## Assistant Executive Engineer,

## Water Resources Department,

## www.pravinkolhe.com

# KINEMATICS OF PARTICLES

**1.** The acceleration of a particle is defined by the relation $a = 9-3t^2$. The particle starts at t = 0 with v = 0 and x = 5m. Determine (a) the time when the velocity is again zero, (b) the position and velocity when t = 4s, (c) the total distance travelled by the particle from t = 0 to t = 4s.

[ 3s : 13m, -28 m/s : 32.5 m ]

**2.** An automobile travels 360m in 30s while being accelerated at a constant rate of $0.5 m/s^2$. Determine (a) its initial velocity, (b) its final velocity, (c) the distance travelled during the first 10s.

[ 4.5 m/s; 19.5 m/s : 70m ]

**3.** Automobiles A and B are travelling in adjacent highway lanes & at t = 0 have the positions and speeds shown. Knowing that automobile A has a constant acceleration of 0.6 $m/s^2$ and that B has a constant deceleration of $0.4 m/s^2$, determine (a) when and where A will overtake B, (b) the speed of each automobile at that time.

[ 13.8 seconds later: $V_A = 18.8$ m/s, $V_B = 10.3$ m/s ]

**4.** Collar A and B starts from rest and move with the following acceleration $a_A = 62.5t$ $mm/s^2$ upward and $a_B = 375/s^2$ downward. Determine (a) the time at which the velocity of block C is again zero, (b) the distance through which block C will have moved at that time.

[ 6s : 750 mm(↓) ]

**5.** A particle moves in an elliptic path defined by the position vector
r = (A cos pt) i + (B sin pt) j. Show that the acceleration (a) is directed
towards the origin, (b) is proportional to the distance from the origin to the particle.

[

**6.** A ball is dropped onto a pad at A and rebounds with a velocity $v_0$ at an angle of $70^0$ with horizontal. Determine the range of values of $v_0$ for which the ball will enter the opening BC.

[ 6.49 m/s ≥ $V_0$ ≥ 4.51 m/s ]

**7.** A nozzle discharges a stream of water in the direction shown with an initial velocity of 7.5 m/s. Determine the radius of curvature of the stream (a) as it leaves the nozzle, (b) at the maximum height of the stream.

[ 7m : 3.85m ]

**8.** A computer tape moves over two drums at a constant speed $v_0$. Knowing that the normal component of the acceleration of the portion of tape in contact with drum B is $480 m/s^2$, determine (a) the speed $v_0$, (b) the normal component of the acceleration of the portion of tape in contacts with drum A.

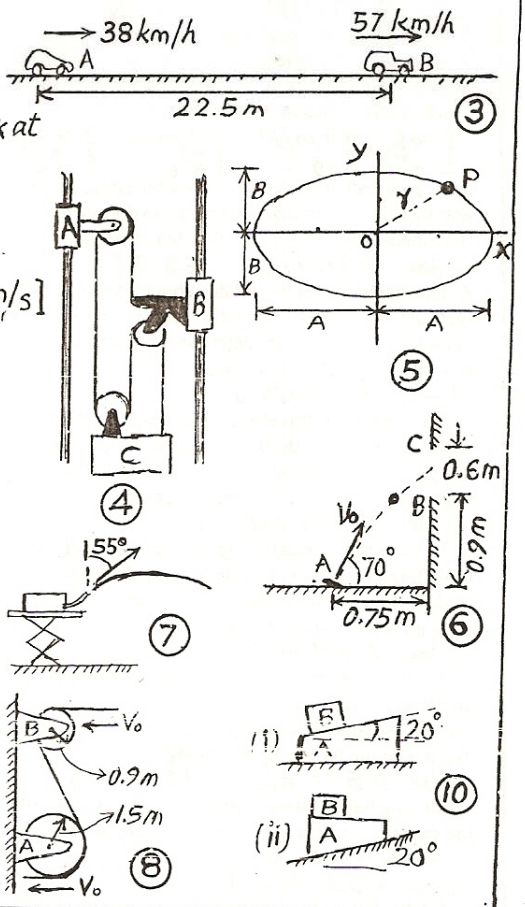
[ $V_0$ = 20.78 m/s : $(a_n)_A = 288 m/s^2$ ]

**9.** During a rainstorm the paths of raindrops appear to form an angle of $30^0$ with vertical and to be directed to the right when observed from a side window of a train moving at a speed of 10 m/s. A short time later, after the speed of the train has increased to 15 m/s the angle between the vertical and the paths of the drops appears to be $45^0$. If the train were stopped, at what angle and with what velocity would the drops be observed to fall?

