

Notes by-

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Formulae at a Glance

Sr. No.	Name of the Topic	Parameter	Formula	Unit (SI)
1	Properties of Fluid Flow	Mass Density	$\rho = m/Vol.$	kg/m ³
		Specific Weight	$\gamma = W/Vol$	N/m ³
		Specific Volume	$Sp. Vol = V/(\rho \text{ or } \gamma)$	m ³ /kg, m ³ /N
		Relative Density	$S = \rho/\rho_w = \gamma/\gamma_w$	No Unit
		Dynamic Viscosity	$\mu = \tau / (dv/dy)$	N-s/m ²
		Kinematic Viscosity	$\nu = \mu/\rho$	m ² /s
		Surface Tension (ST)	σ	N/m
		ST for Spherical Drop	$p = 4\sigma/d$	N/m
		ST for Hollow Bubbles	$p = 8\sigma/d$	N/m
		ST for Liquid Jet	$p = 2\sigma/d$	N/m
		Capillarity	$h = 4\sigma \cos\theta/(\gamma - \gamma_s)d$	m
2	Pressure and its Measurement	Pressure	$p = F/A$ $p = \gamma h$	N/m ²
		Pressure Force on vertical plane surface	$F_p = \gamma A x$	N
		Centre of Pressure	$H = x + I_G/Ax$	m
		Pressure Force on inclined plane surface	$F_p = \gamma A x$	N
		Centre of Pressure	$H = x + I_G(\sin^2\theta)/Ax$	m
		Pressure on Curved Surface	$F = \sqrt{(F_H^2 + F_V^2)}$	N
3	Buoyancy	Metacentric Height	$GM = \pm ((I/V) - BG)$	M
4 MPSC	Relative Equilibrium	Fluid Mass subjected to uniform linear acceleration	$dz/dx = -\alpha_x/(g + \alpha_z)$	
		Liquid containers s. t. constant horizontal acceleration	$p = p_a + (\gamma\alpha/g)(x_0 - x) + \gamma(z_0 - z)$	N/m ²
		Liquid containers s. t. constant vertical acceleration	$p = p_a + \gamma h (1 + \alpha/g)$	N/m ²
		Liquid containers s. t. constant rotation	$p = p_a + \gamma/g (\omega^2 r^2/2) - \gamma(z - z_0)$	N/m ²
5	Fluid Kinematics	Velocity	$V = ds/dt$	M/s
		Continuity Equation (3-D, Compressible, unsteady, nonuniform)	$\partial\rho/\partial t + \partial(\rho u)/\partial x + \partial(\rho v)/\partial y + \partial(\rho w)/\partial z = 0$	

		Continuity Equation 2D, compressible, steady, uniform	$\partial u / \partial x + \partial v / \partial y + \partial w / \partial z = 0$	
		Continuity Equation 1-D, Steady, Incompressible	$Q = A_1 V_1 = A_2 V_2 = A_3 V_3 = C$	m³/s
		Acceleration	$a = dv/dt$	m/s²
		X component	$u(\partial u / \partial x) + v(\partial u / \partial y) + w(\partial u / \partial z) + \partial u / \partial t = 0$	m/s²
		Y component	$u(\partial v / \partial x) + v(\partial v / \partial y) + w(\partial v / \partial z) + \partial v / \partial t = 0$	m/s²
		Z component	$u(\partial w / \partial x) + v(\partial w / \partial y) + w(\partial w / \partial z) + \partial w / \partial t = 0$	m/s²
		Tangential acceleration	$a_s = V_s (\partial V_s / \partial s) + (\partial V_s / \partial t)$	m/s²
		Normal acceleration	$a_n = V_s (\partial V_n / \partial s) + (\partial V_n / \partial t)$	m/s²
		Rotation Z-component	$\omega_z = \frac{1}{2} [(\partial v / \partial x) - (\partial u / \partial y)]$	rad/s
		Rotation X-component	$\omega_x = \frac{1}{2} [(\partial w / \partial y) - (\partial v / \partial z)]$	rad/s
		Rotation Y-component	$\omega_y = \frac{1}{2} [(\partial u / \partial z) - (\partial w / \partial x)]$	rad/s
		Circulation	$\Gamma = [(\partial v / \partial x) - (\partial u / \partial y)] \delta x \delta y$	
		Vorticity	$\zeta = [(\partial v / \partial x) - (\partial u / \partial y)] = 2\omega_z$	Rad/s
		Velocity Potential ϕ	$u = -(\partial \phi / \partial x), v = -(\partial \phi / \partial y), w = -(\partial \phi / \partial z)$	m/s
		Stream Function ψ	$u = -(\partial \psi / \partial y), v = (\partial \psi / \partial x), d\psi = -u \delta y + v \delta x$	m/s
		Laplace eqn. in ϕ	$(\partial^2 \phi / \partial x^2) + (\partial^2 \phi / \partial y^2) + (\partial^2 \phi / \partial z^2) = 0$	
		Cauchy-Riemann Eqns.	$(\partial \phi / \partial x) = (\partial \psi / \partial y), -(\partial \phi / \partial y) = (\partial \psi / \partial x)$	
6	Fluid dynamics	Equation of Motion	$Ma = F_g + F_p + F_v + F_t + F_s + F_e$	N
		Reynolds equation of Motion	$Ma = F_g + F_p + F_v + F_t$	N
		Navier-Stokes equation of Motion	$Ma = F_g + F_p + F_v$	N
		Euler's equation of Motion	$Ma = F_g + F_p$	N
		Euler's equation of Motion (X-component)	$X - 1/\rho (\partial p / \partial x) = u(\partial u / \partial x) + v(\partial u / \partial y) + w(\partial u / \partial z) + \partial u / \partial t$	
		Euler's equation of Motion (Y-component)	$Y - 1/\rho (\partial p / \partial y) = u(\partial v / \partial x) + v(\partial v / \partial y) + w(\partial v / \partial z) + \partial v / \partial t$	
		Euler's equation of Motion (Z-component)	$Z - 1/\rho (\partial p / \partial z) = u(\partial w / \partial x) + v(\partial w / \partial y) + w(\partial w / \partial z) + \partial w / \partial t$	
		Force Potential Ω	$X = -(\partial \Omega / \partial x), Y = -(\partial \Omega / \partial y), Z = -(\partial \Omega / \partial z)$	
		Bernoulli's Equation	$P/\gamma + V^2/2g + z = C$	
		Kinetic Energy Correction Factor	$\alpha = (1/AV^3) \int_A v^3 dA$	
		Discharge through Venturimeter	$Q_{act} = C_d Q_{th}, C_d = [a_1 a_2 \sqrt{(2gh)}] / \sqrt{(a_1^2 - a_2^2)}$	m³/s

