ENGINEERING MECHANICS

Notes by-

Pravin S Kolhe,

BE(Civil), Gold Medal, MTech (IIT-K)

Assistant Executive Engineer,

Water Resources Department,

www.pravinkolhe.com

EM-B

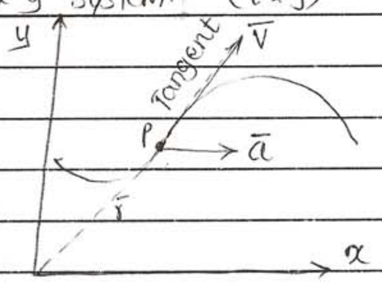
$\dot{x} \rightarrow$ Time derivative
$x' \rightarrow$ x prime - dx/dz

CURVILINEAR MOTION (in a PLANE)

Systems :-

1) x-y (Cartes[illegible] / Rectangular) system (i, j)
2) r-θ (Polar, radial & transverse) system
3) n-t (Normal & Tangential co-ordinate) system.

1) x-y System :- (i & j)



$\bar{r} = f(t)$

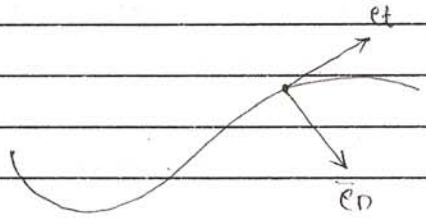
$\boxed{\bar{r} = x\bar{i} + y\bar{j}}$

vel. vector = $\boxed{\bar{V} = \dot{x}\bar{i} + \dot{y}\bar{j} = \dot{\bar{r}}}$

$\boxed{\bar{a} = \dot{\bar{V}} = \ddot{\bar{r}} = \ddot{x}\bar{i} + \ddot{y}\bar{j}}$

Vel is tangential to the Path.
accel^n will be any direct^n but it will be inside the curve @. C.G.
Posit^n not imp. only path is important.

2) n-t (Normal & Tangential co-ordinate) or Path Co-ordinates ($\bar{e}_n$ & $\bar{e}_t$)



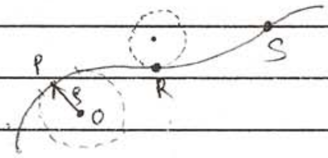
$\bar{e}_t$ Unit vect in tangential direct^n
$\bar{e}_n$ = Unit Vector in normal direct^n
as particle moves $\bar{e}_t$ & $\bar{e}_n$ changes.

Vector = Magnitude x Unit vector in the direct^n

$\boxed{\bar{V} = v \cdot \bar{e}_t}$

$$\boxed{\bar{a} = \dot{v}\,\bar{e}_t + \frac{v^2}{\rho}\,\bar{e}_n}$$

ρ = Radius of curvature.



In a curve, consider any pt. P. Take a short dist. dx. The circle traced by considering dx as a chord is known as circle of curvature.

The pt. O is known as centre of curvature.

ρ is known as Radius of curvature.

The pts. R & S at which direct^n of charro circle changes are known as points of inflection. Or it the pt. at which centre of curvature shifts other side.

$$\boxed{\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}}$$ — This will aways +ve. i.e. towards centre of curvature.

$$\bar{a} = \dot{v}\cdot \bar{e}t + \frac{v^2}{\rho}\cdot \bar{e}n$$

Rate of change of speed.

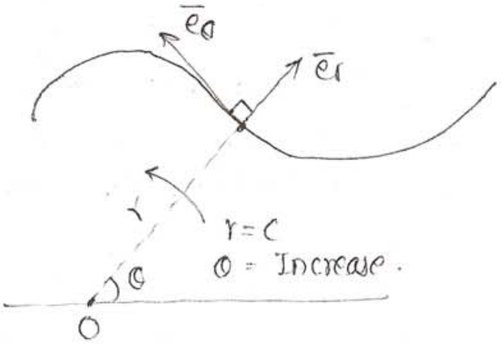
$\dot{v}$ may be + or −.

Particle moving in curvilinear motion will never have zero accel^n unless speed is const. & it is passing th' pt. of inflection.

at The pt of inflect^n : ρ will be ∞ i.e. $\frac{d^2y}{dx^2} = 0$.

ρ

3] r − θ (Radial & Transverse) system:- (e_r & e_θ)



Unit vector in the direct^n of radius in $\bar{e}r$

Unit vector $\perp^{ar}$ to radius is $\bar{e}\theta$

If r = Const.

& Increase θ, The direct^n in which r moves, that di is the direct^n of $\bar{e}\theta$ & direct^n of $\bar{e}r$ will be same as radius.

or if θ = 0, extend r, the direct^n of $\bar{e}r$ will be the same.

$$\boxed{\bar{r} = r\cdot\bar{e}r}$$

$r = f(t)$

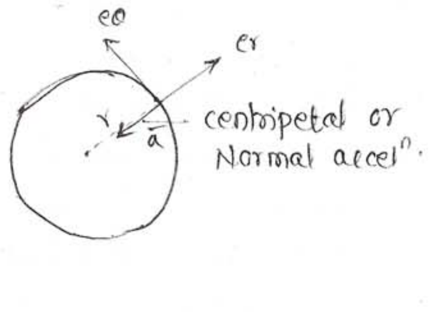
$$\boxed{\bar{v} = \dot{\bar{r}} = \dot{r}\bar{e}r + r\cdot\dot{\theta}\cdot\bar{e}\theta}$$

$$\boxed{\bar{a} = \dot{\bar{v}} = \ddot{\bar{r}} = [\ddot{r} - r\dot{\theta}^2]\bar{e}r + [2\dot{r}\dot{\theta} + r\ddot{\theta}]\bar{e}\theta}$$

$$\dot{\theta}^2 = \left(\frac{d\theta}{dt}\right)^2 \; ; \quad \ddot{r} = \frac{d^2r}{dt^2}$$

eg:-

In case of circle, r = constant.



$$\bar{v} = \dot{r}\bar{e}r + r\cdot\dot{\theta}\bar{e}\theta$$

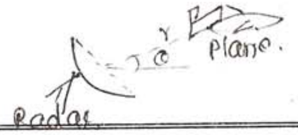
$$\dot{r} = 0$$

$$\boxed{\therefore \bar{v} = r\cdot\dot{\theta}\cdot\bar{e}\theta} \rightarrow \boxed{v = r\omega}$$

$$\bar{a} = [\ddot{r} - r\dot{\theta}^2]\bar{e}r + [2\dot{r}\dot{\theta} + r\ddot{\theta}]\bar{e}\theta$$

