ENGINEERING MECHANICS

Notes by-

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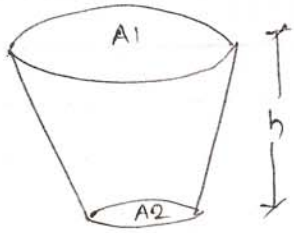
EM-A

EM-T

* Vol. of semisphere = $\frac{4}{3}\pi r^3$ C.G = $\frac{3}{8} \cdot r$

* Vol. of cone = $\frac{1}{3}\pi r^2 h$, C.G. = $\cancel{\frac{1}{6} r}$ $\frac{h}{4}$.

frustum of a cone



$V = \frac{h}{3}\left[A_1^2 + A_2^2 + A_1 \cdot A_2\right.$

DYNAMICS

Syllabus :- Kinematics & Kinetics of particles & rigid bodies.

Space-time relat^n - Kinematics - Geometry of motion - MLT - No Mass involved.

Kinetics - force that cause the motion. - Involves MLT

* Kinematics of Particles :-   L-T - where ; when.

Rectilinear Motion :- x - Movement in straight line

Curvilinear motion

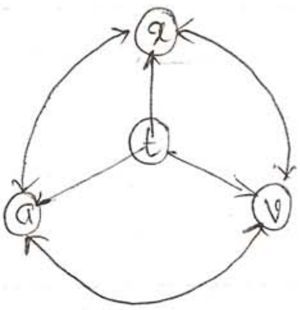
$\boxed{a = \frac{g x^2}{r^2}}$

$$v = \frac{dx}{dt}$$

$$accel^n = \frac{dv}{dt} = \frac{d x^2}{dt^2} = v \cdot \frac{dv}{dx} \quad ; \text{jerk} = \frac{da}{dt}$$

x - Position, v - vel, a = accel^n } all funct^n of time,

at low speed, accel^n is high.



where space	when time	
$x \longrightarrow$	t	Most fundament eq^n.
$v \longrightarrow$	t	fundamental eq^n.
$a \longrightarrow$	t	

$a = f(v)$.

$a = f(x)$

$v\ a = f(x)$

$a = f(v)$; $a = v \cdot \frac{dv}{dx}$.

$\therefore v \cdot \frac{dv}{dx} = f(v)$

Integrate

Pro :- Motion of a particle is defined by, $x = t^3 - 6t^2 + 9t + 5$.

Find i) When $v = 0$

ii) Position, $accel^n$ when $t = 5$ sec.

sol^n:- $v = \frac{dx}{dt} = \frac{d}{dt}(t^3 - 6t^2 + 9t + 5)$

$\therefore = 3t^2 - 12t + 9$

at $v = 0$

$\therefore 3t^2 - 12t + 9 = 0$

$t^2 - 4t + 3 = 0$.

$\therefore (t-1)(t-3) = 0$.

$\therefore \boxed{t = 1 \quad \text{or} \quad t = 3.}$ second.

$\therefore$ Vel. is zero for $t = 1$ s. & $t = 3$ s —— Ans (i)

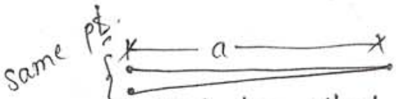
$x_5 = (5)^3 - 6(5)^2 + 9(5) + 5$ ---- at $t = 5$ sec.

$\boxed{x_5 = 25m}$ ---- Ans (ii)

$accel^n$

$a = \frac{dv}{dt}$

$= \frac{d}{dt}(3t^2 - 12t + 9)$



Dist. travelled = 2(a)

$a = 6t - 12$

$\therefore$

$\therefore a_5 = 6(5) - 12$

$\therefore \boxed{a_5 = 18 \ m/s^2}$ --- Ans (ii) change in position = 0

Find the distance travelled in 5 sec.

at $t = 1$ & $t = 3$, velocity $= 0$ i.e. at this pt. change in direction.

$x_0 = 5$ m substitute $t = 0$ in above eq^n.