		Discharge through orifice	$Q_{act} = C_d Q_{th}$ $C_d = [a_1 a_2 \sqrt{2gh}] / \sqrt{(a_1^2 - a_2^2)}$	m^3/s
		Velocity by Pitot Tube	$V = \sqrt{2gh}$	m/s
		Momentum Equation	$\sum F = \rho Q [\beta_2(V_2) - \beta_1(V_1)]$	N
		Momentum Correction Factor	$\beta = (1/AV^2) \int_A v^2 dA$	
7	Conduit Flow	Reynold's number	$R_e = F_i / F_v = \rho V D / \mu$	
		Darcy-Weisbach Equation	$h_f = 4f LV^2 / 2gD$	m
		1. Loss of energy due to sudden enlargement	$h_L = (V_1 - V_2)^2 / 2g$	m
		2. Loss of energy due to sudden contraction	$h_L = 0.5 V^2 / 2g$	m
		3. Loss of energy at the entrance to a pipe	$h_L = 0.5 V^2 / 2g$	m
		4. Loss of energy at the exit from a pipe	$h_L = V^2 / 2g$	m
		5. Loss of energy due to gradual contraction or enlargement	$h_L = k (V_1 - V_2)^2 / 2g$	m
		6. Loss of energy in bends	$h_L = k V^2 / 2g$	m
		7. Loss of energy in various pipe fittings	$h_L = k V^2 / 2g$	m
		Flow through Long pipes	$H = V^2 / 2g (1.5 + f L/D)$	m
		Pipe in series	$H = 4f_1 L_1 V_1^2 / 2gD_1 + 4f_2 L_2 V_2^2 / 2gD_2 + 4f_3 L_3 V_3^2 / 2gD_3$	m
		Equivalent pipe	$L/D^5 = L_1 D_1^{-3} + L_2 D_2^{-3} + L_3 D_3^{-3} + \dots$	
		Pipes in parallel	$h_f = 4f_1 L_1 V_1^2 / 2gD_1 = 4f_2 L_2 V_2^2 / 2gD_2$	
		Flow through a bye-pass	$Q = (Q + q) / 1 + \sqrt{[(D/d)^5 (l + kd)/L]}$	m^3/s
		Branched pipes	$Z_1 = (p/\gamma + Z_d) + h_{f1}$ $(p/\gamma' + Z_d) = Z_2 + h_{f2}$ $(p/\gamma + Z_d) = Z_3 + h_{f3}$ $Q_1 = Q_2 + Q_3$	
		Siphon	$H = V^2 / 2g (1.5 + f L/D)$	
		Head Loss in Tapering Pipes	$h_f = 4f Q L^5 [(1/l^4) - (1/(L+l)^4)] / 2g \pi^2 D_2^5$	m
		Heads Loss due to friction in pipes with side tappings	$h_f = (f/D) (V^2 / 2g) (L/3)$	m
		Time of emptying a reservoir through pipe	$t = [8A \sqrt{1.5 + (f l / D)(H_1^{1/2} - H_2^{1/2})}] / \pi D^2 \sqrt{2g}$	S

