ENGINEERING MECHANICS

Notes by-

Pravin S Kolhe,

BE(Civil), Gold Medal, MTech (IIT-K)

Assistant Executive Engineer,

Water Resources Department,

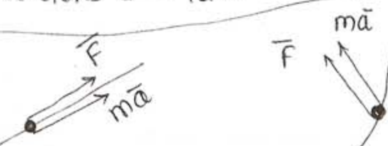
www.pravinkolhe.com

Pro:- (accel^n / time) ← KINETICS OF PARTICLE

(any problem involves accel^n, time etc directly, Use Newtons II^nd law.)

NOTE

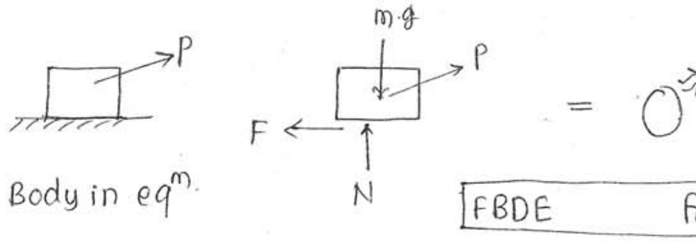
(Forces & dist = WE eq^n)



Newton's II^nd law ⇒ $\bar{F} = m\bar{a}$

$\bar{F}$ = Resultant of ext. Force

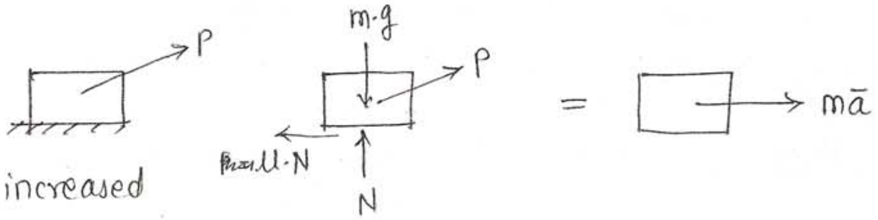
$m\bar{a}$ = eff. force.



→ as body is in eq^m.

Body in eq^m.

FBDE   Free Body Dia. Eq^n.



P increased so body moves.

$$\bar{F} = m \cdot \bar{a}$$

~~Eff. force~~

Resultant of ext. force = Eff. force.

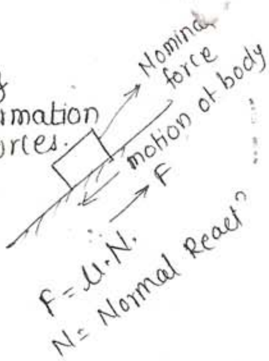
Some thing applied to body ⇒ effective action.

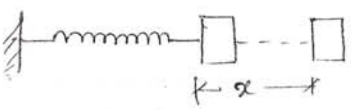
			statics
$\Sigma F_x = m \cdot a_x$	$\Sigma F_n = m \cdot a_n$	$\Sigma F_r = m \cdot a_r$	$\Sigma F_x = 0$
$\Sigma F_y = m \cdot a_y$	$\Sigma F_t = m \cdot a_t$	$\Sigma F_\theta = m \cdot a_\theta$	$\Sigma F_y = 0$

Forces on body :- externally

- gravity — mg (↓) Verti (±) ↕
- Rod — (T or C)
- String — (Tension)
- (±) Spring — (C or T) = $\frac{1}{2}k(x_1^2 - x_2^2)$ (x_1 & x_2 = Initial & Final deformation) not applied forces.
- Friction — (always opposite to motion) (-ve)
- Applied force — (Any)

Normal forces do not do work.





$F = kx$

k = spring const ⇒ ~~Displacement per unit force applied~~ force per unit Displacement (N/m).

x = Deformat^n or ~~di~~ elongation or deformation.