$x_1 = 9$ m

$$x_2 = 7\ m$$
$$x_3 = 5m$$
$$x_4 = 9m$$
$$x_5 = 25m$$

$$|x_1 - x_0| + |x_3 - x_1| + |x_5 - x_3|$$
$$= |9-5| + |5-9| + |25-5|$$
$$= 28\ m.$$

END

Pro:- Accel^n of a particle is defined by $a = -0.4v$, a is mm/s^2 & v is mm/s, If $v = 30$ mm/s at $t=0$, find –

i) Dist travelled before coming to rest.

ii) Time reqd. for that.

iii) Time reqd. for the velocity to be reduced to 1% of its initial value.

Sol^n:-

$$a = -0.4\ v$$

Particle come to rest $\Rightarrow$ vel $= 0$.

$$\therefore \frac{dv}{dt} = -0.4 \cdot v$$

$$\Rightarrow \int dv = -0.4 \int v \cdot dt$$

$$\therefore \cancel{\int v = -0.4 \quad \log v = -0.4}$$

$$\Rightarrow \int_{30}^{v} \frac{dv}{v} = -0.4 \int_0^t dt$$

$$\Rightarrow [\log v]_{30}^{v} = [-0.4t]_0^t$$

$$\therefore \log v - \log(30) = -0.4t$$

$$\therefore \log\left(\frac{v}{30}\right) = -0.4t$$

} Express v in terms of t.

$$\therefore e^{\log\left(\frac{v}{30}\right)} = e^{-0.4t}$$

} take exponentions on both si

but $e^{\log a} = a$.

$$\therefore \frac{v}{30} = e^{-0.4t}$$

$$\therefore \boxed{v = 30\ e^{-0.4t}}$$

$$v = 30 \cdot e^{\frac{1}{0.4t}} \Rightarrow \text{for } v = 0,\ e^{\frac{1}{0.4t}} =$$

$$\Rightarrow t = \infty$$

i) $v = 0$, when $t = \infty$.

A man travells with vel, 1 km / hr, 1/2 km/hr, 1/4 km/hr .....
..... he will never reach at ~~at~~ end.

$1 + 1/2 + 1/4 + 1/8 + 1/16 + 1/32 + \ldots\ldots$ Not possible.

$\therefore$ Particle will never come to rest. — (i)

$$v = 30 e^{-0.4t}$$

at $t=0$, $x=0$

$$\therefore \frac{dx}{dt} = 30 \cdot e^{-0.4t}$$

$$\boxed{\int e^{ax} dx = \frac{e^{ax}}{a}}$$

$$\therefore \int_{x=0}^{x} dx = \int_{t=0}^{t=\infty} 30\, e^{-0.4t}\, dt$$

$$\therefore [x]_0^x = 30 \left[\frac{e^{-0.4t}}{-0.4}\right]_0^\infty$$

$$\therefore x = 30 \left[e^{-\infty} - \frac{e^0}{-0.4}\right]$$

$e^{-\infty} = 0$; $e^0 = 1$.

$$\therefore x = 30 \left[\frac{-1}{-0.4}\right]$$

$$\therefore \boxed{x = 75 \text{ mm}}$$

Vel. is never zero; it is zero at ~~∞~~ $t = \infty$.

$\therefore$ Change in position = Dist. travelled = 75 mm.

Particle does not reverse its path.

iii) Initial value = 30 mm/s.

$v = 1\%$ of $30 = 0.3$

Let $v = 30 \cdot e^{-0.4t}$

$$0.3 = 30 \cdot e^{-0.4t}$$

$$\therefore \frac{0.3}{30} = e^{-0.4t}$$

$$\therefore \log\left(\frac{0.3}{30}\right) = -0.4t$$

$$\boxed{t = 11.515}$$

mathematical $\left\{ \begin{array}{l} \ln \text{ --- } \\ \text{rad --- } \end{array} \right. \left\{ \begin{array}{l} \text{Log} \\ \text{degree} \end{array} \right\}$ Not in math. we used for conviniance.