[ 10.61 m/s (∠43°) ]

**10.** At t = 0, wedge A starts moving to the left with a constant acceleration of 80 $mm/s^2$ and block B starts moving along the wedge towards the right with constant acceleration of 120 $mm/s^2$ relative to the wedge. Determine
(a) the acceleration of block B, (b) the velocity of block B when t = 3s.

[ 52.5 $mm/s^2$ (∠51.4°) ; 157.5 mm/s(∠51.4°) ]




For Probs 2 & 3, motion diagrams are prefered.
For Prob 4. get $a_C$ expression first, then $V_A$, $V_B$, and $V_C$ distance is $\int_0^6 V_C\, dt$.

For Prob. 5, show that $\bar{a} = [\,]\,\bar{r}$
For Prob. 6, check the 'extremes' B&C.

For 7, eqn for ρ may be used. Orelse work through 'n' and 't' directions
For 9, Sketch the vector diagrams for the two instants and then solve the resulting 'trigonometry'. Note that $\bar{V}_{B/A} = \bar{V}_B - \bar{V}_A$
For Prob. 10. Note that $\bar{a}_B = \bar{a}_A + a_{B/A}$. Solve for each case.

# KINETICS OF PARTICLES

**1.** Block A has a mass of 25 kg and block B a mass of 15 kg. The coefficients of friction between all surfaces of contact are $\mu_s = 0.20$ and $\mu_k = 0.15$. Knowing that $\theta = 25^0$ and that the magnitude of the force P applied to block A is 250N, determine (a) the acceleration of block A, (b) the tension in the cord.

[*Ans:* 4.29 m/s² : 213.1 N ]

**2.** Each of the systems shown is initially at rest. Assuming the pulleys to be weightless and neglecting axle friction, determine for each system (a) the acceleration of block A, (b) the velocity of block A after 2s, (c) the velocity of block A after it has moved through 3 m.

[*Ans:* (i) 2.45 m/s², 4.9 m/s, 3.84 m/s
(ii) 7.0 " 14.02 " 6.485 "
(iii) 0.429 " 0.858 " 1.567 "]

**3.** A 2 kg ball evolves in horizontal circle as shown. Knowing that L = 0.9m and that the maximum allowable tension in the cord is 50N, determine (a) the maximum allowable speed, (b) the corresponding value of the angle θ.

[*Ans:* 4.36 m/s : 66.9° ]

**4.** The assembly shown rotates about a vertical axis at a constant rate. Knowing that the coefficient of static friction between the small block A and the cylindrical wall is 0.25, determine the lowest speed v for which the block will remain in contact with the wall.

[*Ans:* 3.13 m/s ]

**5.** A car has been travelling up a long 2 percent grade at a constant speed of 85 km/h. If the driver does not change the setting of the throttle or shift gears as the car reaches the top of the hill, what will be the acceleration of the car as it starts moving down the 3 percent grade?

[*Ans:* 0.491 m/s² ]

**6.** In an iron-ore mixing operation, a bucket full of ore is suspended from a travelling crane which is moving with a speed v = 3 m/s along a stationary bridge. If the crane suddenly stops, determine the additional horizontal distance through which the bucket will move.

[*Ans:* 2.84 m ]

**7.** A 2 kg block is at rest on a spring of constant 350 N/m. A 4 kg block is held above the 2 kg block so that it just touches it, and then is released. Determine (a) the maximum velocity attained by the blocks, (b) the maximum force exerted on the blocks by the spring.

[*Ans:* 0.857 m/s : 98.12 N ]

**8.** An 18 g bullet leaves a fixed rifle barrel 2 ms after being fired. Knowing that the muzzle velocity is 750 m/s and neglecting friction, determine the average power developed by the rifle.

