ENGINEERING MECHANICS

Notes by-

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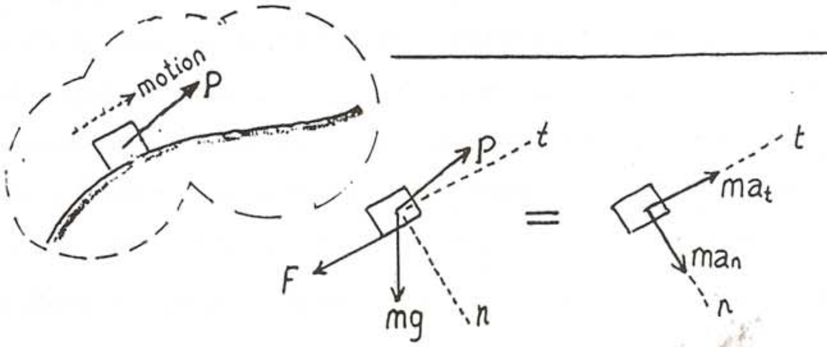
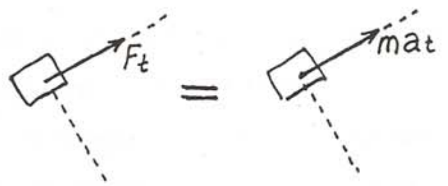
KINETICS OF PARTICLES

Impress strongly that $\bar{F} = m\,\bar{a}$ is the governing principle. Extract from it, the Energy and Momentum methods of attack and treat them only as 'subordinates' though they are powerful. Force-acceleration method, Work-energy principle, and Impulse-momentum principle are to be derived successively and explained clearly. Which one of these three methods works best for a given problem needs to be conveyed.



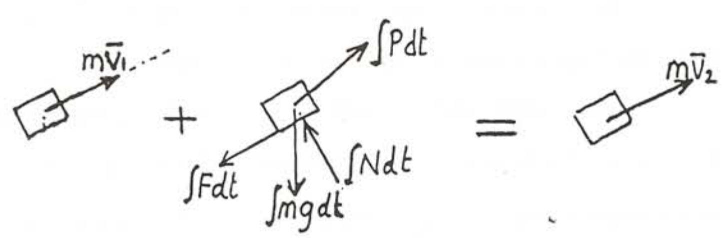
$$\bar{F} = m\bar{a}$$



$$F_t = ma_t$$
$$= m v \frac{dv}{ds}$$
$$\int_1^2 F_t\,ds = \int_{v_1}^{v_2} m v\,dv$$
$$U_{1-2} = \tfrac{1}{2} m v_2^2 - \tfrac{1}{2} m v_1^2$$

$$T_1 + U_{1-2} = T_2$$

Note that $(Imp)_{1-2}$ is nothing but integral of all the external forces in a given time interval, ie $\int F dt$



$$m\bar{V}_1 + (Imp)_{1-2} = m\bar{V}_2$$

Concept of FBDE is to be strongly emphasized, dumping the De'Alembert's principle of dynamic equilibrium. Do not drag in the terms 'Inertia', 'Centripetal force', 'Centrifugal force', 'Potential energy', and 'Conservation of energy'. These are all unwanted terms. FBDE is nothing but a diagrammatic representation of $\bar{F} = m\,\bar{a}$ where the LHS shows all the external forces $\bar{F}$ and RHS shows the effective forces $m\,\bar{a}$. We may split $m\,\bar{a}$ into scalar components, ma_x and ma_y, or ma_n and ma_t, or ma_r and ma_θ depending on the nature of the problem.

Be aware of the fact that the acceleration (as a function of time, position, or velocity of combination any of them) that is dealt with in *Kinematics of particles* is determined by the forces which act on moving particles/bodies and is computed from the equations of Kinetics.

Take up a problem and solve it using $\bar{F} = m\ \bar{a}$, work-energy principle, and impulse-momentum principle so that one can appreciate the superficial and inherent advantages/ disadvantages of each method (the second problem on page no. 11 will serve the purpose).

A clear-cut idea of how the friction force acts on a particle has to be conveyed. Clearly make the distinction between coefficients of static and kinetic friction. Give a briefing on four wheel drive, rear wheel drive, and front wheel drive of vehicles and also how the frictional forces act on the tyres. Distinguish between limiting static equilibrium, non-limiting static equilibrium, and relative equilibrium. Emphasize on the kind of frictional force that acts on a particle ($\mu_s N$ or $\mu_k N$) when it is in an absolute motion or in a relative motion with respect to another particle/rigid body.

Work done by various forces like, gravitation, friction, spring, and applied ones are to be discussed before Problems on work-energy principle are taken up.

A briefing is to be given on Conservative and Non conservative force fields. This may be done without bringing in the concepts of *potential function* and *gradient of a function*. Focus on the advantages of Work-energy principle; (i) computation of accelerations is avoided and instead it leads directly to velocities. (ii) All quantities involved in the equation are scalars and (iii) Forces that do no work are automatically eliminated from the solution.

Be very cautions about absolute and relative velocities of particles while employing the Impulse-momentum principle.

Never assign a special status to *Direct central impact.* It is only a specific phenomenon where the Impulse-momentum principle is used.

When coefficient of restitution (e) is being introduced, make it clear that e is not the modulus of the ratio of relative velocity of two particles after impact to the relative velocity before impact even though e boils down to that ratio. Fundamentally, $e = \int R dt / \int P dt$, i.e., the ratio of magnitudes of the impulse in the restitution period to the impulse in the deformation period.

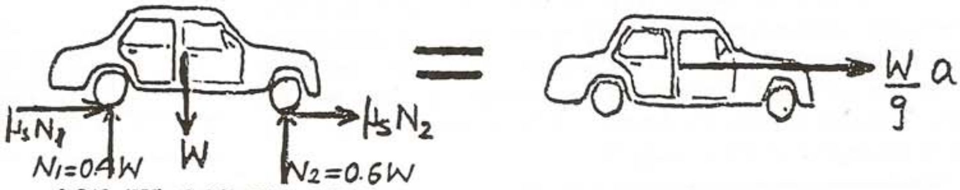
Do not grab any one of the three methods [$\bar{F} = m\ \bar{a}$, $T_1 + U_{1-2} = T_2$, $m\ \bar{V}_1\ (Imp)_{1-2} = mV_2$) as the only method of attack for solving a problem. The shortest solution may demand a judicious choice of one or more of these three methods.

KINETICS OF PARTICLES

Prob.: Determine the maximum theoretical speed that may be achieved over a distance of *60 m* by a car starting from rest, if coefficient of static friction is *0.80* between the tires and the pavement and if *60* percent of the weight of the car is distributed over its front wheels and *40* percent over its rear wheels. Assume *(a)* four-wheel drive, *(b)* front-wheel drive, *(c)* rear-wheel drive.

If W is the weight of the car, 0.6W and 0.4W are the loads on the front and rear wheels respectively. Traction on the car is the frictional force developed on the driving wheels (i.e. $\mu_s N$, where N is the normal reaction on the driving wheels). $\mu_s = 0.8$

(a) Four wheel drive



$\bar{F} = m\bar{a} \Rightarrow$ 0.8(0.4W)+0.8(0.6W) = (W/g)a

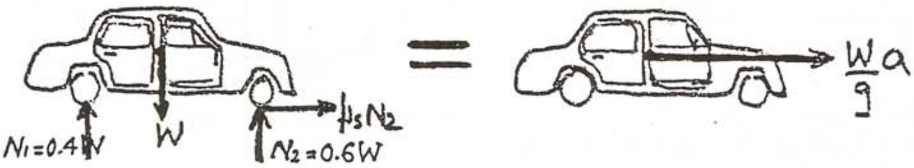
$\Rightarrow$ a = 0.8 g

In a distance of 60m, speed reached, starting from rest, is given by

$V^2 = 0 + 2(0.8g)(60) \Rightarrow$ **V = 30.7 m/s or 110.5 km/h**

(b) Front wheel drive (Traction of $\mu_s N$ acts only on front wheels)

$\bar{F} = m\bar{a}$ gives



$\bar{F} = m\bar{a} \Rightarrow$ 0.8(0.6W) = (W/g)/a

$\Rightarrow$ a = 0.48 g

$V^2 = u^2 + 2as \Rightarrow V^2 = 0 + 2(0.48g)60$

† **V = 23.8 m/s or 85.6 km/h**

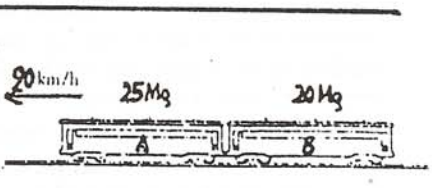
(c) Rear wheel drive

An approach similar to that in (b) gives a speed of

† **V = 19.4 m/s or 69.9 km/h**

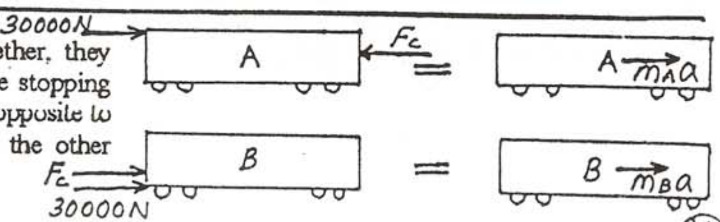
† These speeds are only theoretical (as specified in the question) because we have neglected the effect of rolling friction on the driven wheels; i.e. Rear wheels in (b) and front wheels in (c).