Log → Base 10, ln → base 'e'.

Pro:- $a = 25 - 3x^2$; No initial vel. at $x = 0$.

$a \to mm/s^2$, $x \to mm$

i.e. $v = 0$, $x = 0$

Find - i) vel. when $x = 2$ mm ~~to~~

ii) Position when vel. is again zero.

iii) Position where vel. is max.

sol^n:- $a = 25 - 3x^2$

From given cond^n, $v = 0$. at $x = 0$ & at $t = 0$.

$\therefore \cancel{\frac{d^2x}{dt^2} = 25 - 3x^2}$

$v \cdot \frac{dv}{dx} = 25 - 3x^2$

$\therefore \int_0^v v\,dv = \int_0^x (25 - 3x^2)\,dx$

$\left[\frac{v^2}{2}\right]_0^v = \left[\left(25x - \frac{3x^3}{3}\right)\right]_0^x$

$\boxed{\frac{v^2}{2} = \left(25x - \frac{3x^3}{3}\right)}$

$\therefore$

$v = ?$ at $x = 2$

$\therefore v^2 = 2(25x - x^3) \rightsquigarrow [x = 2]$

$\therefore$

$\therefore \boxed{v = \pm 9.165 \text{ mm/s}}$

$\pm$ Denotes that particle will move as $\longrightarrow$ or $\longleftarrow$.

ii) Position when vel. is again zero.

$\therefore v^2 = 2(25x - x^3)$

$\therefore$ at $v = 0$,

$0 = 2(25x - x^3)$

$\therefore 25x = x^3$

$x^2 = 25$

$a = \frac{dv}{dt}$ for vel is max or min, its derivative will be zero.

$v = \frac{dx}{dt}$ $\therefore$ vel. is max, when accel^n is zero.

iii). Velocity is max.

as $a = \frac{dv}{dt}$

$\therefore a = 25 - 3x^2$

$\because a = 0$ for vel. to be max. or min.

$\therefore 25 - 3x^2 = 0$

$\boxed{x = 2.886 \text{ mm}}$

END

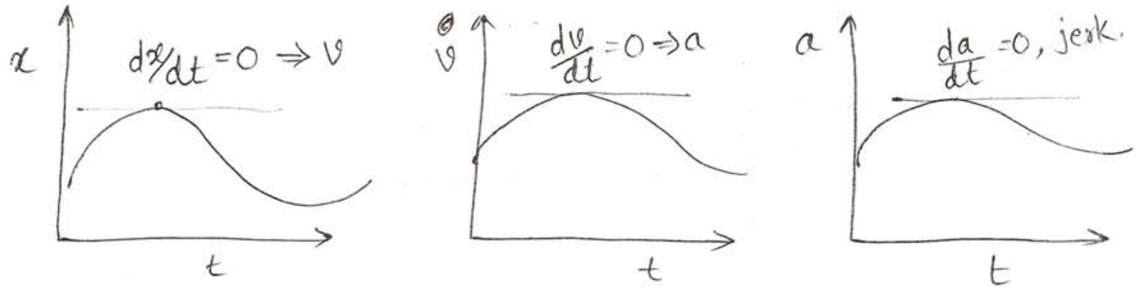
MOTION CURVE :-

The graphical display of fundamental eq^n of motion (i.e. $x \to t$, $v \to t$, $a \to t$) is known as motion curves.

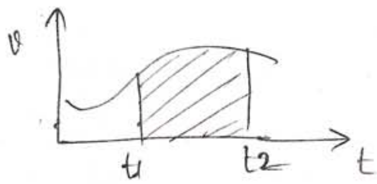
On graph dx/dy means slope of a tangent.

Thus @ $v = \frac{dx}{dt}$ $\frac{dx}{dt} \to$ ~~slope of~~ = slope of x-t cure

$a = \frac{dv}{dt} \to$ slope of v-t curve.