EM-D

Er. Pravin Kolhe (B.E Civil)

(B)

Methods of Tunnelling

Type of soil :-

a) Hard rock or fully self - supporting soil -

i) Igneous

2) Sedimentary

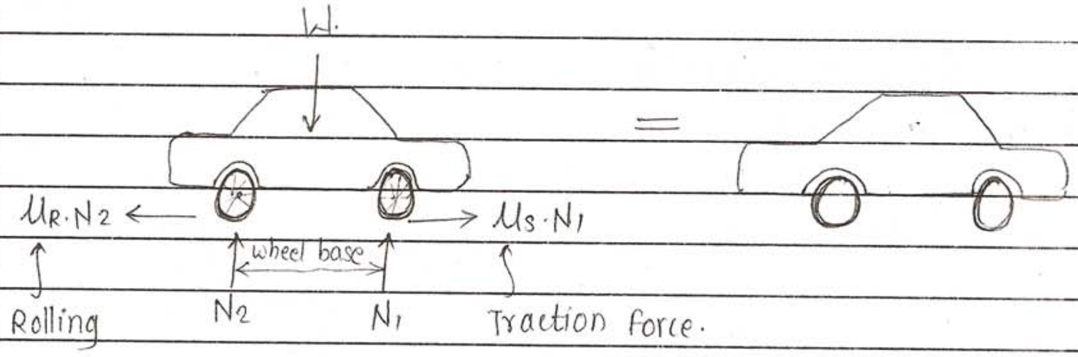
3) Metamorphic.

b) Soft soil requiring temporary supports during & after construction :-

1) Running ground - requiring all side support.

Pro: Determine the max. theoritical speed that may be achieved over a dist. of 60 m by a car starting from rest if $\mu_s = 0.8$ betⁿ tyre ~~of~~ & pavement. & 60% wt. of the car is on its front wheel. Assume front wheel drive.

Solⁿ :- Front wheel drive :-



Neglect friction, $\mu_R = 0$.

$\Sigma F_x = ~~0~~ \cdot m\bar{a}$

$\therefore \mu_s \cdot N_1 = m\bar{a}$

$\therefore 0.8 \times (0.6W) = \frac{W}{g} \cdot a$

$\therefore \boxed{a = 0.48\,g}$

$V^2 = u^2 + 2as$. $u = 0$ ~ at Rest.

$\therefore V^2 = 0 + 2 \times 0.48 g \times 60$

$= ~~7.589\ m/s~~\ 565.056\ m/s$

$= ~~\frac{7.589 \times 10^3}{60 \times 60}~~ \Rightarrow V = 23.77\ m/s$

$V = 85.6\ km/hr.$

29/10/01

Pro

* Mass of aeroplane = 25 ~~kg~~. Mg.   nano Micro Mili   kilo
If it develops thrust = 40 kN   n μ m 0 K M G.
The drag on the plane is $2.25V^2$, D (N), V (m/s)
The plane becomes airborne at a speed of 240 km/hr. Find the length of the runway required. & time for take off.

$t = 0$ $\qquad$ $t = ?$

$V = 0$ $\qquad$ $V = 240$ km/hr $= 66.67$ m/s

$x = 0$ $\qquad$ $x = ?$

$$T - D = m \cdot a$$

$$\therefore 40\times10^3 - 2.25V^2 = 25000 \cdot a \quad \text{--- (a)}$$

$$= 25000 \cdot \frac{dv}{dx} \times v$$

$\left(a = \frac{dV}{dt}\right) \rightarrow$ Calculation of time

$\left(a = v \cdot \frac{dv}{dx}\right) \rightarrow$ calculat^n of 'x'.

~~$40\times10^3 dx - 2.25v^2 dx = 25000 dv$~~

$\therefore$ ~~$25000 \frac{dv}{v^2}$~~

$$\therefore \int_0^x dx = 25000 \int_0^{240/3.6} \frac{v\,dv}{40000 - 2.25v^2}$$

$$\int \frac{f'(x)}{f(x)} = \log f(x)$$

Diff. $\rightarrow$

$$= \frac{25000}{-4.5}\left[\ln(40000 - 2.25v^2)\right]_0^{240/3.6}$$

$$= \frac{25000}{-4.5}\left[\ln\left(\frac{40000 - 2.25\left(\frac{240}{3.6}\right)^2}{40000}\right)\right].$$

$$\therefore \boxed{x = 1598 \text{ m}}$$

Time reqd. for take off :- from (a)

$$dt = \frac{25\times10^3 \cdot dv}{40\times10^3 - 2.25v^2}$$

$$\int_0^t dt = 25\times10^3 \int_0^{66.67} \frac{dv}{40\times10^3 - 2.25v^2}$$

$$\boxed{\int \frac{dx}{a - bx^2} = \frac{1}{\sqrt{ab}} \tanh^{-1}\left(\frac{x\sqrt{ab}}{a}\right)}$$