Prob.: A light train made up of two cars is traveling at 90 km/h when the brakes are applied to both cars. If car *A* has a mass of *25* Mg and car *B* has a mass of *20* Mg, and if the braking force is *30* kN on each car, determine *(a)* the distance traveled by the train before it comes to a stop, *(b)* the force in the coupling between the cars while the train is slowing down.



Since both cars are coupled together, they both have same deceleration, **a** during the stopping operation. Braking force on each car acts opposite to the motion. Each car exerts a force on the other through the coupling.

F = ma for the cars gives



$30000 - F_c = 25000\ a$ (i)

$30000 + F_c = 20000\ a$ (ii)

Solving (i) and (ii) $a = 1.333\ m/s^2$ and $F_c = -3.333$ kN

Negative sign for F_c indicates that assumed direction of coupling force is to be reversed. That is, B 'pulls back' A or A 'drags' B.

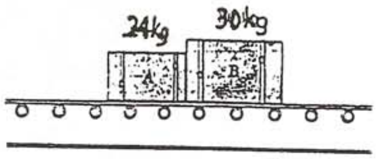
$V^2 = u^2 - 2aS$ can be used to find the stopping distance with V= 0 u = (90/3.6) m/s and $a = 1.333\ m/s^2$

$0 = (90/3.6)^2 - 2(1.333)S$

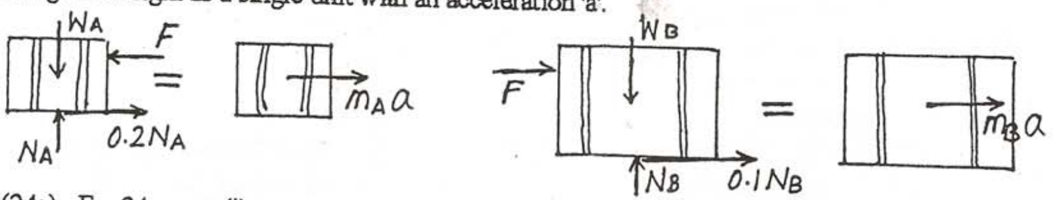
$\Rightarrow$ **S = 234.4 m**

Force in the coupling, **F_c = 3.333 kN (tensile)**

Prob.: Two packages are placed on a conveyor belt which is at rest. The coefficient of kinetic friction is *0.20* between the belt and package *A*, and *0.10* between the belt and package *B*. If the belt is suddenly started to the right and slipping occurs between the belt and the package, determine *(a)* the acceleration of the package, *(b)* the force exerted by package *A* on the package *B*.



When the belt is suddenly started to the right, the tendency of package B to stay in its place is more than that of package A since B is smoother. This results in A exerting a force on B rightward and the packages starting to the right as a single unit with an acceleration 'a'.



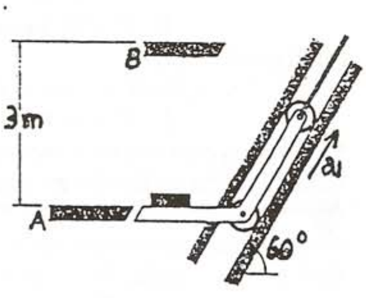
$0.2\ (24g) - F = 24a$ (i) $0.1\ (30g) + F = 30a$ (ii)

From (i) and (ii) **$a = 1.417\ m/s^2$ (→)** and **F = 13.1 N (→)**

Note that frictional force acts in the direction of motion. This is because the relative motion of the packages with respect to the belt is leftward and hence the frictional force rightward.

Prob.: In a manufacturing process, disks are moved from level *A* to level *B* by the lifting arm shown. The arm starts from level *A* with no initial velocity, moves first with a constant acceleration a_1, as shown, then with a constant deceleration a_2, and comes to a stop at level *B*. If the coefficient of static friction between the disks and the arm is *0.30*, determine the largest allowable acceleration a_1 and the largest allowable deceleration a_2 if the disks are not to slide.

If the disks are to be moved from level *A* to level *B* in the shortest possible time without sliding, determine *(a)* the maximum velocity reached by the lifting arm, *(b)* the time required to move each disk.

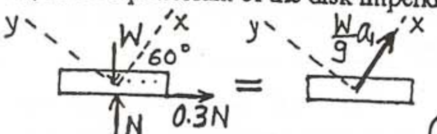


When the constant acceleration reaches a limiting value a_1, static equilibrium of the disk impends to get disturbed with the disk about to slide outwards.

$\Sigma F_x = ma_x \Rightarrow 0.3\ N \cos 60 + N \cos 30 - W \cos 30 = Wa_1/g$

$\Sigma F_y = ma_y = 0 \Rightarrow N \sin 30 - 0.3N \sin 60 - W \sin 30 = 0$

Solving, we get **$a_1 = 12.25\ m/s^2$ (∠60°)**

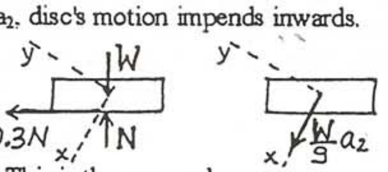


When the constant deceleration reaches a limiting value a_2, disc's motion impends inwards.

$\Sigma F_x = ma_x \Rightarrow 0.3\,N\cos60 - N\cos30 + W\cos30 = Wa_2/g$

$\Sigma F_y = ma_y = 0 \Rightarrow 0.3\,N\sin60 + N\sin30 - W\sin30 = 0$

Solving we get $\mathbf{a_2 = 3.873\ m/s^2}$ (60°)

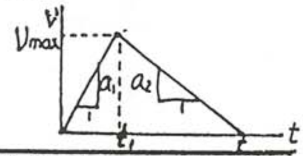


Distance to be covered by the disc is 3/sin60 or 3.464 m. This is the area under the v-t curve.

$V_{max} = a_1t_1 = a_2(t - t_1) \Rightarrow t_1 = 0.24t$

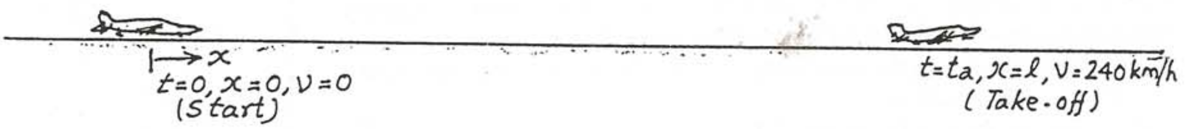
$\frac{1}{2}t(a_1t_1) = 3.464\text{m} \Rightarrow \frac{1}{2}t[(12.25)(0.24t)] = 3.464$ or $\mathbf{t = 1.535\ s}$

and $\therefore \mathbf{V_{max} = 4.51\ m/s}$

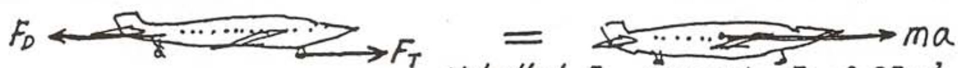


Prob.: An airplane has a mass of *25* Mg and its engines develop a total thrust of *40* kN during take-off. If the drag *D* exerted on the plane has a magnitude $D = 2.25\,v^2$, where v is expressed in metres per second and D in newtons, and if the plane becomes airborne at a speed of *240 km/h*, determine the length of runway required for the plane to take off. Also determine the time required for the plane to take-off.

The plane is acted upon by a force which is a function of velocity and by another constant force. They are the drag force, F_D and the thrust force F_T respectively. Acceleration is obviously a variable.



Consider the motion of the plane on the runway (forces in the direction of motion only are shown.)



Note that $F_T = 40000$ N, $F_D = 2.25\,v^2$

$F = ma \Rightarrow 40000 - 2.25\,V^2 = 25000\,a$ (i)

To find the runway length a is expressed as vdv/dx, variables are separated and integration performed with v running from 0 to 240 km/h and x from 0 to ℓ.

$$\int_0^{\ell} dx = 25000\int_0^{(240/3.6)} \frac{v\,dv}{40000 - 2.25v^2}$$

which on simplification becomes $\ell = 2226\ \ln(40000 - 2.25v^2)$ or $\mathbf{\ell = 1598\ m}$

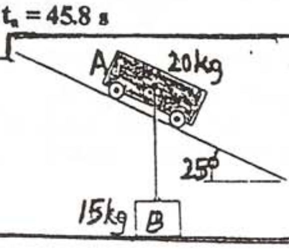
To find the time required a is expressed as dv/dt and integration performed on (i)

Note that $\int \frac{dv}{a^2 - v^2} = \frac{1}{2a}\ln\left(\frac{a+v}{a-v}\right)$

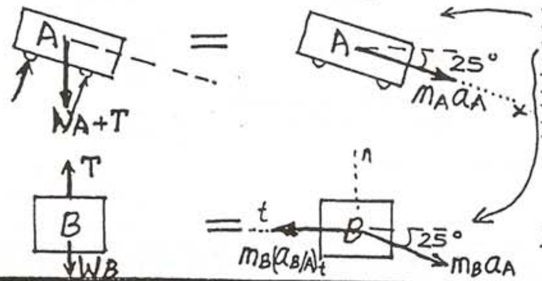
$$\int_0^{t_a} dt = 25000\int_0^{(240/3.6)} \frac{dv}{40000 - 2.25v^2}$$

$$\text{or } t_a = 25000 \cdot \frac{1}{2\sqrt{40000}}\left|\ln\left(\frac{\sqrt{40000} + v}{\sqrt{40000} - v}\right)\right|_0^{\frac{240}{3.6}} \Rightarrow \mathbf{t_a = 45.8\ s}$$

Prob.: A *15-kg* block *B* is suspended from a *2.5-m* cord attached to a *20-kg* cart *A*. Neglecting friction, determine *(a)* the acceleration of the cart, *(b)* the tension in the cord, immediately after the system is released from rest in the position shown.