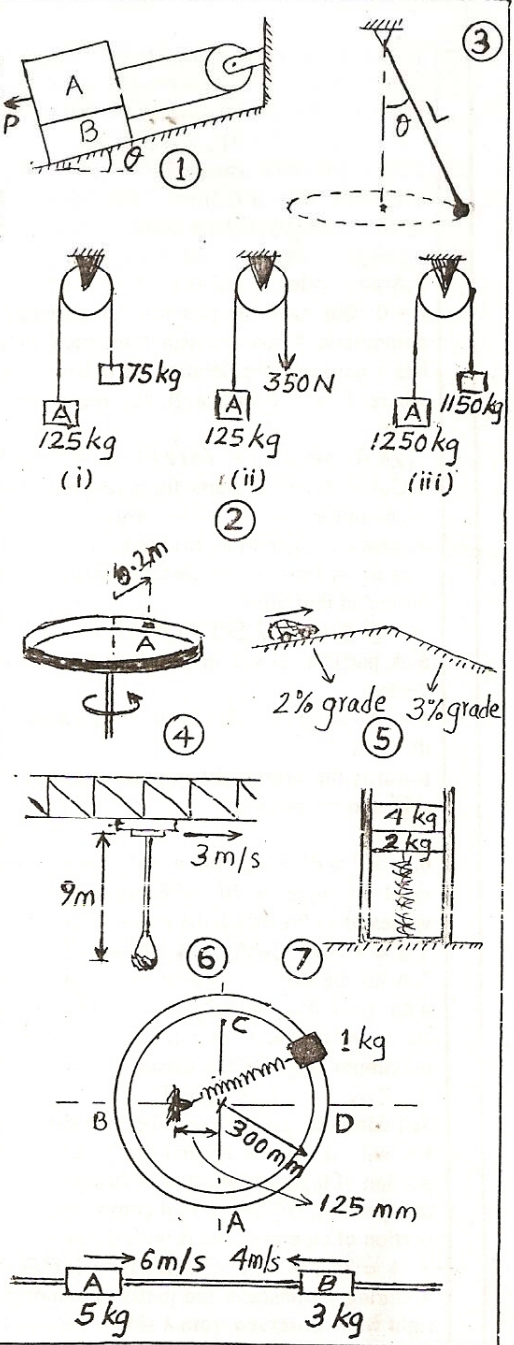
[*Ans:* 2.531 MW ]

**9.** A 1 kg collar attached to a spring and slides without friction along a circular rod which lies in horizontal plane. The spring has a constant k = 250 N/m and undeformed when collar is at B. Knowing that the collar passes through point D with a speed of 1.8 m/s, determine the speed of the collar as it passes through (a) point C, (b) Point B.

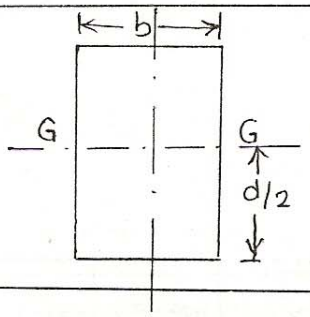
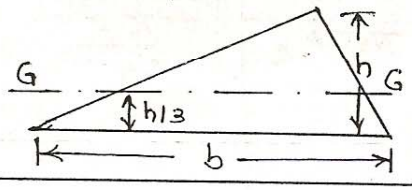
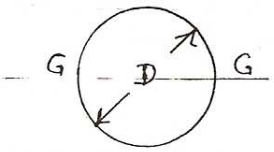
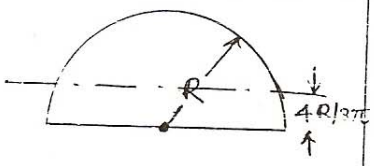
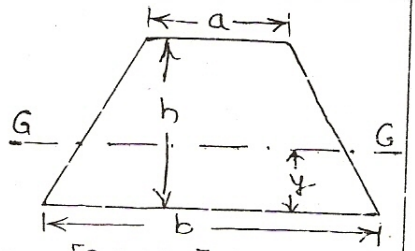
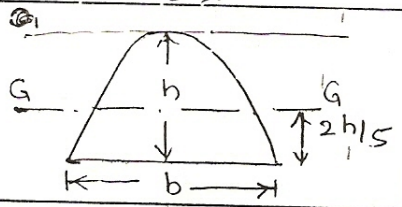
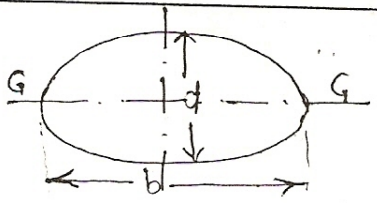
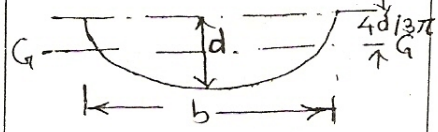
[*Ans:* 3.639 m/s : 4.343 m/s ]