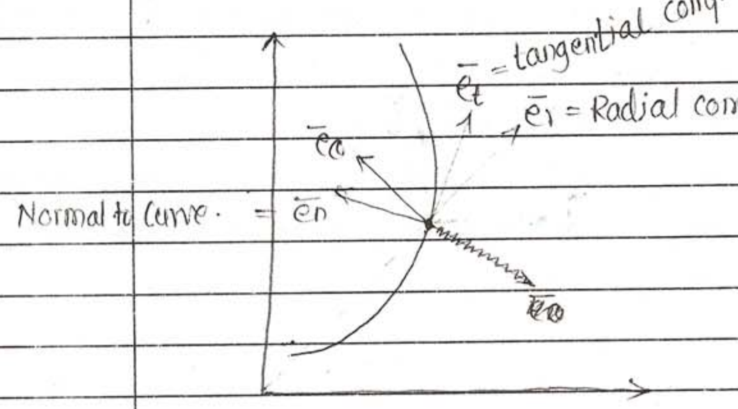
$$\boxed{\bar{a} = r\dot{\theta}^2\bar{e}r + r\ddot{\theta}\bar{e}\theta}$$ → Coriolis compound.

$$\boxed{\bar{a} = r\omega^2}$$



In case of Radar we follow $r-\theta$ system.

Selectⁿ of any system depends only on conviniance.



If we know $\bar{r}, \bar{v}$ & $\bar{a}$ we can find all co-ordinate.

V. Imp. Projectile is a body from resisting fluid medium.

System.	Radius	velocity	accelⁿ
$x-y$	$\bar{r} = x\bar{i} + y\bar{j}$	$\bar{v} = \dot{x}\bar{i} + \dot{y}\bar{j}$	$\bar{a} = \ddot{x}\bar{i} + \ddot{y}\bar{j}$
$n-t$	~~$v = \dot{x}\bar{i} + \dot{y}\bar{j}$~~	$\bar{V} = v \cdot \bar{e}_t$	$\bar{a} = \dot{v}\bar{e}_t + v^2/\rho \cdot \bar{e}_n$
$r-\theta$	$\bar{r} = r \cdot \bar{e}_r$	$\bar{V} = \dot{r}\bar{e}_r + r \cdot \dot{\theta}\bar{e}_\theta$	$\bar{a} = [\ddot{r} - r\dot{\theta}^2]\bar{e}_r + [2\dot{r}\dot{\theta} + r\ddot{\theta}]\bar{e}_\theta$

Pro:- A motion is defined by $x = (t+1)^2$ & $y = (4)(t+1)^{-2}$. $x, y \to m$ & $t \to$ sec. Examine the path & find velocity & accelⁿ when $t = 0.5$ sec.

Solⁿ:-

$$\bar{r} = x\bar{i} + y\bar{j}$$
$$= (t+1)^2\bar{i} + 4(t+1)^{-2}\bar{j}$$
$$\bar{V} = 2(t+1)\bar{i} - 8(t+1)^{-3}\bar{j}$$
$$\bar{a} = 2\bar{i} - 8 \times -3\,(t+1)^{-4}\bar{j}$$

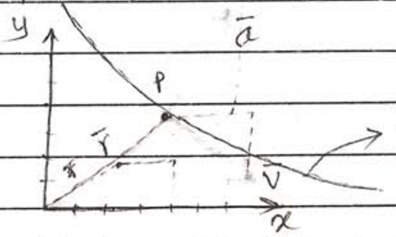
$$\frac{d}{dx}(x^{-2}) = -2 \cdot \frac{x^{-2}}{x} = -2 \times x^{-2-1}$$

$$\frac{d}{dx}(x^2) = 2x^{2-1}$$

at $t = 0.5$ sec.

$$\left.\begin{aligned} \bar{V} &= 3\bar{i} - 2.37\bar{j} \\ \bar{a} &= 2\bar{i} + 4.74\bar{j} \\ \bar{r} &= 2.25\bar{i} + 1.78\bar{j} \end{aligned}\right\} \text{at } t = 0.5 \text{ sec:}$$

Magnitude $r = \sqrt{x^2 + y^2}$
$= \sqrt{2.25^2 + 1.78^2}$
$= 2.87$ m.



$x = (t+1)^2$
$y = 4/(t+1)^2$
$= 4/x$
$\therefore xy = 4 \to$ Hyperbola. passing through P,
$\bar{v}$ is tangential & $\bar{a}$ is towards accelⁿ centre.

Plot ordinates x & y of each Parameter.

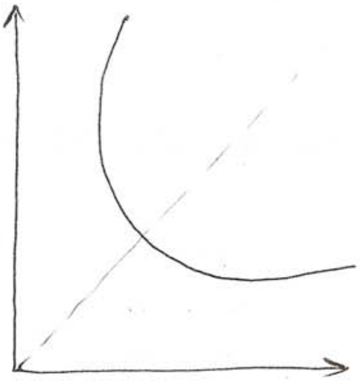
Pro: Pro:- A particle has component vel. $V_x = (5-3t)$ m/s & $V_y = t$, m/s

at $t=0$, $x=y=0$

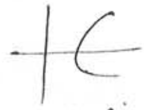
Obtain expression for dist. r from the origin.

Find the fatherest (largest) dist. from the origin in the time period 0 to 2.6 sec. At what time does this occur? State the significance of the instant $t = 2.5$ sec.

Sol^n:-



$y = ax^2$



$x = ay^2$

$V_x = 5-3t$ $\qquad$ $V_y = t$

$\int V_x \cdot dx = \int (5-3t)\,dt$ $\qquad$ $\int V_y\,dy = \int t\,dt$

$\therefore x = [5t - 3t^2/2] + C_1$ $\qquad$ $\therefore y = 0.5t^2 + C_2$.

at $\cancel{x}\cdot t=0$, $x=y=0$ $\qquad$ $0 = 0 + C_2$

$\Rightarrow 0 = [0-0] + C_1 \Rightarrow C_1 = 0$ $\qquad$ $\Rightarrow C_2 = 0$.