Acceleration of the cart ($\bar{a}_A$) is obviously down the incline whereas acceleration of the block ($\bar{a}_B$) is to be analyzed as consisting of two parts. B being attached to A has as acceleration $a_{B/A}$. This has two components ($a_{B/A})_n$ and ($a_{B/A})_t$. This is because B swings in a circular path with respect to A. Besides this, B also has an acceleration $\bar{a}_A$ due to its being connected to A. Thus $\bar{a}_B = (\bar{a}_{B/A})_n + (\bar{a}_{B/A})_t + \bar{a}_A$



$\Sigma F_x = ma_x \Rightarrow (W_A + T)\sin 25 = m_A a_A$ (i)

$\Sigma F_n = ma_n \Rightarrow T - W_B = -m_B a_A \sin 25$ (ii)

Note that $(a_{B/A})_n = 0$ since $V_B = 0$ at release.

In (i) and (ii) $W_A = (20g)$N, $m_A = 20$ kg,

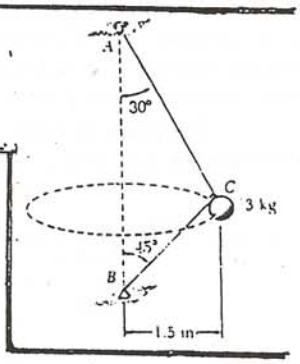
$W_B = (15g)$N, $m_B = 15$kg

From (i) and (ii)

$\mathbf{a_A = 6.4\ m/s^2}$ (↘25°) and $\mathbf{T = 106.6\ N}$

Note that cord length is a redundant data.

Prob.: Two wires *AC* and *BC* are tied at *C* to a sphere which revolves, at a constant speed *v* in the horizontal circle shown. Determine the range of the allowable values of *v* if both wires are to remain taut and if the tension in either of the wires is not to exceed *75 N*.



The sphere undergoes a horizontal circular motion. The only acceleration it possesses is normal (directed towards the centre of the circle). Tangential direction is perpendicular to the plane of this paper. Binormal direction is z.

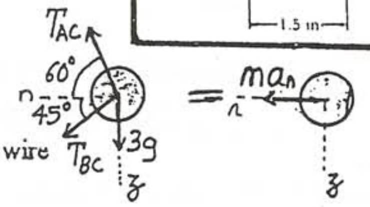
(a) When v reaches v_{max} the only 'danger' is from the tension in either of the wires exceeding 75 N

(i) Let tension in wire AC touch 75 N.

$\Sigma F_n = ma_n \Rightarrow 75\cos 60 + T_{BC}\cos 45 = 3\,v^2_{max}/1.5$

$\Sigma F_z = ma_z = 0 \Rightarrow -75\sin 60 + T_{BC}\sin 45 + 3g = 0$

Solving $T_{BC} = 50.2$ N and $v_{max} = 6.04$ m/s



(ii) If the tension in wire BC touches 75 N (keen observation of the wire geometry can avoid this check)

$\Sigma F_n = ma_n \Rightarrow T_{AC}\cos 60 + 75\cos 45 = 3\,v^2_{max}/1.5$

$\Sigma F_z = ma_z = 0 \Rightarrow -T_{AC}\sin 60 + 75\sin 45 + 3g = 0$

Solving $T_{AC} = 95.2$ N and $v_{max} = 7.09$ m/s

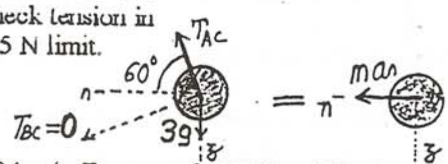
$V_{max} = 7.09$ m/s cannot be accepted since $T_{AC} > 75$ N. So $\mathbf{v_{max} = 6.04\ m/s}$

(b) When v reaches v_{min} the only 'danger' is from the tension in wire BC touching zero (i.e. no more taut). However as a casual check tension in AC corresponding to that v_{min} has to be checked for the 75 N limit.

$\Sigma F_n = ma_n \Rightarrow T_{AC}\cos 60 = 3\,V^2_{min}/1.5$

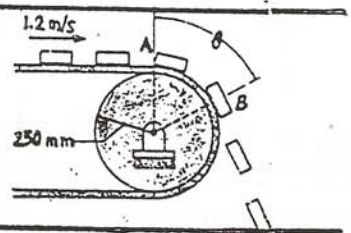
$\Sigma F_z = ma_z = 0 \Rightarrow -T_{AC}\sin 60 + 3g = 0$

Solving $T_{AC} = 34$ N and $\mathbf{V_{min} = 2.91\ m/s}$



Range is thus **6.04 m/s > V > 2.91 m/s**. If v crosses 6.04 m/s, T_{AC} exceeds 75 N and if v drops below 2.91 m/s wire BC becomes loose.

Prob.: A series of small packages, each with a mass of *340 g*, are discharged from conveyor belt as shown. Knowing that the coefficient of static friction between each package and the conveyor belt is *0.40*, determine *(a)* the force exerted by the belt on a package just after it has passed point *A*, *(b)* the angle *θ* defining the point *B* where the package first slip relative to the belt.



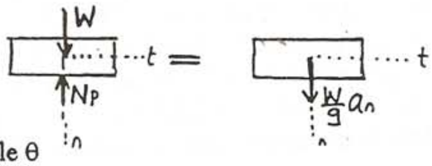
Just after the package has passed point A it is on a curvilinear path. In general it has two acceleration components a_n and a_t. However $a_t = dv/dt = 0$ since the belt moves with a constant speed of 1.2 m/s.

(a) (N_p is the force exerted by the belt on the package)

$\Sigma F_n = ma_n \Rightarrow W - N_p = (W/g)\ v^2/r$

where $W = 0.34$ g, $v = 1.2$ m/s, and $r = 0.25$ m

$\Rightarrow$ **Np=1.377 N**



(b) Let the packages starts slipping at a point B defined by angle θ

Packages are acted upon by frictional force, $\mu_s N_p$ at the instant of slip

$\Sigma F_n = ma_n \Rightarrow W\cos\theta - N_p = (W/g)\ v^2/r$

or $0.34\, g \cos\theta - N_p = 0.34(1.2)^2/0.25$ (i)

$\Sigma F_t = ma_t \Rightarrow W\sin\theta - 0.4N_p = 0$

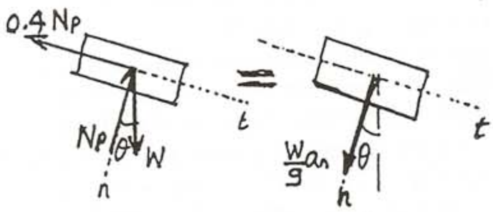
or $0.34\, g \sin\theta - 0.4N_p = 0$ (ii)

Eliminating N_p between (i) and (ii) $\cos\theta - 2.5\sin\theta = 0.5872$

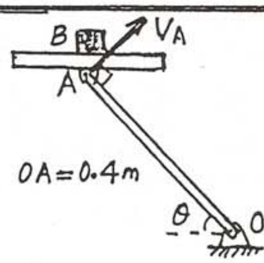
or $6.25\,(1 - \cos^2\theta) = (0.5872 - \cos\theta)^2$

$7.25\cos^2\theta - 1.1743\cos\theta - 5.9052 = 0$

Solving for $\cos\theta$ and thereafter θ, we get **$\theta = 9.24^0$**



Prob.: A small block *B* is supported by a platform at *A* to rod *OA*. Point *A* describes a circle in a vertical plane at the constant speed V_A while the platform is constrained to remain horizontal throughout its motion by the use of a special linkage (not shown in the figure). The coefficient of friction between the block and the platform are $\mu_s = 0.40$ and $\mu_k = 0.30$. Determine *(a)* the maximum allowable speed V_A if the block is not to slide on the platform, *(b)* the values of θ for which sliding is impending.



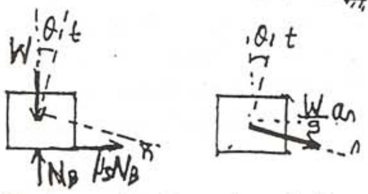
The block moves in a circular path of 0.4 m radius. We consider the limiting static equilibrium state of the block relative to the platform at the instant when sliding impends.

$\Sigma F_n = ma_n \Rightarrow W\sin\theta - N_B\sin\theta + \mu_s N_B\cos\theta = (W/g)\ V_A^2/0.4$ (i)

$\Sigma F_t = ma_t = 0 \Rightarrow -W\cos\theta + N_B\cos\theta + \mu_s N_B\sin\theta = 0$ (ii)

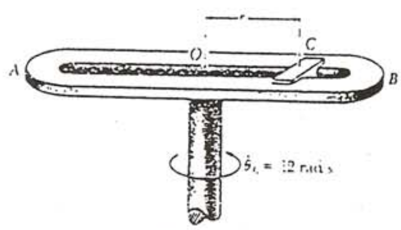
Expressing N_B in terms of W using (ii) and then substituting it in (i) we get

$$V_A^2 = \frac{0.4 g \mu_s}{\cos\theta + \mu_s \sin\theta} \quad \text{(iii)}$$



Equation (iii) gives the maximum possible V_A for which static equilibrium exists for a given θ. But we have to select that value of V_A which is smallest in the entire journey along the circular path. So we maximize the denominator of the right hand side of (iii). $f^1(\theta) = -\sin\theta + \mu_s\cos\theta = 0$ when $(\cos\theta + \mu_s \sin\theta)$ is maximum i.e. $\theta = \tan^{-1}\mu_s$ or $\theta = 21.8^0$. The other angle follows from symmetry $\theta = 180 - 21.8 = 158.2^0$. It is also obtained from (iii) by assigning a negative sign to μ_s and then maximizing the denominator as done before. $(V_A)_{max}$ is obtained by substituting $\theta = 21.8^0$ in (iii). **$(V_A)_{max} = 1.207$ m/s , $\theta = 21.8^0$, 158.2^0**

Prob.: Slider *C* has a mass of *200* g and may move in a slot cut in arm *AB*, which rotates at the constant rate $\dot{\theta}_0 = 12$ rad/s in a horizontal plane. The slider is attached to a spring of constant k = 36 N/m, which is unstretched when *r = 0*. If the slider passes through the position *r = 400* mm with a radial velocity $v_r = +1.8$ m/s, determine at that instant *(a)* the radial and transverse components of its acceleration, *(b)* its acceleration relative to arm *AB*, *(c)* the horizontal force exerted on the slider by arm *AB*.