$\tanh^{-1}(0) = 0.$

$$\therefore t = 25\times10^3 \times \frac{1}{\sqrt{40\times10^3\times2.25}} \left| \tanh^{-1} \frac{v\sqrt{40\times10^3\times2.25}}{40\times10^3} \right|_0^{66.67}$$

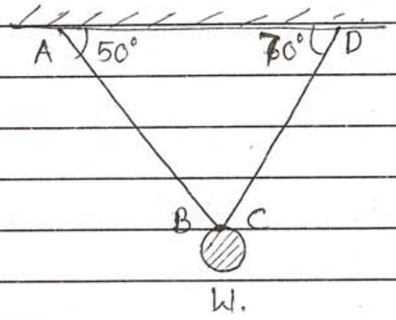
$$= 83.333 \left[\tanh^{-1} \left(\underbrace{66.67\times7.5\times10^{-3}}_{0.5} \right) \right] - 0$$

$$\boxed{t = 45.77 \text{ Sec.}}$$

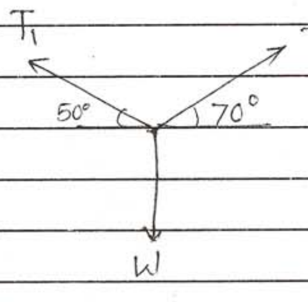
Pro:- A small sphere of wt 'w' is held as shown by two wires AB & CD if AB is cut find the tension in the CD -

i) Before AB is cut.

ii) Immediately after AB is cut.



Soln:-



$-T_1\cos50 + T_2\cos70 = 0 \quad \text{(a)}$

$T_1\sin50 + T_2\sin70 = W \quad \text{(b)}$

$\therefore T_2 = T_1\left(\frac{\cos50}{\cos70}\right)$

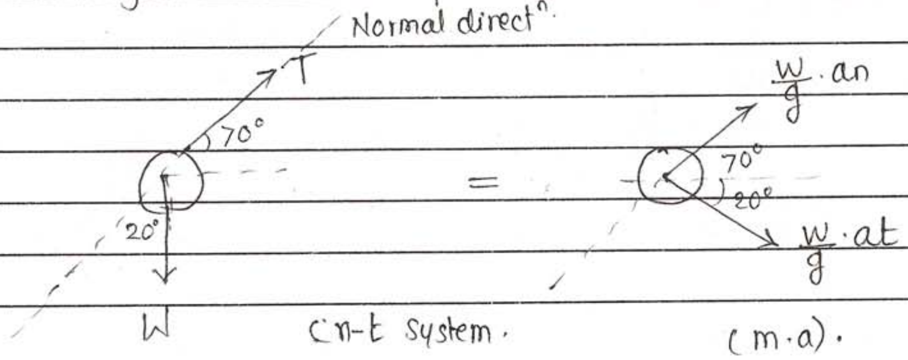
$\therefore T_1\sin50 + T_1\left(\frac{\cos50}{\cos70}\right)\sin70 = W.$

$\therefore T_1 = 0.3949\,W.$

$\boxed{T_2 = 0.742\,W.}$

After the cut :-

A C goes on circular path.



(n-t system.

(m.a).

$\therefore T\sin 70 - W = \frac{W}{g} \cdot a_n \cdot \sin 70 - \frac{W}{g} \cdot a_t \times \sin 20$

$\therefore$ or resolving along normal direction.

$\therefore T - W\cos 20 = W/g \cdot a_n$

but $a_n = \frac{v^2}{\rho}$, $a_t = \dot{v}$

$\bar{a} = \dot{v} e_t + \frac{v^2}{\rho} \cdot \bar{e}_n$

$= a_t \cdot \bar{e}_t + a_n \cdot \bar{e}_n$

Immediately after cut, $v = 0 \Rightarrow a_n = 0$

$\therefore T = W\cos 20$

$\therefore \boxed{T = 0.94W}$

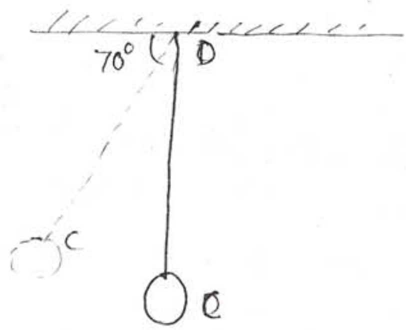
Note:-

If accel^n is to be calculated, that rate of change of speed, Resolve along tangential direct^n.