$\therefore x = 5t - 1.5t^2$; $y = 0.5t^2$

$\therefore$ $\Rightarrow t = \sqrt{y/0.5}$

$\therefore x = 5\sqrt{y/0.5} - 1.5(y/0.5)$

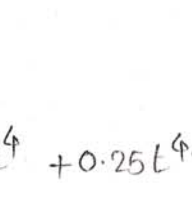
$y = 0$,

$\therefore x = 7.07\sqrt{y} - 3y$

$\therefore \cancel{x^2 + 5xy + 9y^2}$ $(x+3y) = 7.07\sqrt{y}$

$\cancel{x^2+}$ $(x+3y)^2 = (7.07\sqrt{y})^2$

$\therefore x^2 + 6xy + 9y^2 = 50y \longrightarrow$ Eq^n of parabola



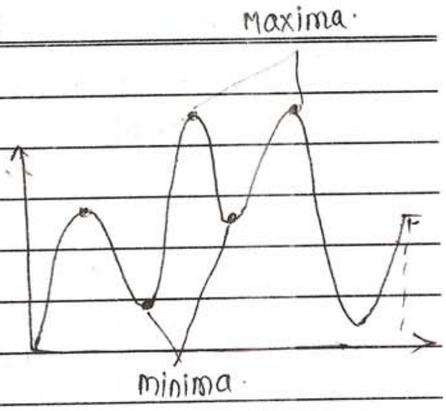
$r = \sqrt{x^2+y^2}$

$\therefore r^2 = (x^2+y^2)$

$r^2 = (5t - 1.5t^2)^2 + (0.5t^2)^2 \Rightarrow 25t^2 - 15t^3 + 2.25t^4 + 0.25t^4$.

$= \cancel{25t^2 - 15}$ $\quad \therefore r^2 = 25t^2 - 15t^3 + 2.5t^4$

$\boxed{r^2 = 2.5(t^4 - 6t^3 + 10t^2)} \longrightarrow$ Ans. (i) r will be max, when $\frac{d}{dt}(r^2) = 0$



$\therefore \cancel{0 = \frac{d}{dy} 2.5(t^4 - 6t^3 + 10t^2)}$
$\cancel{= 2.5}$

$\therefore 0 = \frac{d}{dy}[2.5(t^4 - 6t^3 + 10t^2)]$

$= 2.5[4t^3 - 18t^2 + 20t]$

$= 2.5t[4t^2 - 18t + 20]$

$\therefore t = 0$. or $t = 2.5$, $t = 2$

for r to be max, $\frac{d}{dt}(r) = 0$

$\therefore \cancel{0 = 2.5(}$ $\therefore \frac{d^2}{dt^2}(r^2) = 2.5[12t^2 - 36t + 20]$

For $r \to$ max, $t \Rightarrow -ve$
$r \to$ min, $t \Rightarrow +ve$.

at $t = 0$, $\frac{d^2(r^2)}{dt^2} = 20 \Rightarrow$ Min.

at $t = 2.5$, $\frac{d^2(r^2)}{dt^2} = +12.5 \Rightarrow$ Min.

at $t = 2$, $\frac{d^2(r^2)}{dt^2} = -8 \Rightarrow$ Max.

$\therefore$ Particle will be at farthest dist at $t = 2$ sec.

$\therefore r^2 = 2.5(t^4 - 6t^3 + 10t^2)$

at $t = 2$ sec,

$\boxed{r = 4.472\ m}$ ← from above eq^n. — Ans. (ii)

If we get two values of max, Put it in eq^n & check.

Significance At ~~di~~ $t = 2.5$ sec, particle will be at closest dist — Ans. (iii)

i.e. $\boxed{r = 4.42\ m}$

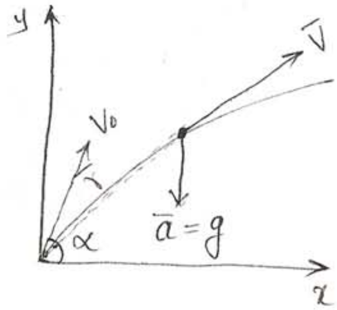
V_0

V_0 Imp.

Projectile (x,y)

Anything projected in resisting fluid medium is projectile.
Forces on particle : Drag / SW.

If we consider air is the resistanceless medium, only force will be SW. Thus accel^n due to gravity.



α = Angle at the time of project^n which particle makes with horizontal at initial vel. V_0

α = Angle of projection.

x-y ⇒ Never changes.

n-t & r,θ ⇒ changes every time.

General accel^n ⇐ $\boxed{\bar{a} = -g\bar{j}}$ → Not f^n of time, always const.

$(\bar{V})_0$ = at time of projection
$= V_0\cos\alpha\, i + V_0 \sin\alpha\, j$

General velocity ⇐ $\boxed{(\bar{V})_t = V_0\cos\alpha \cdot i + \underbrace{(V_0\sin\alpha - gt)}_{} \cdot j}$ → Vel. at any instant 't'. Time dependent.

$V = u + at$
= Initial vel. + vel. due to accel^n
$a = -g$

Integrating above eq^n

$\therefore \boxed{\bar{r} = (V_0\cos\alpha \cdot t)\bar{i} + (V_0\sin\alpha\, t - gt^2/2)\bar{j}}$ → General Position

$x \to f(t)$ & $y \to f(t)$

$\therefore x = V_0\cos\alpha \cdot t$ $\qquad$ $y = V_0\sin\alpha \cdot t - gt^2/2$

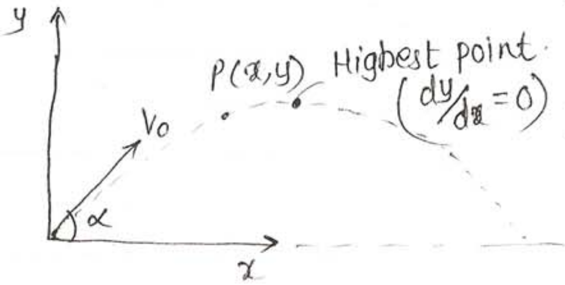
$t = x/V_0\cos\alpha$

$$\therefore y = V_0 \sin\alpha \cdot \left(\frac{x}{V_0\cos\alpha}\right) - \frac{g}{2}\left(\frac{x}{V_0\cos\alpha}\right)^2$$

$$\boxed{\therefore y = x\tan\alpha - \frac{gx^2}{2V_0^2\cos^2\alpha}}$$

⇒ Eq^n of TRAJECTORY.
⇒ $y \to f(x)$, A Time independent.

$y = [\ ]x + [\ ]x^2$ ⇒ Eq^n of Parabola.



$\left(dy/dx = 0\right)$; $\rho = \dfrac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$

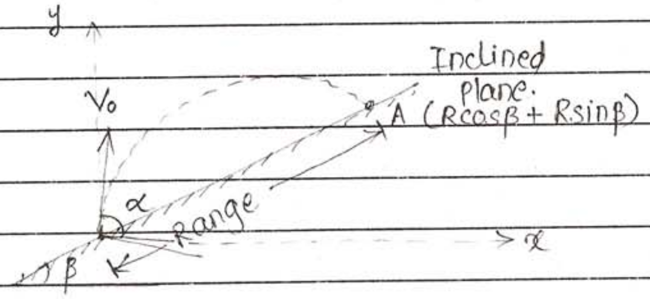
$\therefore dy/dx = 0$.