When the slider C is at 0.4 m from the centre, the force on the slider (kx) is 36 N/m x 0.4 m or 14.4 N acting towards the centre. This is in the negative radial direction.

$F_r = ma_r \Rightarrow -14.4 = 0.2\, a_r$ or $\mathbf{a_r = -72\ m/s^2}$

At the instant shown $r = 0.4$ m, $\dot{r} = 1.8$ m/s, $\dot{\theta} = 12$ rad/s, $\ddot{\theta} = 0$ since $\dot{\theta}$ is constant

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2\,(1.8)\,(12) \Rightarrow \mathbf{a_\theta = 43.2\ m/s^2}$

$a_r = \ddot{r} - r\dot{\theta}^2 \Rightarrow -72 = \ddot{r} - 0.4\,(12)^2 \Rightarrow \mathbf{\ddot{r} = -14.4\ m/s^2}$

Note that $\ddot{r}$ is the acceleration of the slider relative to the arm AB.

Horizontal force exerted on the slider by arm AB is in the transverse direction only due to the presence of the slot. We are hence required to find F_θ

$F_\theta = ma_\theta \Rightarrow \mathbf{F_\theta = 8.64\ N}$

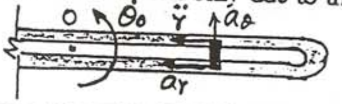


Figure shows the direction of the acceleration components.

Prob.: If the slider in the previous Problem is released with no radial velocity in the position r=500mm and if friction is neglected, determine for the position r= 300mm *(a)* the radial and transverse components of the velocity of the slider, *(b)* the radial and transverse components of its acceleration (c) the horizontal force exerted on the slider by arm *AB.*

Force in the radial direction is only due to the spring. $F_r = ma_r \Rightarrow F_r = m(\ddot{r} - r\dot{\theta}^2)$ (i)

In (i) $F_r = -kr$ when the slider is at a distance r from the centre of the arm. $\ddot{r}$ can be written as

$$\ddot{r} = \frac{d\dot{r}}{dt} = \frac{d\dot{r}}{dr}\frac{dr}{dt} = \dot{r}\frac{d\dot{r}}{dr}$$

Equation (i) takes the form $-kr = m\left(\dot{r}\frac{d\dot{r}}{dr} - r\dot{\theta}^2\right)$ or

$$-kr\,dr = m\,(\dot{r}d\dot{r} - \dot{\theta}^2 r\,dr)$$ (ii)

It is to be noted that $\dot{r}$ is the radial velocity. Such a step is necessary to enable us to integrate (ii). As r changes from 0.5 m to 0.3 m, the radial velocity $\dot{r}$ changes from zero to the required v_r. Integration of (ii) thus yields

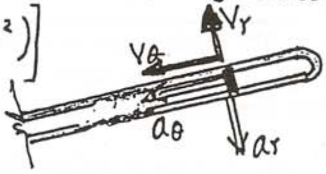
$$-\frac{36}{2}(0.5^2 - 0.3^2) = 0.2\left[\left(\frac{0 - V_r^2}{2}\right) - \frac{12^2}{2}(0.5^2 - 0.3^2)\right]$$

$$\Rightarrow \mathbf{v_r = -2.4\ m/s}$$

$V_\theta = r\dot{\theta} \Rightarrow \mathbf{v_\theta = 3.6\ m/s}$ since $r = 0.3$ m, $\dot{\theta} = 12$ rad/s

$a_r = F_r/m \rightarrow \mathbf{a_r = -54\ m/s^2}$ since $F_r = 14.4$ N as seen earlier

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \Rightarrow \mathbf{a_\theta = -57.6\ m/s^2}$ since $\ddot{\theta} = 0$, $\dot{r} = -2.4$ m/s

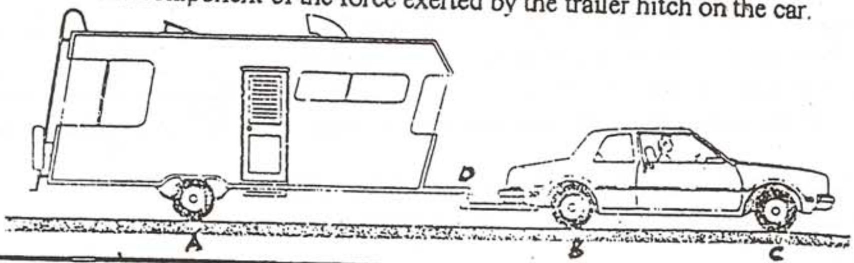


$F_\theta = m a_\theta$

Horizontal force exerted on the cylinder is the transverse component of the Force, i.e. $\mathbf{F_\theta = 11.52\ N}$

Figure shows the velocities and accelerations of the slider.

Prob.: A 1200 Kg trailer is hitched to a 1400 Kg car. The car and trailer are traveling at 72 km/h when the driver applies the braaks on both the car and the trailer .If the breaking force exerted on the car and the trailer are 5000N and 4000N, respectively, determine (a) the distance traveled by the car and trailer before they come to a stop, (b) the horizontal component of the force exerted by the trailer hitch on the car.



To find the stopping distance, car and the trailer are together taken as the system. Total breaking force of (5000N+4000N) does work on the system which has a mass of the (1200Kg +1400Kg) and brings it to rest from a speed of (72 /3.6) m/s

$T_1 = ½(1200+1400)(72/3.6)^2$, $T_2 = 0$, $U_{1\text{-}2} = -(5000 + 4000)d$

Where negative sign shows that breaking force opposes the direction of motion and d is the stopping distance.

$T_1 + U_{1\text{-}2} = T_2 \Rightarrow$ **d = 57.8m**

To find the force exerted on the car by the trailer hitch, we consider the car as the system

$T_1 = ½(1400)(72/3.6)^2$

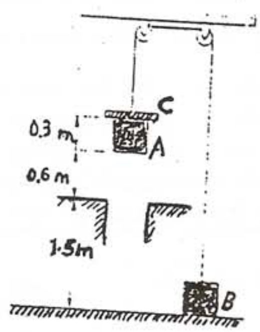
$T_2 = 0$

$U_{1\text{-}2} = (F - 5000)\,57.8$

$T_1 + U_{1\text{-}2} = T_2 \Rightarrow F = 154$ N

So the force exerted is 154N (→)

Prob.: Two blocks A and B, with masses of 4.5 Kg and 5Kg, respectively, are connected by a cord which passes over pulleys as shown. A collar C is placed on block A and the system is released from rest. After the blocks have moved 0.9m, collar C is removed and the blocks continue to move. Knowing that collar C has a mass of 2.5Kg, determine the speed of the block A just before it strikes the ground. Also determine the smallest mass of collar C for which block A will reach the ground.



Blocks A and B together is taken as the system †

For the first 0.9 m of the motion, work is done by gravity on the system as well as the collar C. At the start $T_1=0$ At the end of this 0.9m motion A, B and C possess kinetic energy

$T_2 = ½(9.5 + 2.5)V_1^2$, $U_{1\text{-}2} = -m_Bg(0.9) + m_Ag(0.9) + m_cg(0.9)$

$T_1 + U_{1\text{-}2} = T_2 \Rightarrow 0 + 4.5g(0.9) - 5g(0.9) + 2.5g(0.9) = ½(9.5 + 2.5)V_1^2$

$\Rightarrow V_1 = 1.716$ m/s, V_1 is the speed of A and B at the removal of collar C.

Now the second phase of the motion begins with initial KE of $T_1 = ½(9.5)(1.716)^2$. Work done $U_{1\text{-}2}$ is due to gravity force acting on A and B, A and B move through a distance of 1.2 m before A touches ground. $U_{1\text{-}2} = -m_Bg(1.2) + m_Ag(1.2)$. Final KE, $T_2 = (1/2)(9.5)V^2$

$T_1 + U_{1\text{-}2} = T_2 \Rightarrow ½(9.5)(1.716)^2 + (4.5 - 5)1.2g = ½(9.5)V^2$

$\Rightarrow$ **V= 1.306 m/s (↓)**

† Let m be the mass of the collar which makes A just reaches the ground i.e. V = 0

Up to removal of collar C $T_1 + U_{1\text{-}2} = T_2$ gives

$0 + (4.5g - 5g + mg)0.9 = (½)(9.5+m)V_1^2$

Where V_1 is the speed of A, B and C on removal. After removal,considering the motion, (i)

$T_1 + U_{1\text{-}2} = T_2$ gives

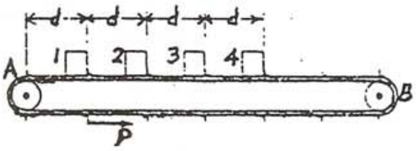
$½(9.5)V_1^2 + (4.5g - 5g)1.2 = 0$ (ii)

Eliminating V_1^2 between (i) and (ii) **m = 1.25 Kg**

† Isolation of blocks unnecessarily drags in the tension in the cord into the computations.