		Transmission of power through pipe	$P = \gamma Q (H - h_f)$, $H = 3 h_f$ for P_{max}	m
8	Boundary Layer Theory <i>MPSC Not</i>	Nominal Thickness	δ is y at which $u = 0.99V$	m
		Displacement Thickness	$\delta^* = \int (V-u) dy$	m
		Momentum Thickness	$\theta = \int (u/V) [1-(u/V)] dy$	m
		Energy Thickness	$\delta_E = \int (u/V) [1-(u^2/V^2)] dy$	m
		Prandtl's Boundary Layer Equations	$1) \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ $2) u(\frac{\partial u}{\partial x}) + v(\frac{\partial u}{\partial y}) = -1/\rho(\frac{\partial p}{\partial x}) + v(\frac{\partial^2 u}{\partial y^2})$ $3) -1/\rho(\frac{\partial p}{\partial y}) = 0$	
		Momentum Integral Equation of Boundary Layer	$d(V^2\theta)/dx + dV/dx (V\delta^*) = \tau_0 / \rho$	
		Laminar BL	$u/V = y/\delta$ $\delta/x = k \sqrt{(xu/V)}$ $\tau_0 = \sqrt{(\mu \rho V^3 / x)}$ $C_f = 1.328 / \sqrt{Re_L}$ $\delta^*/x = 1.729 / \sqrt{Re_x}$ $\theta/x = 0.664 / \sqrt{Re_x}$	
		Turbulent BL	$u/V = (y/\delta)^3$ $\delta/x = 0.376 / (Re_x)^{1/2}$ $C_f = 0.074 / (Re_f)^{1/4}$	
		Laminar Sublayer	$\delta^* = 11.6 v / \sqrt{(\tau_0 / \rho)}$	
		Hydrodynamically smooth boundary	$(V * k_s) / v < 5$	
		Transition	$5 < (V * k_s) / v < 70$	
		Hydrodynamically smooth boundary	$(V * k_s) / v > 70$	
9	Laminar Flow <i>MPSC-Not</i>	Relation between shear and pressure gradient	$\frac{\partial \tau}{\partial y} = \frac{\partial p}{\partial x}$	
		Laminar Flow through circular pipe, shear stress	$\tau = (- \frac{\partial p}{\partial x}) r/2$ $\tau_0 = (- \frac{\partial p}{\partial x}) R/2$	N/m^2
		Velocity distribution	$v = 1/4 \mu (- \frac{\partial p}{\partial x}) (R^2 - r^2)$	m/s
		Max. velocity	$v_{max} = 1/4 \mu (- \frac{\partial p}{\partial x}) (R^2)$	m/s
			$v = v_{max} [1 - (r/R)^2]$	m/s
		Discharge	$Q = 1/128 \mu (- \frac{\partial p}{\partial x}) (D^4)$	m^3/s
		Average velocity	$V = 1/32 \mu (- \frac{\partial p}{\partial x}) (D^2)$	m/s
			$V = v_{max}/2$	m/s
		Pressure difference	$(p_1 - p_2) = 32 \mu V L / D^2$	N/m^2
		Pressure difference	$(p_1 - p_2) = 128 \mu Q L / \pi D^4$	N/m^2
		Head loss due to friction	$h_f = 32 \mu V L / \gamma D^2$	m
		Friction factor	$f = 64/Re$	
		Shear velocity	$\sqrt{(\tau_0 / \rho)} = V^* = V \sqrt{(f/8)}$	

	Laminar flow through flat plates: Plane Poiseuille's flow		
	Velocity distribution	$v = 1/2\mu (- \partial p / \partial x) (By - y^2)$	m/s
	Max. velocity	$v_{max} = B^2/8\mu (- \partial p / \partial x)$	m/s
	Discharge per unit width	$q = B^3/12\mu (- \partial p / \partial x)$	$m^3/s/m$
	Average velocity	$V = B^2/12\mu (- \partial p / \partial x)$	m/s
		$V = 2v_{max}/3$	m/s
	Pressure difference	$(p_1 - p_2) = 12 \mu V L / B^2$	N/m ²
	Head loss due to friction	$h_f = 12 \mu V L / \gamma B^2$	m
	shear stress	$\tau = (- \partial p / \partial x) (B/2 - y)$	N/m ²
		$\tau_0 = (- \partial p / \partial x) (B/2)$	N/m ²
	Laminar flow through flat plates: Couette flow		
	Velocity distribution	$v = (U/B)y - 1/2\mu (- \partial p / \partial x) (By - y^2)$	m/s
	Discharge per unit width	$q = B^3/12\mu (- \partial p / \partial x)$	$m^3/s/m$
	Average velocity	$V = B^2/12\mu (- \partial p / \partial x)$	m/s
		$V = 2v_{max}/3$	m/s
	Pressure difference	$(p_1 - p_2) = 12 \mu V L / B^2$	N/m ²
	Head loss due to friction	$h_f = 12 \mu V L / \gamma B^2$	m
	shear stress	$\tau = \mu (U/B) + (- \partial p / \partial x) (B/2 - y)$	N/m ²
		$\tau_0 = (- \partial p / \partial x) (B/2)$	N/m ²
	Laminar flow through porous media	$V = K I$	m/s
	Stoke's law	$F_D = 3 \pi \mu V D$	N
	Terminal velocity	$V = (D^2/18\mu) (\gamma_s - \gamma)$	m/s
	Coefficient of drag	$C_D = 24/Re$	