$\therefore \rho = \rho_{min} = \dfrac{1}{d^2y/dx^2}$

$\cos(A\pm B) = \cos A\cos B \mp \sin A \sin B$
$\sin(A\pm B) = \sin A \cos B \pm \cos A \sin B.$

Fix up co-ordinate system.
from eq^n f. $y = f(x)$ - big eq^n.

SH.



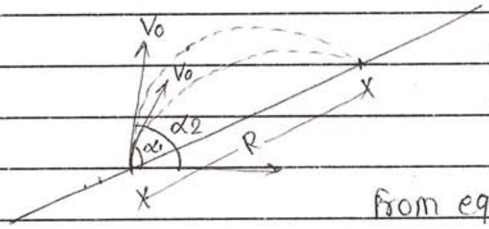
$$y = x\tan\alpha - \frac{g x^2}{2V_0^2\cos^2\alpha}.$$ $x = R\cos\beta$, $y = R\sin\beta.$

$$R\sin\beta = R\cos\beta\cdot\tan\alpha - \frac{g\,R^2(\cos^2\beta)^2}{2V_0^2\cos^2\alpha} \Rightarrow R = \frac{2V_0^2\cos\alpha\cdot\sin(\alpha-\beta)}{g\cos^2\beta}$$

$\therefore R = f(\alpha, \beta, V_0)$ $\boxed{R = K\cdot\boxed{\cos\alpha\cdot\sin(\alpha-\beta)}}$ — (a)

Thus Range (R) can be calculated. Thus only 2 values for R & of α are possible
V_0, const, $\Rightarrow R = K\cdot\cos\alpha\cdot\sin\alpha$. — (a)

There are two possibilities of α for same V_0 to have the same Range.



If target to reach at B, the vel. is same, Range is same, this can be achieved at only two angles α_1 & α_2.

$R = K\cos\alpha\cdot\sin\alpha \Rightarrow \cos\alpha\cdot\sin\alpha = \sin 2\alpha.$
from eq^n (a), Thus α ranges from 0 to 90.
2 values of α are possible having, $\boxed{\alpha_1 + \alpha_2 = 90^\circ} \Rightarrow$ Horizontal.
$\alpha = 45^\circ$ for horizontal, to have R be max.

To maximize R, set $\frac{dR}{d\alpha} = 0$

$\therefore \cos\alpha\cdot\cos(\alpha-\beta) - \sin\alpha\cdot\sin(\alpha-\beta) = 0$

$\therefore \cos[\alpha + (\alpha-\beta)] = 0$

$\therefore \cos(2\alpha-\beta) = 0$

$\therefore 2\alpha - \beta = \pi/2$

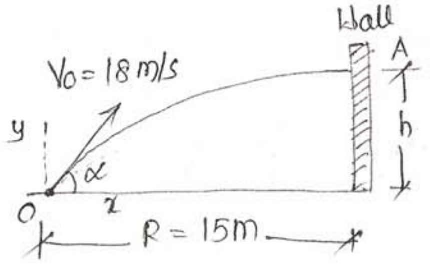
$\therefore \boxed{\alpha = \pi/4 + \beta/2}$ $\therefore \boxed{\alpha = 45^\circ + \beta/2}$

for hori. plane, $\beta = 0$, $\alpha = 45^\circ$

Pro: MPSC '99 11·102 Beer & Johnston

A player throws a ball, initial vel. = 18 m/s towards a vertical wall 15 m away. Find the max. ht. at which the ball can strike the wall ? & the corresponding angle of projection.

Sol^n :-



V_0 = const, R = const, Thus α changes.

$\therefore$ Develop relation bet^n α & h.

$$y = x\tan\alpha - \frac{g x^2}{2 V_0^2 \cos^2\alpha}$$

$$\frac{d}{dx}(\sec^2\alpha) = 2\sec\alpha \cdot \frac{d}{d\alpha}(\sec\alpha)$$

$$= 2\sec\alpha \cdot \sec\alpha \cdot \tan\alpha$$

$$= 2\sec^2\alpha \cdot \tan\alpha$$

$$\therefore h = 15\tan\alpha - \frac{g(15)^2}{2\times(18)^2\cos^2\alpha}$$

For $h \to$ max, $\frac{dh}{d\alpha} = 0$

$$\therefore 15\sec^2\alpha - \frac{g(15)^2}{2(18)^2} \times 2\sec^2\alpha \cdot \tan\alpha = 0$$

$$\therefore 15 - 0.694\, g \cdot \tan\alpha = 0$$

$$\therefore 15 = 6.81 \tan\alpha$$

$$\tan\alpha = 2.20$$

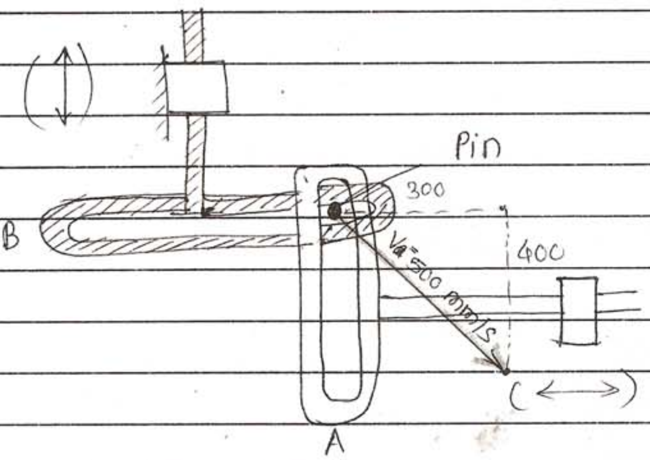
$$\boxed{\therefore \alpha = 65.59^\circ}$$

$$h = 15\tan(65.59) - \frac{9.81 \times 15^2}{2\times 18^2 \times \cos^2(65.59)}$$

$$\therefore \boxed{h = 13.1 \text{ m}}$$

Any pt. except highest pt. has two values of α.

Pro :- A particle Pin 'P' moves along a curved path. Determine by motions of A & B at the instant shown $V_A = 300$ mm/s ($\rightarrow$) $a_A = 250$ mm/s^2 ($\rightarrow$), $V_B = 400$ mm/s ($\downarrow$), $a_B = 125$ mm/s^2 ($\downarrow$)
Find radius of curvature of the path.