Prob.: Guide angles have been attached to conveyor belt at equal distances d = 300 mm. Four packages, each having a mass of 4 kg, are placed as shown on the belt which is rest. If a constant force P of magnitude 60N is applied to the belt, determine the velocities of packages 1 and 2 as they fall off the belt at point A. Assume that the mass of the belt and pulleys is small compared with the mass of the packages.

If the package 2 has a velocity of 2.29 m/s as it falls off the belt, determine the velocities of packages 3 and 4 as they fall off the belt at point A.



Packages move in horizontal direction. Gravity is hence inactive on the system. Only P does work on the system which consists of 4 packages, 2 pulleys and the belt. Initial KE, T_1=0. After 0.3m of P's movement package 1 fall off the belt at A. Work done is p x 0.3.

† Final KE $T_2 = (½)(4 \times 4) V_1^2$ where V_1 is the speed of all the packages.

$T_1 + U_{1\text{-}2} - T_2 \Rightarrow 0 + 60\,(0.3) - (½)(4 \times 4)V_1^2 \Rightarrow$ $\mathbf{V_1 = 1.5}$ **m/s**

Now the system has only 3 packages. Initial KE, $T_1 = (½)(12)(1.5)^2$. Final KE, $T_2 = (½)(3 \times 4)V_2^2$, where V_2 is the speed of all the three packages after 0.3 m of motion. This also corresponds to the velocity with which package 2 falls off.

$T_1 + U_{1\text{-}2} = T_2 \Rightarrow (½)(12)(1.5)^2 + 60\,(0.3) = (½)(12)V_2^2 \Rightarrow$ $\mathbf{V_2 = 2.291}$ **m/s**

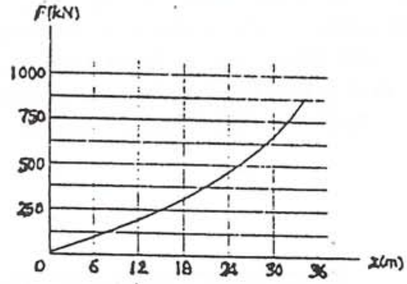
Similarly for the next two phases of the motion

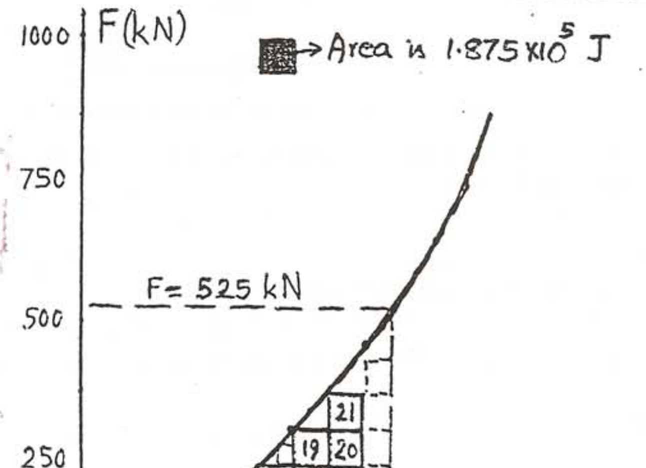
$(½)(8)(2.291)^2 + 60(0.3) = (½)(8)V_3^2 \Rightarrow$ $\mathbf{V_3 = 3.122}$ **m/s**

And $(½)(4)(3.122)^2 + 60\,(0.3) = (½)(4)V_4^2 \Rightarrow$ $\mathbf{V_4 = 4.33}$ **m/s**

† KE possessed by the pulleys due to their rotation and the KE possessed by the belt are neglected because their masses are small.

Prob.: A 7000 Kg airplane lands on an aircraft carrier and is caught by an arresting cable which is characterized by the force-deflection diagram shown. if the landing speed of the plane is 145 Km/h, determine (a) the distance required for the plane to come to rest, (b) the maximum rate of deceleration of the plane.





The plane possesses a kinetic energy, $T_1 = (½)(7000)(145\,/\,3.6)^2$ at landing i.e. $T_1 = 5.678 \times 10^6$ J. Kinetic energy at stop is zero. Work is done by the arresting cable. This work is characterized by the force- deflection diagram reproduced below. Change in KE has to be found and equated to that value of x up to which the area under the force -deflection diagram equals the change in KE.

$T_1 + (\text{Area})_{F-x} = T_2$

$(\text{Area})_{F-x} = T_2 - T_1$

$= 5.678 \times 10^6$ J

Each square of 5mm in the graph represents 1.875×10^5 J

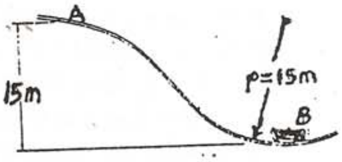
$\dfrac{5.678 \times 10^6}{1.875 \times 10^6}$ i.c. 30.3 such squares are counted (roughly) up to x = 26 m.

∴ Stopping distance, **l = 26m**

At any instant the declaration of the plane (a) is obtained from the equation F = ma.a is maximum when F is maximum. That is obviously at x = 26m. F_{max} = 525KN from the graph $525 \times 10^3 = 7000a \Rightarrow$ **a = 75 m/s²**

Prob.: A roller coaster starts from rest at A and rolls down the track shown. Assuming no energy loss and the radius of curvature of the track at B to be 15m, determine the apparent weight of a 67kg passenger at B.

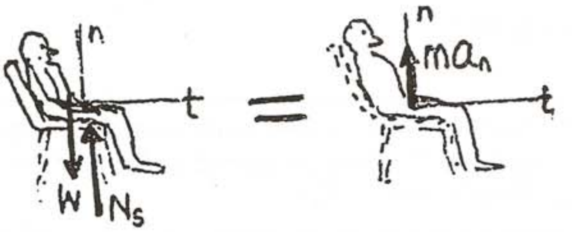
The breaks are suddenly applied as the car passes through point B, causing the wheels of the car to slide on the track ($\mu_k = 0.25$). Assuming no energy loss between A and B, determine the normal and tangential components of the acceleration of the car just after the brakes have been applied.



Between A and B the only agency that does work is gravity. $T_1 = 0$ at A, $U_{1\text{-}2} = mg(15)$ where m is the mass of the roller coaster.

$T_1 + U_{1\text{-}2} = T_2 \Rightarrow 0 + mg(15) = (\frac{1}{2})mV$ where V is the coaster speed at B $\Rightarrow V = 17.16$ m/s

It is to be borne in mind that apparent weight of any body is a direct. account of the reaction offered on it by the surface on which it rests. In the present case reaction offered by the seat on the passenger. At B he is on a circular path of 15m radius. Let N_s be the reaction



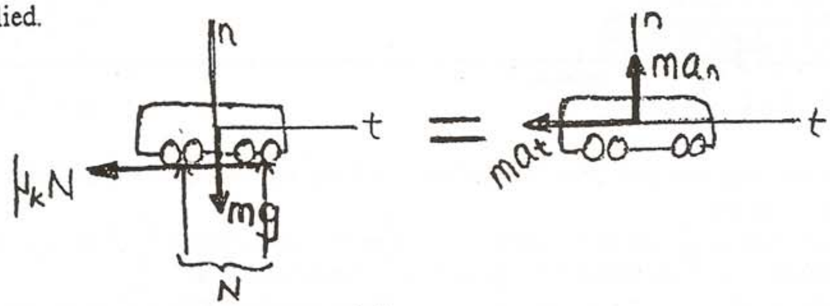
$F_n = ma_n \Rightarrow N_s - W = m\,a_n$

$\Rightarrow N_s - 67g = 67\,\frac{(17.16)^2}{15}$

$\Rightarrow$ **N_s = 1973 N**

N_s is his apparent weight. Note that his real weight is only 657N

Note that N is the net reaction on the rollercoaster and mg is its weight. The above FBDE is valid just after the brakes are applied.



$a_n = v^2/\rho = (17.16)^2/15 \Rightarrow$ **a_n = 19.63 m/s²(↑)**

$\Sigma F_n = ma_n \Rightarrow N - mg = m\,v^2/\rho$

$\Sigma F_t = ma_t = \mu_k N = -ma_t$

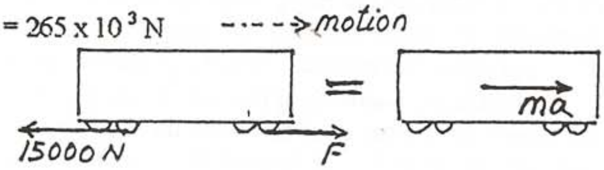
Eliminating N between these two equations **a_t = 7.362 m/s² (←)**

Prob.: A train of total mass equal to 500 Mg starts from rest and accelerates uniformly to a speed of 90 km/h in 50 s. After reaching this speed, the train travels with a constant velocity. The track is horizontal and axel friction and rolling resistance result in a total force of 15kN in direction opposite to the direction of motion. Determine the power required as a function of time.