~~Slope~~ $\int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v \cdot dt$



$\int^{2} dt = \int$

$x_2 - x_1 =$ Area under v-t curve

$\Rightarrow$ & change in position ~~[velocity]~~ is the area under v-t curve.

change in velocity is the area under a-t curve.

Position $\to$ Area (v-t)
velocity $\to$ Area (a-t)

schedule

Oct - 20 to 30 - George sir - Str - except 26,	6.30 to 9.30
Nov - 1 - to 30 - except 7, 11, 12, 13, 14, 21, 28.	
Dec - 1 to 11 - except - 5,	

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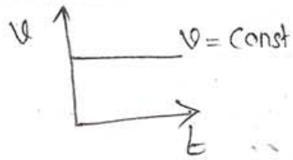
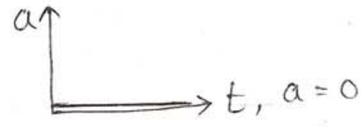
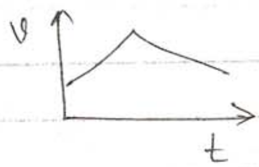
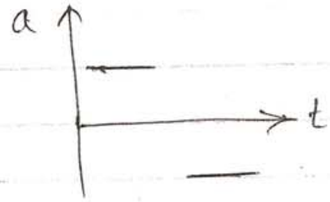
~~For~~ Uniform ~~accl^n~~ vel.





Unifor ~~m~~ accl^n





Pro: Two cars are travelling adjacent HW lanes. 'B' is stopped when it is passed by 'A', which travells at a const speed of 24 km/hr. 2 sec. later 'B' starts accelerates at a const rate of $0.9\ m/s^2$. Find when & where B will overtake A & speed of B at that tim.

Sol^n $V_a = 24\ km/hr = 6.67\ m/s$

$t_b = t_a - 2$ sec

B starts after 2 sec. $\qquad a_A$ & a_B = const.

$$\left.\begin{aligned} s &= ut + \tfrac{1}{2}at^2 \\ v &= u + at \\ v^2 &= u^2 + 2as \end{aligned}\right\}$$

$s = ut + \frac{1}{2}at^2$.

$u = 6.67$ sec = const $\Rightarrow a = 0$

$\therefore s = 6.67\,t \quad$ — (i)

also, $s = ut + \frac{1}{2}at^2$

$u = 0$ at $a = 0.9\ m/s^2$.

at $t = t - 2$

$\therefore s = 0 + \frac{1}{2}(0.9)(t-2)^2$ — (ii)

equating (i) & (ii)

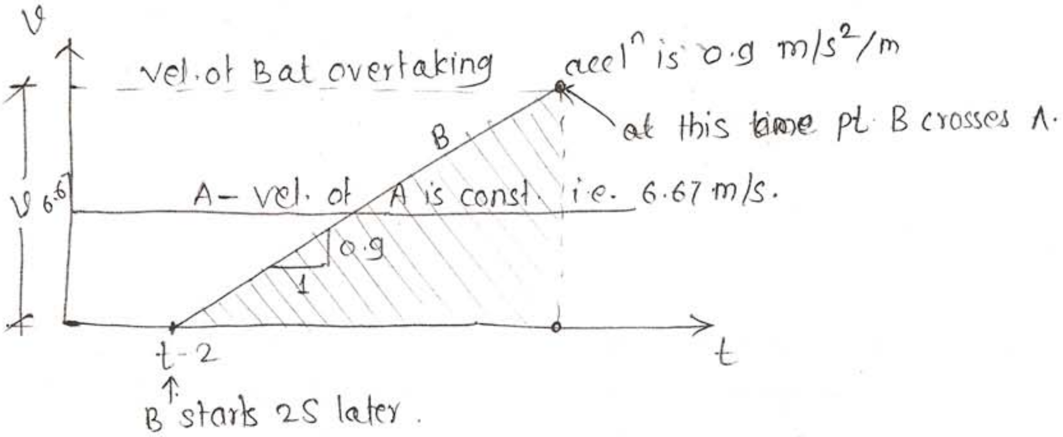
$\therefore 6.67t = \frac{1}{2} \times 0.9 \times (t-2)^2$

$\therefore$ find t

Let $v = u + at$

$= 0 + 0.9\,(t-2)$

amoung three curve $x-t$, $v-t$, $a-t$ select $v-t$ curve because its slope i.e. dv/dt is accel^n & its area under curve is 'x'.