Sol^n :-

$$\boxed{\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left(\frac{d^2y}{dx^2}\right)}} \Rightarrow \rho = \text{Radius of Curvature.}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{\dot{y}}{\dot{x}} \quad \text{— chain Rule.}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{\dot{y}}{\dot{x}}\right)$$

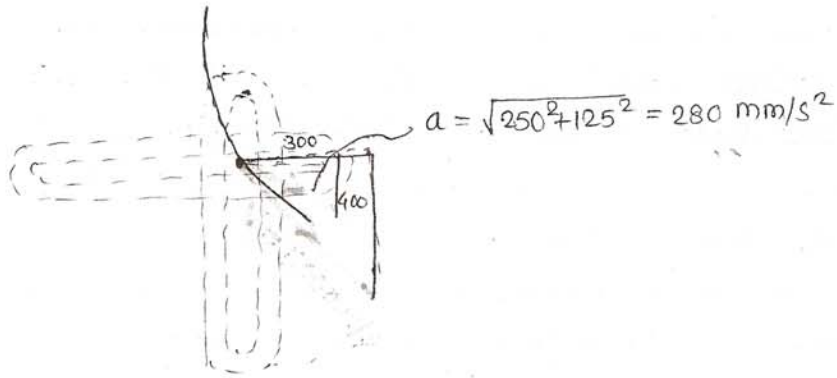
$$= \frac{d}{dt}\left(\frac{\dot{y}}{\dot{x}}\right)\left(\frac{dt}{dx}\right) = \frac{1}{\dot{x}}\left[\frac{\dot{x}\ddot{y} - \dot{y}\ddot{x}}{\dot{x}^2}\right] \rightarrow \text{Division Rule.}$$

$\dot{x} = +300$ mm/s $\qquad \dot{y} = -400$ mm/s
$\ddot{x} = 250$ mm/s^2 $\qquad \ddot{y} = -125$ mm/s^2

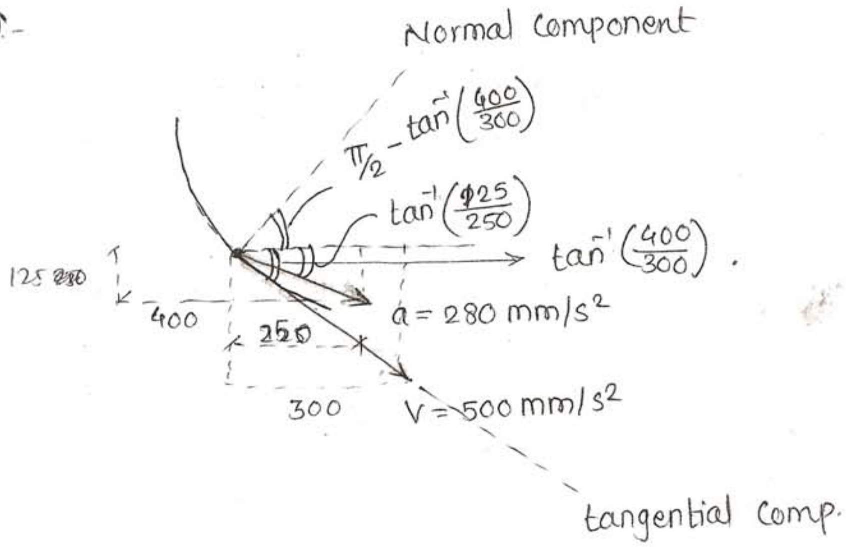
$$\therefore \frac{dy}{dx} = -1.333 \ \&, \ \frac{d^2y}{dx^2} = \frac{1}{300}\left[\frac{300\times(-125) - (-400)(250)}{(300)^2}\right]$$

$$= 2.315\times10^{-3}$$

$$\therefore \rho = \frac{\left[1 + (-1.333)^2\right]^{3/2}}{(2.315\times10^{-3})} = 1999.26$$



Or :-



Eq^n for accel^n $\Rightarrow$ $\bar{a} = \dot{v}\,\bar{e}_t + \frac{v^2}{\rho}\,\bar{e}_n$

Coeff. of normal component $= v^2/\rho$.

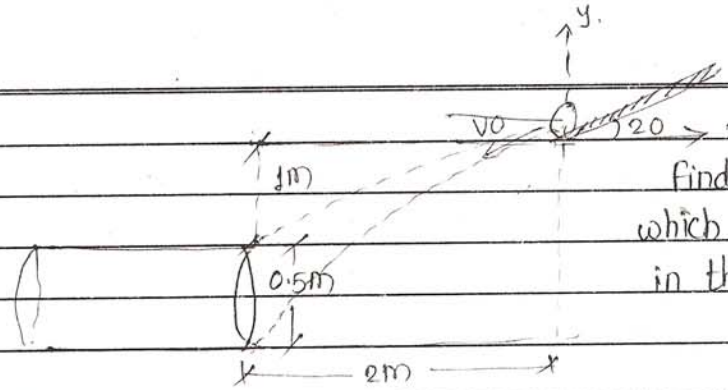
$\therefore$ ~~280~~ Resolving accel^n along en & et & compairing,

$$\sqrt{125^2+250^2}\,\cos\left[\frac{\pi}{2} - \tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{1}{2}\right)\right] = \frac{V^2}{\rho} = \frac{500^2}{\rho}$$

$$\therefore \boxed{\rho = 2600\ mm}$$

Imp.

Pro.



Find the range of Vo for which the ball will enter in the pipe.

~~What will be the range. for sphere enter in cylindrical pipe.~~

Sol^n:- $(V_0)_{max}$. Ball will strike at A.

$$\therefore -1 = 2\tan(-20) - \frac{g(2)^2}{2V_0^2\cos^2(-20)}$$

h is -ve ↓.

α is -ve.

For $(V_0)_{min}$, ie. at B.

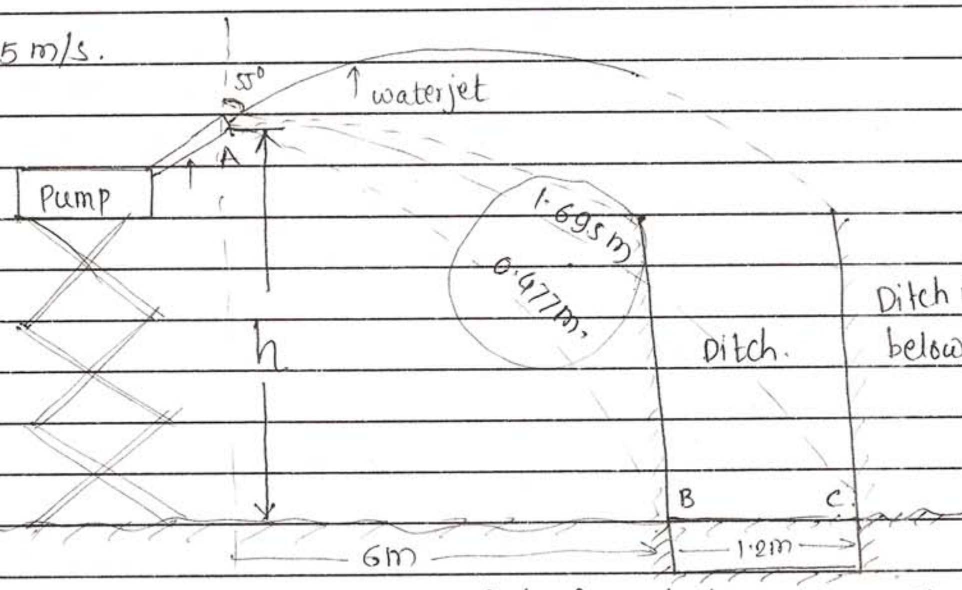
$$\therefore -1.5 = 2\tan(-20) - \frac{g(2)^2}{2V_0\cos^2(-20)}$$