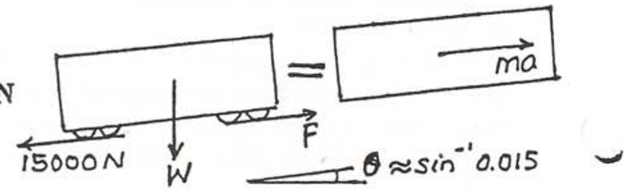
If the train is traveling up a 1.5 percent grade, determine the power required as a function of time.

Power developed at any instant is the product of the force exerted by the train and its velocity. In the acceleration phase: $v = u + at \Rightarrow 90 \times (5/18) = 0 + a(50)$ or $a = 0.5\ m/s^2$.
At any time t, hence the velocity is $v = 0.5t$

$F = ma$ give $(F - 15000) = 500 \times 10^3\,(0.5)$ or $F = 265 \times 10^3$ N
Power developed = F. V = $265 \times 10^3\,(0.5t)$ J/s
or **$P = 132.5$ kW for $t \le 50$s**
$P = 6.625$ MW at $t = 50$s
In the constant velocity phase:
$a = 0\ \therefore F = ma$ gives $F = 15000$ N
Power = F. v $\Rightarrow P = 15000 \times (90 \times 5/18)$ J/s
or **$P = 375$ kW for $t > 50$s**



$F = ma \Rightarrow (F - 15000 - W\sin\theta) = ma$
Knowing that $a = 0.5\ m/s^2$ and $\sin\theta \approx 0.015$, **$F = 338.6$ kN**
Power = F. V $\Rightarrow$ power = 338.6 (0.5t)
$P = 169.3t$ kW for $t \le 50$s
$P = 8.465$ MW at $t = 50$s
For $t > 50$s $a = 0\ \therefore F = 15000 + 500 \times 10^3\,(0.15\,g) = 88575$ N
$P = F.\,V \Rightarrow$ **$P = 2.214$ MW**

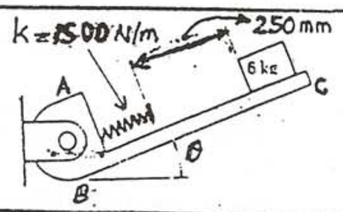


Prob.: The fluid transmission of a 15- Mg truck allows the engine to deliver an essentially constant power of 50kW to driving wheels. Determine the time required and the distance traveled as the speed of the truck is increased (a) from 36km/h to 54 km/h to 72km/h,(b)from 54 km/h to 72 km/h.

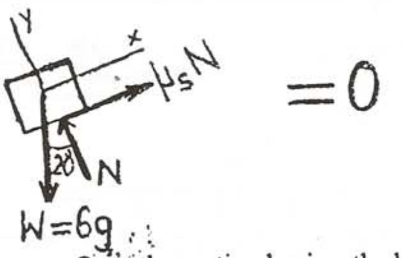
Power developed remains constant at 50000W. The force F which drives the truck hence varies since the velocity changes i.e. $50000 = F\,v$. When velocity and time are to be related the above equation may be written as $50000 = (m\,dv/dt)^v$ and when velocity and position are to be related the equation taken the form $50000 = (m\,v\,dv/dx)^v$. Note that m = 15000kg. On substitution $dt = \frac{15000}{50000} v.dv$ and $dx = \frac{15000}{50000} v^2\,dv$

† (a) Velocity is to be changed from $V_1 = 36$ km/h or 10 m/s to $V_2 = 54$ km/h or 15m/s. Time varies from zero to t and x varies from 0 to d
$\int_0^t dt = 15000 / 50000 \int_{10}^{15} V\,dv \Rightarrow$ **$t = 18.8$ s** $\int_0^d dx = 15000 / 50000 \int_{10}^{15} V^2\,dv \Rightarrow$ **$d = 238$ m**
† (b) Velocity changes from 54 km/h to 72 km/h i.e. 15m/s to 20 m/s
$\int_0^t dt = 15000 / 50000 \int_{10}^{15} v\,dv \Rightarrow$ **$t = 26.3$ s** $\int_0^d dx = 15000 / 50000 \int_{10}^{15} v^2\,dv \Rightarrow$ **$d = 463$ m**
† Change in speed in both cases is 18 km/h. But note how the 'changing procedure' is slower at a higher speed. In simple terms we say that acceleration rate becomes slower at higher speeds.

Prob.: As the bracket ABC is slowly rotated, the 6-kg block starts to slide towards the spring when $\theta = 20^0$. The maximum deflection of the spring is observed to be 50mm. Determine the values of coefficients of static and kinetic friction.



When the motion of the block impends down, the static equilibrium of the block is just about to end and FBDE is as given below. It is a pure 'statics' problem.



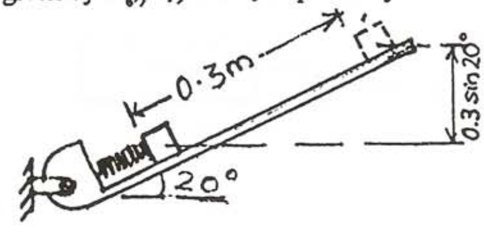
$\Sigma F_x = 0 \Rightarrow \mu_s N - 6g \sin 20 = 0$ (i)

$\Sigma F_y = 0 \Rightarrow N - 6g \cos 20 = 0$ (ii)

From (i) and (ii)

$\mu_s = \tan 20$ or $\mu_s = \mathbf{0.364}$

Once the motion begins, the block covers a total distance of 250mm + 50mm i.e. 0.3 m before coming to a stop. Initial and final KE are zero. Work is done on the block by gravity, frictional force and spring force, given by U_g, U_f, and U_s respectively



$U_g = 6g\,(0.3 \sin 20)$

$U_s = \frac{1}{2} k\,(x_1^2 - x_2^2)$

$= \frac{1}{2}(1500)(0 - 0.05^2)$

$U_f = -\mu_k N(0.30)$

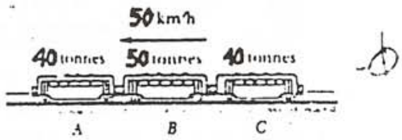
But N = 6g cos20 considering the equilibrium of the block in a direction normal to the bracket

$T_1 + U_{1\text{-}2} = T_2 \Rightarrow 0 + (U_g + U_s + U_f) = 0$

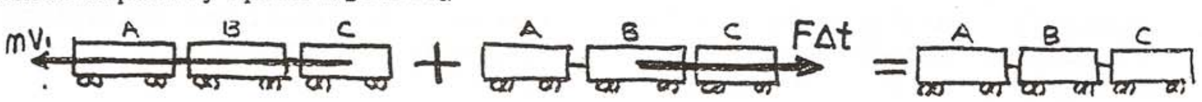
or $6g\,(0.3 \sin 20) + \frac{1}{2}(1500)(-0.05^2) - \mu_k(6g \cos 20)(0.3) = 0$

$6.039 - 1.875 - 16.593\,\mu_k = 0$ or $\mu_k = \mathbf{0.251}$

Prob.: The subway train shown is traveling at speed of 50 km/h when the brakes are fully applied on the wheels of car B and C, causing them to slide on the track, but are not applied on the wheels of car A. Knowing that the coefficient of kinetic friction is 0.35 between the wheels and track, determine (a) the time required to bring the train to a stop, (b) the force in each coupling.



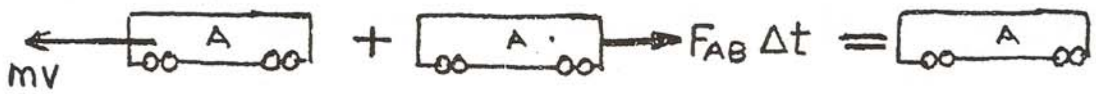
Speed of the train = 50 x 5/ 18 = 13.89 m/s. brakes are applied on cars B and C and hence frictional forces act on them. They are respectively equal to $\mu_k N_B$ and $\mu_k N_C$. N_B and N_C are the net reactions on the wheels which in turn are respectively equal to W_B and W_C.



$mv_1 + \text{Imp}_{1\text{-}2} = mv_2 \Rightarrow (40000 + 50000 + 40000)\,13.89 + 0.35(50000g + 40000g)\,\Delta t = 0$

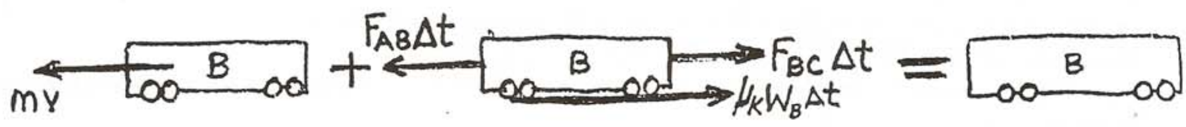
$\Rightarrow \Delta t = \mathbf{5.84s}$

To find the coupling forces we isolate the cars and consider the impulse momentum of A and B.



F_{AB} is the force in the coupling between A and B. Note that there is no frictional force on A.

Thus $- 40000\,(13.89)\ !F_{AB}\,(5.84) = 0 \Rightarrow F_{AB} = \mathbf{95.1\ kN\ (tensile)}$ †

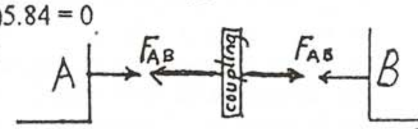


F_{BC} is the force in the coupling between B and C and μmg is the frictional force. Use F_{AB} = 95100N.