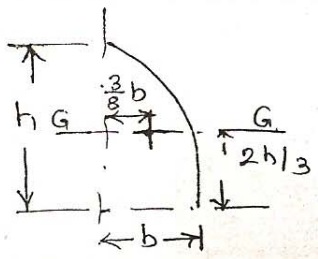
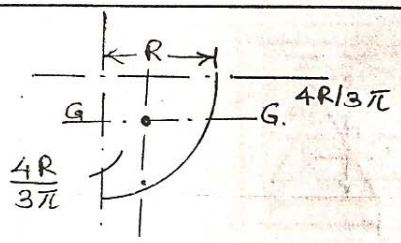
**10.** The coefficient of restitution between the two collars is known to be 0.80. Determine (a) their velocities after impact, (b) the energy loss during impact.

[*Ans:* 0.75 m/s (←) : 7.25 m/s(→) : 33.75 J ]




In Prob. **1**, consider the FBDE of each block. In Prob **2**, note that 350 N is a force in (ii). In Prob **3**, tangential direction is along the tangent to the circle. acceleration along the binormal direction is always zero. In prob **4**, at the limiting minimum speed, the block's motion impends (relative to the wall) down. In Prob **5**, if gears aren't changed, the tractive effort remains unchanged. Also note the term 'grade'. For small slopes $\tan\theta_1 \approx \sin\theta_1 \approx 0.02$ and $\tan\theta_2 \approx \sin\theta_2 \approx 0.03$ in the context of the problem. In Prob **6**, consider the KE of the bucket after the stop. In Prob **7**, consider the initial deformation of the spring. In Prob **8**, consider the energy change in the '2 millisecond' interval. In Prob **9**, use the WE principle between the concerned points. In Prob. **10**, use $e = (V_B' - V_A')/(V_A - V_B)$ and the conservation of momentum principle.

| Plane | Surface | Area | $I_{G-G}$ |
|---|---|---|---|
| Rectangle |   | $b \cdot d$ | $\frac{bd^3}{12}$ |
| Triangle |   | $\frac{1}{2} b \cdot h$ | $\frac{b h^3}{36}$ |
| Circle |   | $\frac{\pi D^2}{4}$ | $\frac{\pi D^4}{64}$ |
| Semi-Circle |   | $\frac{\pi R^2}{2}$ | $0.11 R^4$ |
| Trapezium |   $y = \left[\frac{2a+b}{a+b}\right]\frac{h}{3}$ | $(a+b)\frac{h}{2}$ | $\left[\frac{a^2+4ab+b^2}{36(a+b)}\right]h^3$ |
| Paraboia |   | $\frac{2}{3} b \cdot h$ | $\frac{8}{175} bh^3$ |
| Ellipse |   | $\frac{\pi}{4} b \cdot d$ | $\frac{\pi b d^3}{64}$ |
| Semi-Ellipse |   | $\frac{1}{2} \frac{\pi b \cdot d}{4}$ | $0.055 bd^3$ |

| Plane | Surface | Area | $I_{G-G}$ |
|---|---|---|---|
| Semi-~~ellipse~~ parabola |   | $\frac{2}{3} b \cdot h$ | $\frac{8}{175} bh^3$ |
| Quarter Circle |   | $\frac{\pi R^2}{4}$ | $0.55 R^4$ |