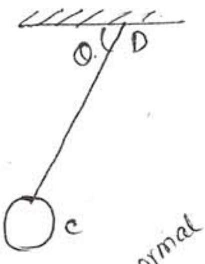
$\therefore W\sin 20 = W/g \cdot a_t$

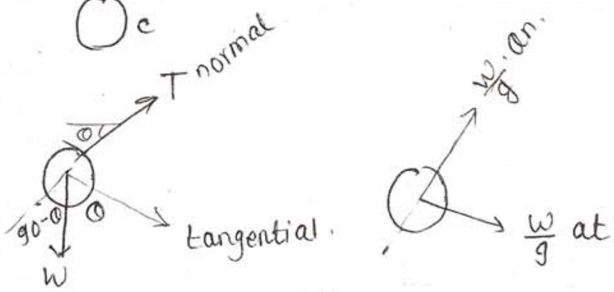
$\therefore \boxed{a_t = g\sin 20}$

Pro: for the previous problem find the speed of the sphere as CD reaches a vertical position.



General Position.





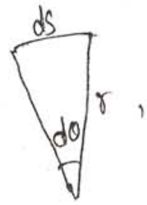
$\therefore W\cos\theta = W/g \cdot a_t$

$\therefore a_t = g\cos\theta$.

$\therefore g\cos\theta = a_t$

$= \frac{dv}{dt}$ → as $\frac{ds}{d\theta} = r$ -- for conviniance

$= \frac{dv}{d\theta} \left(\frac{d\theta}{ds}\right) \frac{ds}{dt}$



, $r = \frac{ds}{d\theta}$.

$\therefore g\cos\theta = \frac{dv}{d\theta} \cdot \frac{1}{r} \cdot v$.

Speed ⇒ No direct^n only magnitude
vel. ⇒ direct^n + Magnitude.

$r \cdot g\cos\theta \, d\theta = v \cdot dv$.

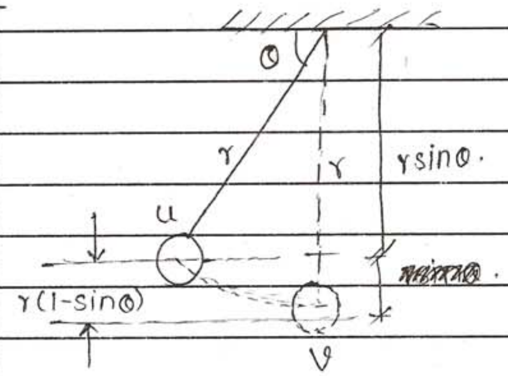
$\therefore gr \int_{70}^{90} \cos\theta \, d\theta = \int_0^v v \cdot dv$.

$\therefore gr \left[\sin\theta\right]_{70}^{90} = v^2/2$

$\therefore gr(1-\sin 70) = \frac{v^2}{2}$ ⇒ $\boxed{v^2 = 2gr(1-\sin\theta)}$

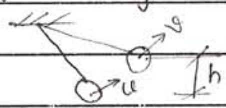
$\therefore \boxed{v = 0.347\sqrt{g \cdot r}}$

Imp. Note:-



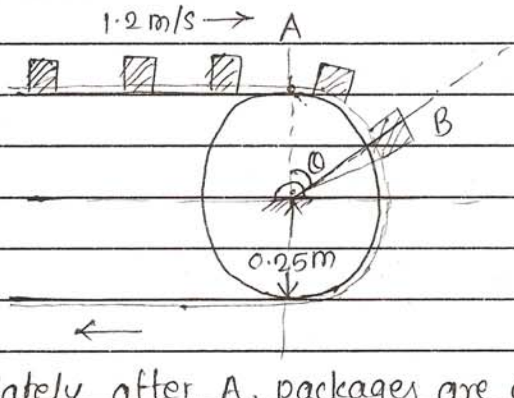
$\boxed{v^2 = u^2 + 2gr(1-\sin\theta)}$

$v^2 = u^2 + 2gh$.

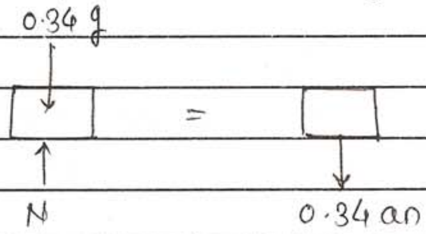


In such case, the motion is same as free fall. accel^n is not const. but final result will same.