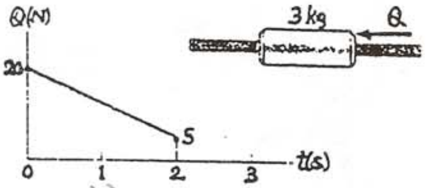
Thus $-50000\,(13.89) - 951000\,(5.84) + F_{BC}\,(5.84) + 0.35\,(50000g)5.84 = 0$

$\Rightarrow F_{BC} = \mathbf{42.3\ kN\ (tensile)}$ †

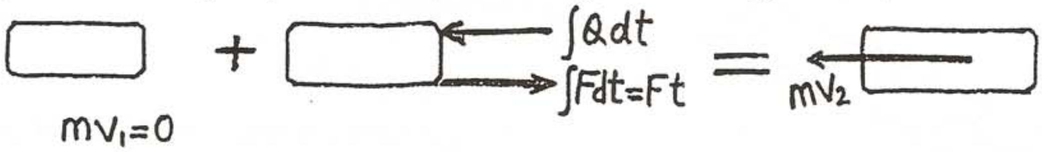
† Nature of force can be easily understood if the force on each coupling is analyzed. For example



Prob.: The 3 kg collar is initially at rest and is acted upon by the force Q which varies as shown. if $\mu_k = 0.25$, determine the velocity of the collar at (a) t = 1s, (b) t= 2s. Also determine the maximum velocity reached by the collar and the corresponding time. At what time does the collar come to rest.

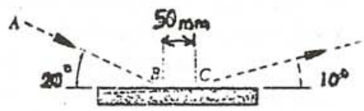


Force acting on the collar is a variable one. Hence impulse of the Q is not $Q\Delta t$ but $\int Q\,dt$
Impulse = $\int(-7.5t + 20)dt = -3.75t^2 + 20t$. Note that $Q(t) = -7.5t + 20$ as observed from the given graph. Using the impulse momentum principle for the collar of mass 3kg and considering the frictional force 0.25mg.



$mv_1 + (\text{Imp})_{1\text{-}2} = mv_2 \Rightarrow 0 + (-3.75t^2 + 20t) - 0.25(3g)t = 3V$
Or $v = 4.214t - 1.25t^2$ (i)
From (i)
At t = 1s **v = 2.964 m/s** At t = 2s **v = 3.428 m/s**
When v is maximum, dv/dt = 0 and from (i) we get $4.214 - 2.5t = 0$ corresponding to that **t = 1.69 s** and **V_{max} = 3.55 m/s.** v=0 when $4.214 - 1.25t = 0$, from (i) or **t = 3.37 s**. But this cannot be accepted since Q does not exist at t=3.37s. from t=2s, IM principle gives $3(3.428) - 0.25(3gt) = 0 \Rightarrow t = 1.4s$. So the collar comes to a stop at t=3.4s.

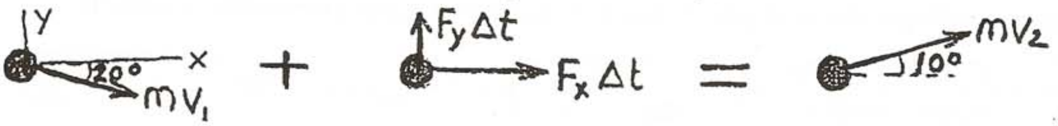
Prob.: A 30kg steel-jacketed bullet is fired with a velocity of 600 m/s towards a steel plate and ricochets along path CD with velocity of 400 m/s. knowing that the bullet leaves a 50 mm scratch on the surface of the plate and assuming that it has an average speed of 540 m/s while in contact with the plate, determine the magnitude and direction of the impulsive force exerted by the plate on the bullet.



Initial and final momentums of the bullet are known. The change in momentum occurs due to the impulsive force exerted by the plate on the bullet in the short time interval Δt of impact. This short time interval is found knowing that bullets travel for 50mm along the plate at speed of 540 m/s.

$\Delta t = \dfrac{50 \times 10^{-3}\,m}{540\,m/s}$ or **$\Delta t = 9.259 \times 10^{-5}$ s**

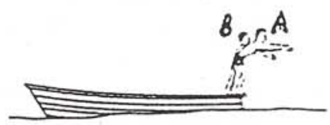
we have to use the impulse momentum principle on the bullet along two mutually perpendicular directions since the impulsive force has two components F_x and F_y †



In x direction $mv_1 \cos 20 + F_x\Delta t = mv_2 \cos 10$
Or $0.03(600)\cos 20 + F_x(9.259 \times 10^{-5}) = 0.03(480)\cos 10 \Rightarrow$ **$F_x = -29520$ N**
In y direction $-mv_1 \sin 20 + F_y\Delta t = mv_2 \sin 10$
Or $-0.03(600)\sin 20 + F_y(9.259 \times 10^{-5}) = 0.03(480)\sin 10 \Rightarrow$ **$F_y = 93497$ N**
Impulsive force $F = \sqrt{F_x^2 + F_y^2}$ or **F = 98kN (↖ 72.5°)**

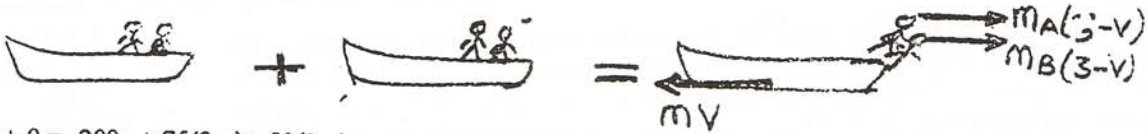
† Noting the change in the direction of the velocity, we have to understand that the impulsive force is in some unknown direction.

Prob.: Two swimmers A and B of masses 75kg and 50kg, respectively, dives off the end of a 200kg boat. Each swimmer dives so that his relative horizontal velocity with respect to the boat is 3 m/s. if the boat is initially at rest, determine its final velocity, assuming that (a) the two swimmers dives simultaneously, (b) swimmer A dives first, (c) swimmer B dives first.



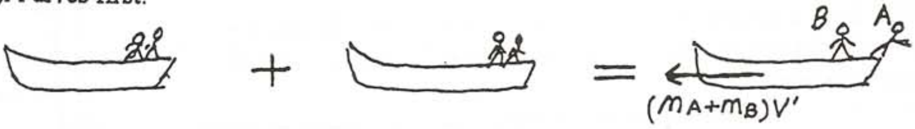
Boat and the swimmers are together considered to be the system. M_A = 75kg, M_B = 50kg, mass of the boat m =200 kg. There are no impulsive forces in the horizontal direction if the water resistance is neglected. Each swimmer can dive only at an absolute horizontal velocity of (3 - v) m/s where v is the leftward velocity of the system he leaves behind while jumping. Impulse momentum principle is used. $mv_1 + Imp_{1\text{-}2} = mv_2$

(a)Both dive simultaneously.



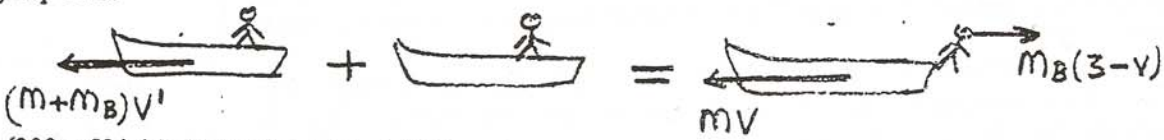
$0 + 0 = -200v + 75(3-v) + 50(3-v) \Rightarrow$ **v = 1.154 m/s(←)**

(b)A dives first.



$0 + 0 = -(200 + 50)\,v' + 75\,(3 - v') \Rightarrow v' = 0.692$ m/s

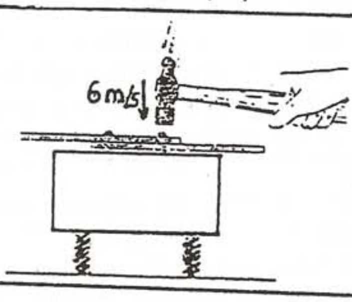
The boat together with swimmer B is now set in motion leftward with a speed of 0.692 m/s. then comes the jump of B.



$-(200 + 50)(0.692) + 0 = -200v + 50(3 - v) \Rightarrow$ **v = 1.292 m/s (←)**

Case (c) when B dives first also is to be tackled similar to case (b). It results in **v= 1.28 m/s (←)**

Prob.: A small rivet connecting two pieces of sheet metal is being clinched by hammering. Determine the impulse exerted on the rivet and the energy absorbed by the rivet under each below, knowing that the head of the hammer has a mass of 750 g and that it strikes the rivet with a velocity of 6m/s. Assume that the hammer does not rebound and that the anvil is supported by spring and (a) has an infinite mass (rigid support), (b) has a mass of 4kg.



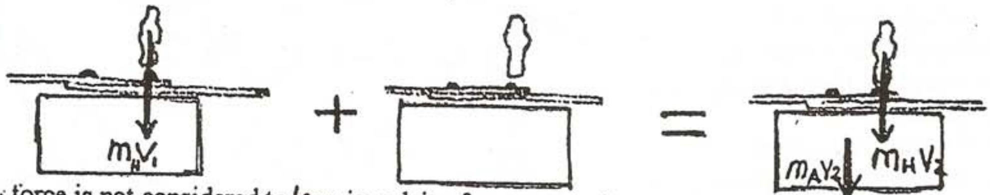
(a) When the anvil is supported by spring and has an infinite mass there is no movement to it after the below. We consider the hammer, it does not rebound after the below. This means $v_2 = 0$.

$Mv_1 + Imp_{1\text{-}2} = mv_2 \Rightarrow -0.75(6) + F\Delta t = 0 \Rightarrow$ **Impulse = 4.5Ns**

KE possessed by the hammer is completely absorbed by the rivet since the head does not rebound.KE of the hammer is ½ (0.75)(6)² or 13.5 J

∴Energy absorbed by the rivet = **13.5 J**

(b) When the anvil has a 4 kg mass and rests on spring,there is a movement for the anvil and the hammer after every below. System is (anvil + hammer)



Spring force is not considered to be an impulsive force. m_H and m_A are the masses of hammer and anvil.