Equating area under A & area under B.

$$\therefore 6.67 \times t = \frac{1}{2}(t-2)(v)$$

But $\frac{v}{t-2} = \frac{0.9}{1}$ → Triangle law. $\Rightarrow v = 0.9(t-2)$

$$\therefore 6.67t = \frac{1}{2}(t-2)(t-2)\times 0.9$$

$$\therefore 6.67t = 0.45(t^2 - 4t + 4)$$

$$\therefore 6.67t + 1.8t - 0.45t^2 - 1.8 = 0$$

$$\therefore -0.45t^2 + 8.47t + 1.8 = 0$$

$$\boxed{t = 18.6S \; ; \; t = 0.21S}$$

$$\frac{v}{t-2} = \frac{0.9}{1} \Rightarrow v = 0.9(t-2)$$

$$= (18.6-2)\times 0.9$$

$$\boxed{v = 14.98 \text{ m/s.}}$$

$$S = 6.67 \times t$$

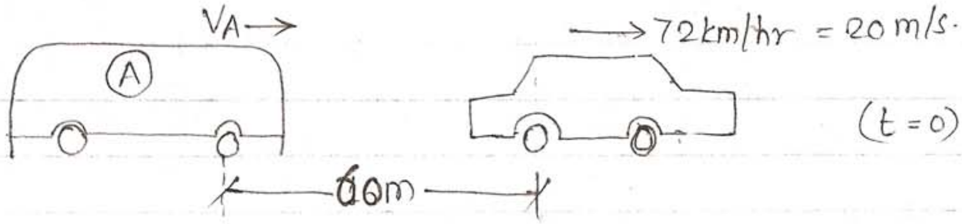
$$= 6.67 \times 18.6$$

$$\boxed{S = 124.062 \text{ m}}$$

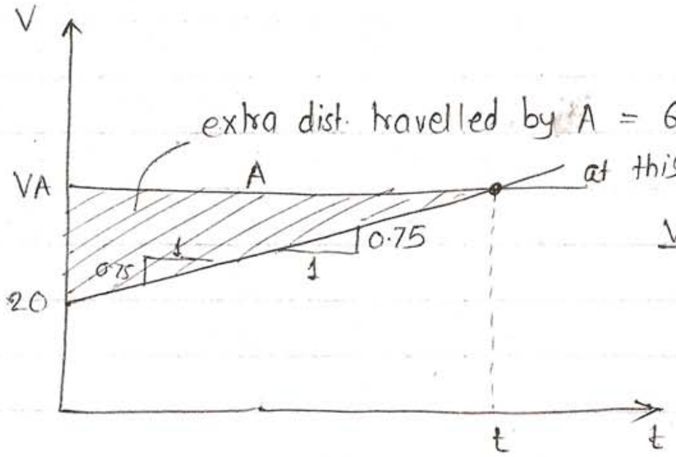
∴ Car B will overtake car A after 18.6 Sec at a velocity of 14.98 m/s & at a dist of 124.06 m from original place.