$\therefore (V_0)_{max} = 9.04$ m/s

$(V_0)_{min} = 5.36$ m/s.

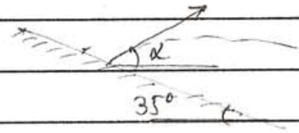
Thus ball will enter in the pipe $(9.04 \geq V_0 \geq 5.36)$ m/s.

Pro: $V_0 = 7.5$ m/s.



What will be the range of h for which water will enter in the ditch. (Ans: $1.695 \geq h \geq 0.477$ m.)

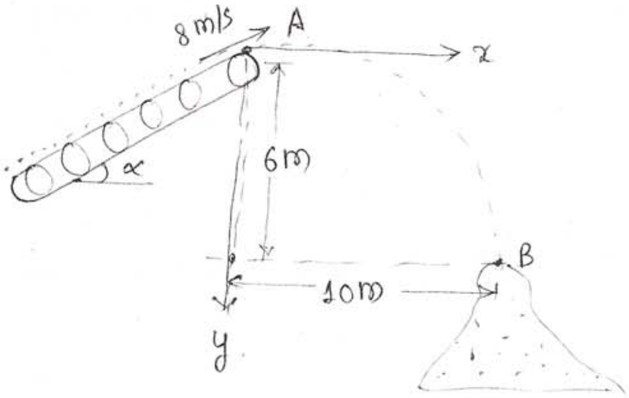
$\alpha = +35° = 90 - 55°$.



$\alpha = 35°$

$\beta = -35°$

Pro: The conveyor will moves at 8 m/s. Find α for which the sand is deposited at B ?



$$-6 = 10\tan\alpha - \frac{g(10)^2}{2(8)^2} \times \sec^2\alpha \qquad \frac{1}{\cos^2\alpha} = \sec^2\alpha = 1+\tan^2\alpha$$

$$\therefore -6 = 10\tan\alpha - \frac{9.81\times100}{128}(1+\tan^2\alpha)$$

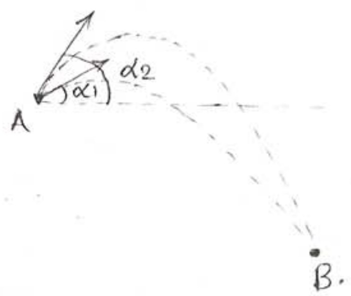
$$\therefore \cancel{-6 = 10} \quad -6 = 10\tan\alpha - 7.66 - 7.66\tan^2\alpha$$

$$\therefore 7.66\tan^2\alpha + 1.66 - 10\tan\alpha = 0$$

$\therefore$ Solving,

$$\tan\alpha = 0.195,\ 1.11$$

$$\boxed{\therefore \alpha = 11.03^\circ,\ 47.98^\circ}$$ ⇒ This proves that there are two answers.

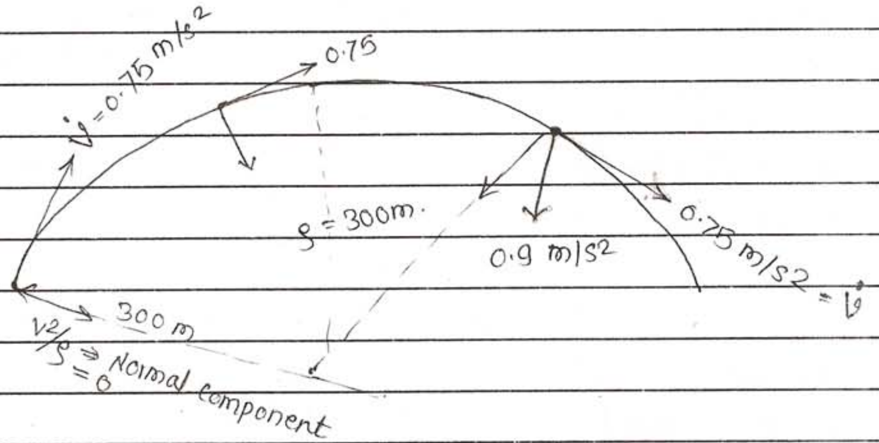


$$\boxed{t = \frac{x}{V_0\cos\alpha}}$$

For α is min, t = Max.

α is max, t = min.

Pro: A bus starts from rest on a 300 m radius curve & increases its speed at $0.75\ m/s^2$. Find dist. & time the bus will travel before its acceln is $0.9\ m/s^2$.



(Path Variable, x-y ⇒ Not reqd.)

$$\bar{a} = \dot{v}\cdot\bar{e}_t + v^2/\rho\cdot\bar{e}_n$$

$$\therefore a = \sqrt{\dot{v}^2 + (v^2/\rho)^2} \Rightarrow \text{Magnitude of acceln.}$$

$$\therefore a^2 = \dot{v}^2 + (v^2/\rho)^2$$

$$\therefore 0.9^2 = 0.75^2 + (v^2/300)^2$$

$$\therefore \frac{v^2}{300} = 0.4975$$

$$\boxed{\therefore v = 12.216\ m/s}$$ To find time & dist. at this vel.

$$v = u + at$$

$$\therefore 12.216 = 0 + 0.75t$$

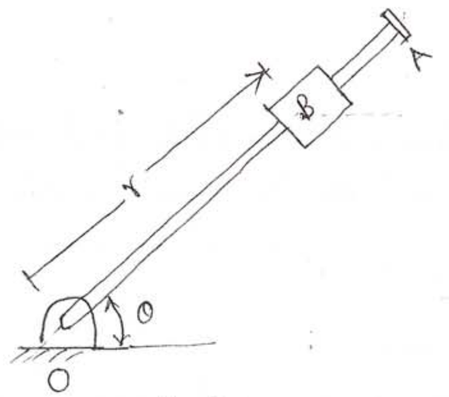
$$\therefore \boxed{t = 16.29\ S}$$

$$v^2 = u + 2as$$

$$\therefore 12.216^2 = 0 + 2(0.75)\times S$$

$$\therefore \boxed{S = 99.49\ m}$$

Pro:- OA rotates about O such that $\theta = 2t^2$. B slides on OA such that $r = 60t^2 - 20t^3$, t (s), r (mm), θ (rad) at $t = 1\ S$, find vel. & acceln of the collar. Also find acceln of the collar relative to the rod.