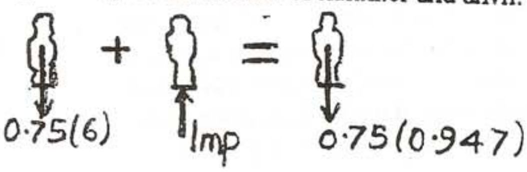
$mv_1 + Imp_{1\text{-}2} = mv_2 \Rightarrow -0.75(6) + 0 = -0.75v - 4v$

or **v= 0.947 m/s**

The system now goes down at a speed of 0.947 m/s.

Consider the hammer now.

$mv_1 + Imp_{1\text{-}2} = mv_2 \Rightarrow -0.75(6) + Imp = -0.75(0.947)$



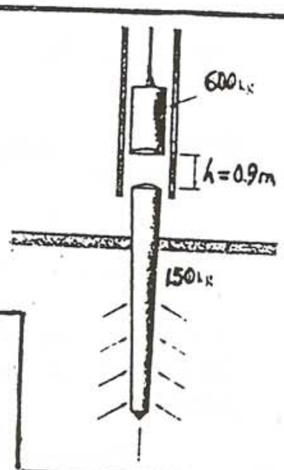
or **Impulse = 3.79 Ns**

In each blow, KE lost by hammer = ½ (0.75) $(6^2 - 0.947^2)$ = 13.16J

KE gained by the anvil = ½ (4) $(0.947)^2$ = 1.794 J. The remaining KE i.e. 13 · or **11.37 J** has been absorbed by rivet.

Prob.: It is desired to drive the 150kg pile into the ground until the resistance to its penetration is 100-kN. Each below of the 600-kg hammer is the result of 0.9 m free fall onto the top of the pile. Determine how far the pile will be driven into the ground by single blow as the 100-kN resistance is being achieved. Assume that the impact is perfectly plastic.

If the hammer falls from a height of 0.9 m onto the top of the pile driving it100mm into the ground, determine the average resistance of the ground to penetration. Assume plastic impact.



In each case blow of the hammer, H,the pile is imparted a velocity V with which it penetrates the soil. The soil offers resistance to the penetration which goes on increasing as the pile gets deeper and deeper into the soil due to the increasing surface area of contact of the pile. Penetration therefore goes on decreasing as the blows progress.

We are interested in finding the penetration, d of the pile when the resistance R is 100kN. On falling from a height of 0.9 m the hammer has a velocity $V_H = \sqrt{2g(0.9)}$ = 4.2m/s . Since $e = 0$ for the impact

$m_H V_H + m_P V_P = (m_H + m_P)$ V,

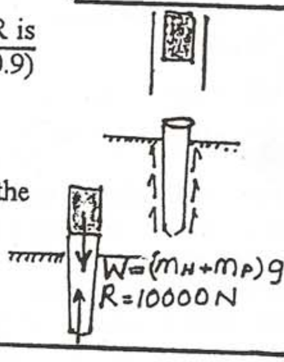
600(4.2) + 0 = (600 + 150)V

or V= 3.362m/s,where $V_P = 0$ and V is the velocity of pile and hammer after the impact. If d is the penetration $T_1 + U_{1\text{-}2} = T_2$ gives ½(W/g) V^2 + (W – R) d = 0

½ (750) $(3.362)^2$ + (750g – 100000) d = 0 ⇒ **d** = 0.046 m or **46 mm**

If d = 0.1 m and all other data remain unchanged

½ (750)$(3.362)^2$ + (750g – R)0.1 = 0 ⇒ **R = 49.7 kN**



Prob.: A 15000 kg airplane lands on aircraft carrier and caught by an arresting cable. The cable is inextensible and paid out at A and B from mechanism located below deck and consisting of pistons moving along oil-filled cylinders. Knowing that the piston-cylinder system maintains a constant tension of 425 kN in the cable during the entire landing, determine the landing speed of the airplane if it travels a distance d = 28m after being caught by the cable.

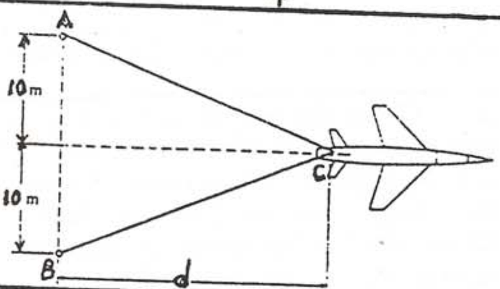
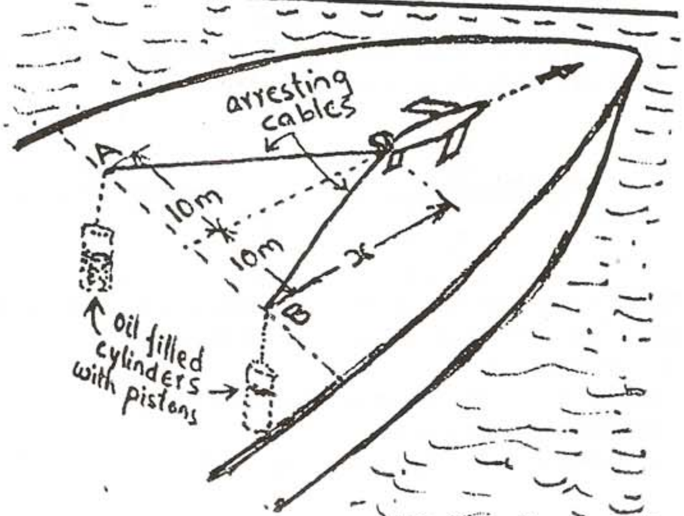
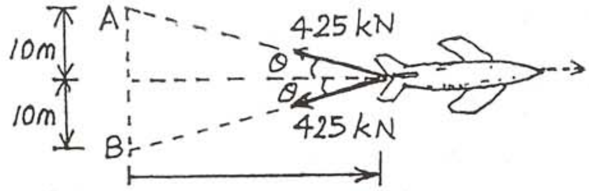


Figure is only a rough sketch of the arresting mechanism employed in an aircraft carrier to allow aircrafts complete their landing in a relatively short distance.

The tension in the cable is constant, 425 kN. However the net pull on the plane is a variable since it depends on the position of the plane. At any position of the plane defined by x, the force exerted on the plane by the cables is 2(425000 Cosθ) Newtons. Elementary work done by this force as the plane moves by a distance dx is given by



$dU = -2\,(425000\ \cos\theta\,)dx$. Negative sign is because tension component opposes the motion direction. Noting that $\cos\theta = x / \sqrt{10^2 + x^2}$ and that $T_1 = ½\,(15000)v^2$, $T_2 = 0$

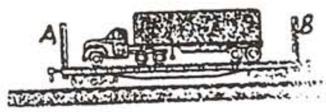


$† \; T_1 + U_{1\text{-}2} = T_2 \Rightarrow ½\,(15000)v^2 + \int_0^{28} -2\,(425000)\,x / \sqrt{10^2 + x^2}\,dx = 0$

or $7500v^2 = 85000\ \left|\sqrt{100 + x^2}\right|_0^{28} \Rightarrow$ **$v = 47.29$ m/s or 170km/h**

† As the plane position changes from x=0 to x= 28m, its speed changes from v to zero.

Prob.: A 12-Mg truck and 30-Mg railroad flatcar are both at rest with their brakes released. An engine bumps the flatcar and causes the flatcar alone to start moving with a velocity of 1.4 m/s to the right. Assuming e=1 between the truck and the ends of the car flatcar and neglecting friction, determine the velocities of truck and of the flatcar after (a) end A strikes the truck, (b) the truck strikes end B.



(a) Brakes of the truck are released. This causes the factor alone to move rightward at 1.4 m/s thus $V_F = 1.4$ m/s, $V_1 = 0$ before impact. After the impact of end A and the truck, let the velocities of the flatcar and the truck be V_F and V_T respectively. Conservation of momentum implies that

$M_F V_F + M_T V_T = M_F V_F + M_T V_T$

Or $30000\,(1.4) + 0 = 30000\ V_F + 12000\ V_T$

Or $3.5 = 2.5\ V_F + V_T$ (i)

Relative velocity along the line of impact implies that

$e = (V'_F - V'_T) / (V_T - V_F)$ or $1 = (V'_F - V'_T) / (0 - 1.4)$ (ii)

From (i) and (ii) $V_F = 0.6$ m/s, $V_T = 2$ m/s

Since +ve sign indicates rightward the **flatcar** and the **truck** have velocities **0.6 m/s (→) and 2 m/s (→)**

(b) Now the truck moving at 2 m/s (→) strikes end B of the flatcar moving at 0.6 m/s (→)

Conservation of momentum implies that

$12000(2) + 30000\,(0.6) = 12000\ V'_T + 30000\ V'_F$

or $V'_T + 2.5\ V'_F = 3.5$ (iii)

Consideration of relative velocity along the line of impact implies that

$e = (V'_F - V'_T) / (V_T - V_F)$ or $1 = (V'_F - V'_T) / (2 - 0.6)$

Or $V'_F - V'_T = 1.4$ (iv)

From (iii) and (iv) $V'_F = 1.4$ m/s $V'_T = 0$

Thus the **flatcar** and the **truck** have velocities **1.4 m/s (→)** and **0**.