Pro:- A is travelling at a constant speed V_A. It approaches B moving in same direction 72 km/hr. B notices 'A' when it is 60 m behind 'B'. & then he accelerate at ~~0.75 m/s~~ a constant $0.75\ m/s^2$ to avoid being passed by A. If the closest that A come to 6 m find V_A. 21/10 S

Sol^n:-



at a 60 m dist. B notices 'A'
i.e. $V_A > V_B$.
at some time dist. bet^n A & B is 6 m. Find the V_A.



at this pt. accl^n of B becomes constant & A is already constant

$$\frac{V_A - 20}{t} = \frac{0.75}{1}$$

$$\Rightarrow t = \frac{V_A - 20}{0.75}$$

Hatched area $= \frac{1}{2}(V_A - 20)\cdot t$

$$\therefore 54 = \frac{1}{2}(V_A - 20)\,t$$

$$= \frac{1}{2}\frac{(V_A - 20)(V_A - 20)}{0.75}$$

$$\therefore 108(0.75) = (V_A - 20)^2$$

$$\therefore 81 = (V_A - 20)^2$$

$$\therefore V_A - 20 = 9$$

$$\therefore \boxed{V_A = 29\ m/s}$$

Analytically:-
exp. for S_A — (i)
S_B — (ii)
$S_B - S_A = \ldots$
$\frac{d}{dt}(S_B - S_A) = 0.$

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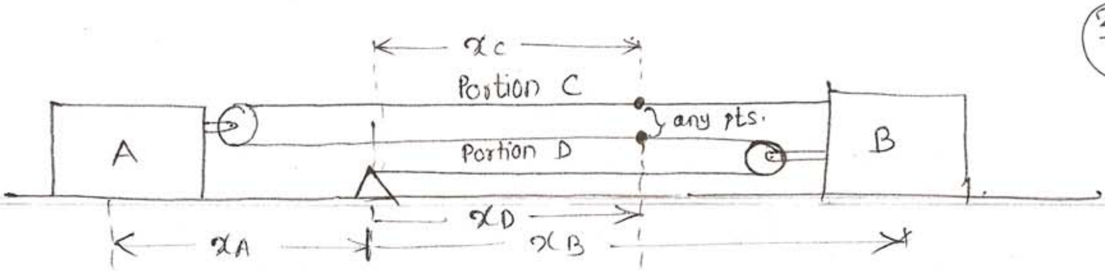

As peg remains at same point, consider as ref. pt.
fix.

We can select any point outside the blocks or system.

Length of string = constant.

$\therefore x_B + (x_A + x_B) + (x_A + x_B) = K$

$\therefore 2x_A + 3x_B = K.$

$2\ddot{x}_A + 3\ddot{x}_B = 0$

$\therefore 2V_A + 3V_B = 0$

B moves to the right i.e. $V_B = +450$ mm/s.

$\therefore 2V_A + 3(450) = 0$

$\therefore \boxed{V_A = -675 \text{ mm/s}}$ — (i)

$\therefore$ VA moves by 675 mm/s towards right

(ii) Vel. of portion 'D' of cable:-

Consider any arbitary point on portion D at x_D from fixed point.

Point moves but length of string does not changes.

holding lower portion.

$\therefore x_B + (x_B - x_D) = k$

$\therefore 2x_B - x_D = k$

$\therefore 2V_B = V_D$

$\therefore V_D = 450(2)$

$\therefore \boxed{V_D = +950 \text{ mm/s}}$

$\therefore$ VD moves with 950 mm/s towards right

iii) Relative vel. of A w.r.t. B.

$V_A = -675$ m/m/s $(\longrightarrow)$

$V_B = 450$ mm/s $(\longrightarrow)$

$\therefore \boxed{V_{A/B} = 225 \text{ mm/s}}$ $(\longleftarrow)$

$V_{A/B} = 450$ mm/s $\rightarrow$

-675 mm/s $\rightarrow$

225 mm/s $\leftarrow$

iv) <u>Relative vel. of C w.r.t. D</u>

$$\therefore x_B - x_C = k$$

$$\therefore V_B = V_C$$

$$\boxed{\therefore V_C = 450 \text{ mm/s} \quad (\longrightarrow)}$$

$$
\begin{array}{rll}
V_{C/D} = & 450 \text{ mm/s} & \longrightarrow \\
- & 950 \text{ mm/s} & \longrightarrow \\
\hline
& -450 \text{ mm/s} & \longleftarrow
\end{array}
$$