(r-θ system)

θ is changing,
r is changing @ $r = 60t^2 - 20t^3$
$\therefore$ curvilinear motion.
If θ = Const, ⇒ Rectilinear
r = const & θ changes ⇒ Circular.

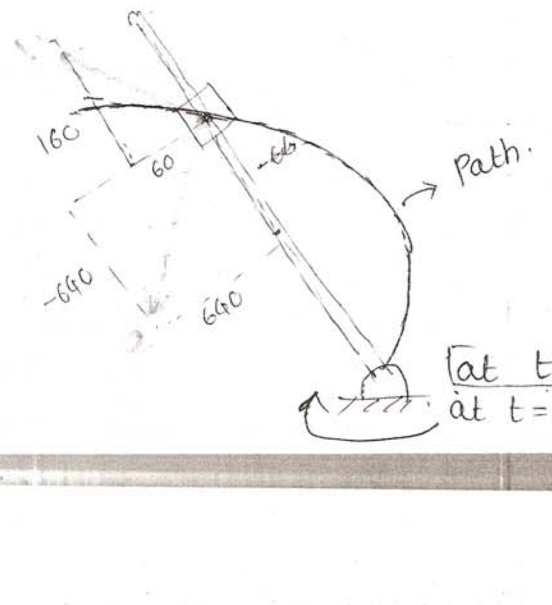
Sol^n :- $\left.\begin{array}{l}\bar{v} = \dot{r}\,\bar{e}_r + r\dot{\theta}\,\bar{e}_\theta \\ \bar{a} = [\ddot{r} - r\dot{\theta}^2]\,\bar{e}_r + [2\dot{r}\dot{\theta} + r\ddot{\theta}]\,\bar{e}_\theta\end{array}\right\}$ . . . . (a)

Let $\boxed{r = 60t^2 - 20t^3}$ at $t = 1s$ $r = 40$ mm
$\dot{r} = 120t - 60t^2$ at $t = 1s$, $\dot{r} = 60$ mm/s
$\ddot{r} = 120 - 120t$ at $t = 1s$, $\ddot{r} = 0$

$\boxed{\theta = 2t^2}$ at $t = 1s$ $\theta = 2$ rad
$\dot{\theta} = 4t$ at $t = 1s$, $\dot{\theta} = 4$ rad/s
$\ddot{\theta} = 4$ at $t = 1s$, $\ddot{\theta} = 4$ rad/s.

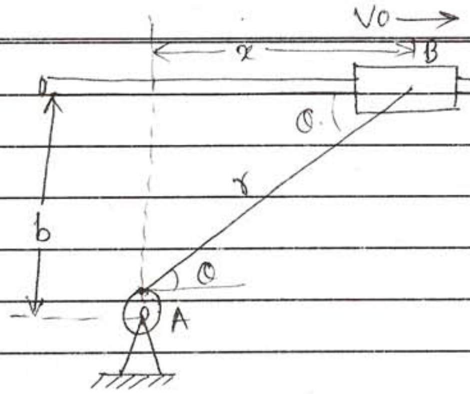
$\therefore \left.\begin{array}{l}\bar{v} = 60\,\bar{e}_r + 160\,\bar{e}_\theta \quad (170.88) \\ \bar{a} = [0 - 40\times4^2]\,\bar{e}_r + [2\times60\times4 + 40\times4]\,\bar{e}_\theta \\ \quad = -640\,\bar{e}_r + 640\,\bar{e}_\theta \quad (905.1)\end{array}\right\}$ from eq^n (a)

$\therefore v = \sqrt{60^2 + 160^2} = 170.88$ m/s
$a = \sqrt{640^2 + 640^2} = 905.1$ m/s^2



Initial condition.
$\boxed{\text{at } t = 0, r = 0, \theta = 0.}$
at $t = 1$, $r = 40$ mm, $\theta = 2$ rad.

Pro:-



collar moving (⟶)
Wire winding at A.

V_0 = const. Find $\ddot{\theta}$ in terms of V_0, b & θ.

$x = r\cos\theta$, $b = r\sin\theta$ $r = b/\sin\theta$.

∴ $x = b\cos\theta/\sin\theta = b\cot\theta$. ← diff. w.r.t 't'

eq (a) ⟵ $\dot{x} = -b\,\mathrm{cosec}^2\theta\ \dot{\theta}$ → (dθ/dt) → Multiplicatⁿ of two fⁿ..

$\ddot{x} = -b\{[2\mathrm{cosec}\theta \times [-\mathrm{cosec}\theta\cdot\cot\theta]\dot{\theta}]\ \dot{\theta} + \mathrm{cosec}^2\theta \times \ddot{\theta}]\}$

∴ $\ddot{x} = 0$ as V_0 = const.

∴ $0 = -b\,\mathrm{cosec}^2\theta\,[-2\cot\theta\cdot\dot{\theta}^2 + \ddot{\theta}]$

$\mathrm{cosec}^2\theta \neq 0$, as $\frac{1}{\sin\theta^2} \neq 0$, $\theta \neq \infty$.

∴ $-2\cot\theta\cdot\dot{\theta}^2 + \ddot{\theta} = 0$

∴ $\ddot{\theta} = 2\cot\theta\cdot\dot{\theta}^2$

$= 2\cot\theta \times [\dot{\theta}^2]$

From (a), $\dot{\theta} = \frac{\dot{x}}{-b\,\mathrm{cosec}^2\theta} = \frac{V_0}{-b\,\mathrm{cosec}^2\theta}$