Pro:- Series of small packages each of 0.34 kg is discharged from a conveyor belt. If $\mu_s = 0.4$, $\mu_k = 0.3$ bet^n each packet & belt Find the force exerted by the belt on a package just after it passes pt. 'A'. Also find θ, where each packet will leave the belt.



Sol^n:- Immediately after A, packages are on circular path.

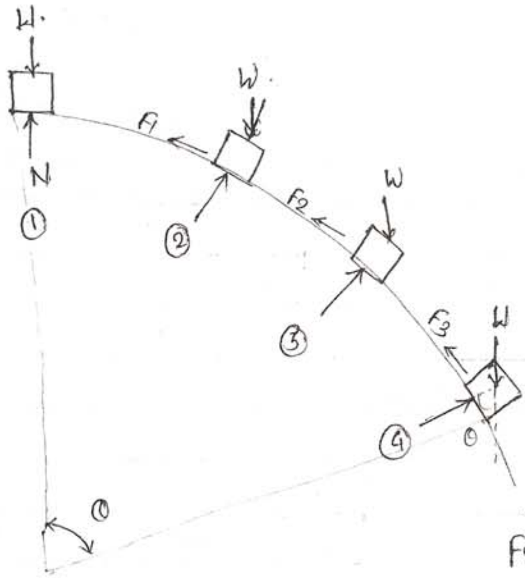


$\therefore 0.34g - N = 0.34\,a_n$

$a_n = \frac{v^2}{\rho} = \frac{1.2^2}{0.25} = 5.76$

$\therefore -N = 0.34 \times 5.76 - 0.34 \times g$.

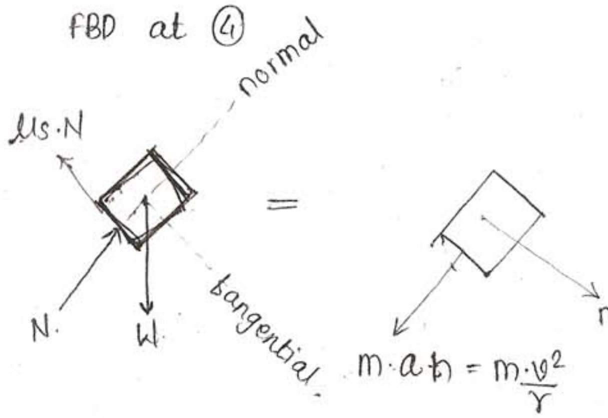
$\boxed{\therefore N = 1.337 \text{ N}}$



Contact is about to get lost
contact is still there but will be lost on next moment.
Limiting value of frictⁿ = μ_s.

$F_3 = \mu_s \cdot N$.

FBD at ④



$m \cdot a_n = m \cdot \frac{v^2}{r}$

$m a_t = 0 \Rightarrow$ since it is still part of belt which is moving with a speed of 1.2 m/s.
∴ No change of speed.

$\therefore N - W\cos\theta = -m\frac{v^2}{r}$ & $\mu_s \cdot N - W\sin\theta = 0$.

$\therefore$ ~~$\mu_s\ N = \frac{mv^2}{r}$~~ $N = W\cos\theta - m\frac{v^2}{r} \Rightarrow$

$\Rightarrow \mu_s\left[W\cos\theta - \frac{mv^2}{r}\right] - W\sin\theta = 0$

$\therefore \mu_s\left[W\cos\theta - \frac{W}{g}\cdot\frac{v^2}{r}\right] - W\sin\theta = 0$

$\therefore \sin\theta = \mu_s\left[\cos\theta - \frac{v^2}{g \cdot r}\right]$

$\mu_s = 0.4,\ v = 1.2,\ r = 0.25$.

$\therefore \sin\theta = 0.4\left[\cos\theta - \frac{1.2^2}{9.81 \times 0.25}\right]$

$\therefore \sin\theta = 0.4\cos\theta - 0.234$ ⇒ $\sin^2\theta = (0.4\cos\theta)^2 + (0.234)^2 - 2(0.4\cos\theta \times 0.234)$

$\therefore \sin\theta - 0.4\cos\theta + 0.236 = 0$