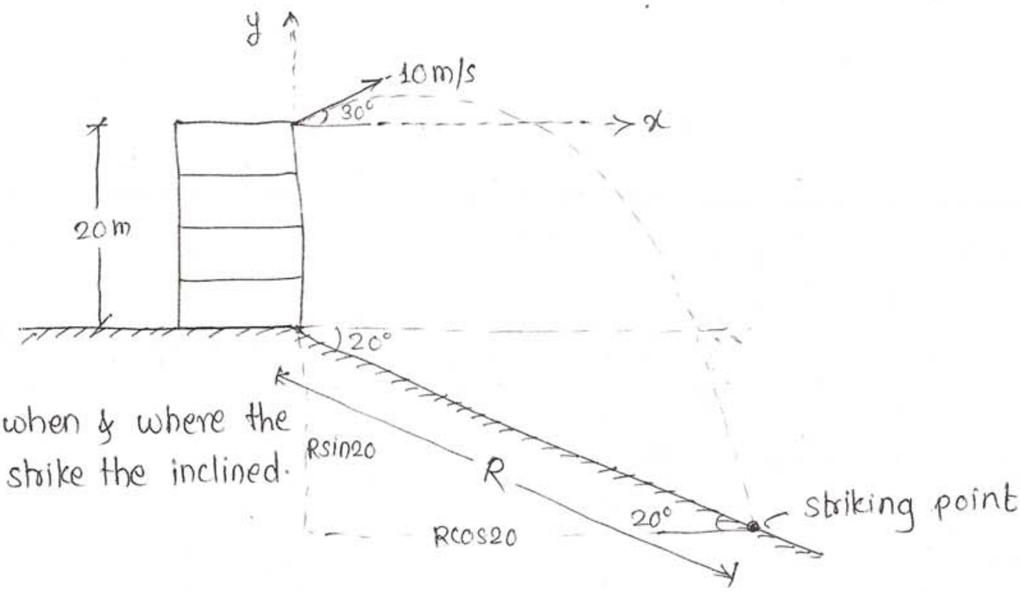
∴ $\ddot{\theta} = 2\cot\theta\left[\frac{\cancel{x}\ \cancel{\dot{x}}\ V_0}{-b\,\mathrm{cosec}^2\theta}\right]^2$

$= \frac{2\cos\theta}{\sin\theta} \times \left(\frac{-V_0}{b}\right)^2 \sin^4\theta$

$$\boxed{\ddot{\theta} = \frac{2V_0^2\ \sin^3\theta\cdot\cos\theta}{b^2}}$$

Pro:-

Pro:-



Determine when & where the stone will strike the inclined.

$x = +R\cos 20$

$y = \cancel{R} - (20 + R\sin 20)$

$$y = x\tan\alpha - \frac{g x^2}{2 v_0^2 \cos^2\alpha}$$

$$\therefore \cancel{R} - (20 + R\sin 20) = R\cos 20 \cdot \tan 30 - \frac{g\,(R\cos 20)^2}{2(10)^2 \cdot \cos^2 30}$$

$$\therefore R\cos 20 \cdot \tan 30 + R\sin 20 - R^2(0.0577) + 20 = 0$$

$\therefore \cancel{0.0577R^2 + 0.884R + 20 = 0}$

$\therefore \cancel{R = 0.0578R^2 - 0.2005R - 20 = 0}$

$\therefore \cancel{R = -16.96m}, R = 20.44\ m$

$\Rightarrow \boxed{0.0577R^2 - 0.884R - 20 = 0}$ → Remember. *

$\therefore R = (27.8, -12.47)\ m$

$R = -12.47$ is not possible.

$$\text{time} = \frac{\text{Hori. Dist}}{\text{Hori. Vel.}}$$ } only for horizontal direction as vel. is const. in hori direct^n & not for vertical direct^n., $V \neq C$

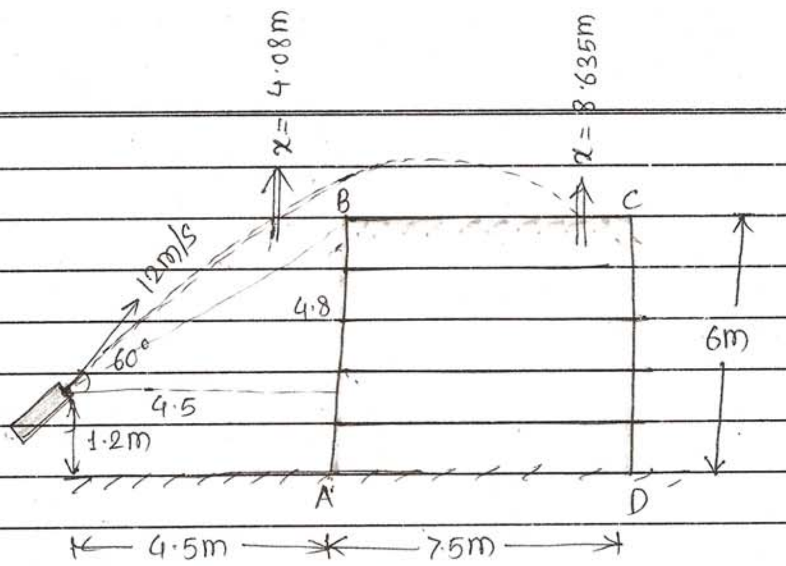
$$= \frac{R\cos 20}{10\cos 30}$$

$$= \frac{27.8 \times \cos 20}{10\cos 30} \quad \text{or} \quad \cancel{\frac{12.47\cos 20}{10\cos 30}}$$

$= 3$ Sec. $\quad \cancel{= 1.353\ \text{Sec}}$.

$\therefore \boxed{R = 27.8m\ ;\ t = 3\ \text{Sec}}$ ← Ans.

Pro:-



Find Where the stream will strike the roof.
Check wheather it clears the edge of the roof.
Also find the range of values of initial vel. for which will fall on the roof.

$y = 4.8m$

$y = 4.8 = x \tan 60 - \frac{g x^2}{2(12)^2 \cos^2 60}$

$\Rightarrow 4.8 - 1.732x + 0.1362 x^2 = 0$

$\therefore x = 8.635m, 4.08m.$

But $x = 4.08$ is not possible.

$\therefore x = 8.635m.$

check that $y_{x=4.5} > 4.8.$

$\therefore y_{x=4.5} = 4.5 \tan 60 - \frac{g(4.5)^2}{2(12)^2 \cos^2 60}$

$= 5.035 > 4.8.$

Vo for striking B, $\therefore x = 4.5, y = 4.8m$

$\therefore 4.8 = 4.5 \tan 60 - \frac{g(4.5)^2}{2(V_0)^2 \cos^2 60}$

$\therefore \boxed{V_0 = 11.52 \text{ m/s.}}$

Vo for striking C, $x = 12, y = 4.8$

$\therefore 4.8 = 12 \tan 60 - \frac{g(12)^2}{2(V_0)^2 \cos^2 60}$

$\boxed{\therefore V_0 = 13.29 \text{ m/s.}}$

$\therefore$ Range of Vo $\Rightarrow 11.52 \cancel{\leqslant} V_0 \cancel{\leqslant} 13.29$ m/s.

$11.52 \leq V_0 \leq 13.29$ m/s.