Numerical Methods

Notes by-

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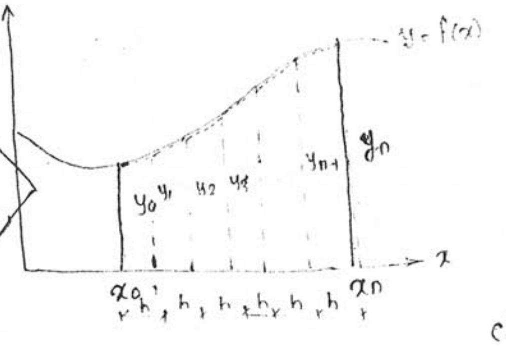
[Numerical Methods]

20/11/2004
Pg. 1.

In the majority of physical problem & all engg. problems govered by DE

* Integration :

Finding Area using trapezoidal Rule :-



Area bet^n x_0 & x_n is,

$$I = \int_{x_0}^{x_n} y\,dx \quad \ldots \text{ Basic eq}^n$$

Divide the area in to number of ~~widt~~ sections or strips having same width.

$$I = \frac{h}{2}\left[y_0 + 2\,(y_1 + y_2 + \cdots + y_{n-1}) + y_n\right]$$

choice of h depends on us, How many accuracy we want.

No. of divisions may be odd or even.

Evaluate $\int_0^1 \frac{1}{1+x}\,dx$ using trapezoidal rule :-

$$\int_0^1 \frac{1}{1+x}\,dx = \left|\cancel{\log}\ \ln x\right|_0^1 = \ln 2 = 0.69317$$

h = 0.2

x	0	0.2	0.4	0.6	0.8	1.0
$y = \frac{1}{1+x}$	1	1/1.2	1/1.4	1/1.6	1/1.8	0.5

$$\therefore I = \frac{h}{2}\left[y_0 + 2\,(y_1 + y_2 + \cdots + y_{n-1}) + y_n\right]$$

$$= \frac{0.2}{2}\left[1 + 2\left(\frac{1}{1.2} + \frac{1}{1.4} + \frac{1}{1.6} + \frac{1}{1.8}\right) + 0.5\right]$$

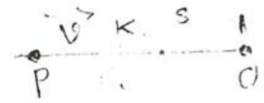
$$= 0.6956$$

$$\% \text{ err error} = \left[\frac{0.6956 - 0.6931}{0.6931}\right] \times 100$$

$$= 0.36\ \%$$

$$e = \lim_{x \to 0} (1+x)^{1/x}$$

$x = 0.01$, $e = 2.7048$
$x = 0.001$, $e = 2.7169$
$x = 0.0001$, $e = 2.7181$



$V \propto S$

i.e. $V = -KS$

Dist. is from P1. to left
V is from A to left.

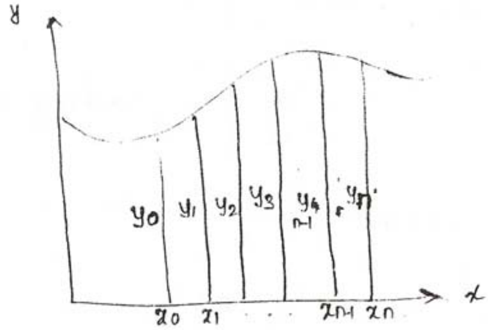
$$\therefore \frac{ds}{dt} = -ks$$

$$\frac{ds}{s} = -k\,dt$$

$$\therefore \int \frac{ds}{s} = s = -kt$$

$$s = e^{-kt}$$

* Simpson's Rule [Simpson's 1/3rd Rule] :-



$$I = \frac{h}{3}\left[y_0 + 4\,(y_1 + y_3 + y_5 + \cdots + y_{n-1}) + 2\,(y_2 + y_4 + y_6 + \cdots + y_{n-2}) + y_n\right]$$

where n is even No.

$$\therefore I = \frac{h}{3}\left[1 + 4\,(\right.$$

h = 0.125

x	0	0.125	0.25	0.375	0.5	0.625	0.75	0.875	1.0
y	1	0.889	0.8	0.727	0.667	0.615	0.571	0.533	0.5

$\therefore I = \frac{0.125}{3}\left[1 + 4(0.8889 + 0.7272 + 0.6154 + 0.5333) + 2(0.8 + 0.6667 + 0.5714) + 0.5\right]$

$= 0.6931$

$I = \int_3^4 \frac{2x}{2+\sin^2 x}\,dx$. 1 and 4.8112

x	3	3.25	3.5	3.75	4.0
y	4.4556	4.8466	4.9457	4.8352	4.663

Open end solutions. (Does not have exact sol^n)
Closed form solution: (Have exact Sol^n)

$\therefore I = \frac{0.25}{2}\left[4.4556 + 2(4.8466 + 4.8352 + 4.9457) + 4.663\right]$

$= 4.8115$

V.V.Imp

Solution of equations :

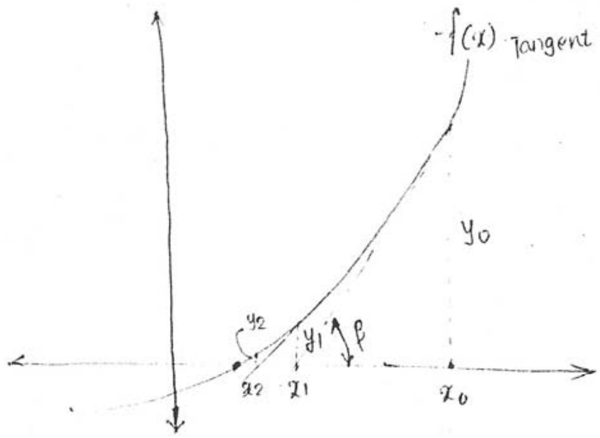
Roots of an eq^n using Newton Raphson mtd.

$3x^2 - 2x + e^x - 3\sin x$ = $\boxed{LIATE}$ Logarithmic, Inverse, Algebraic, Trignometric, Expo.

such eq^n containing LIATE are known as <u>Transcendental Eq^n</u>.

i.e. $f(x) = 0$

This mtd. is most efficient for sol^n of transcendental eq^ns & is based on a simple approach of slope of the tangent of a function.



Select x_0,
find y_0 by substituting 'x_0' in $f(x)$
Draw y_0 as shown.
Draw tangent at intersection which cuts x axis at x_1
Repeat mtd until we reach to reqd. point.

$\tan\beta = \frac{y_0}{x_0 - x_1} = \frac{f(x_0)}{x_0 - x_1} = f'(x_0)$ --- Slope of tangent

$\therefore \frac{f(x_0)}{f'(x_0)} = x_0 - x_1 \Rightarrow \boxed{x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}}$ ... Working formula.

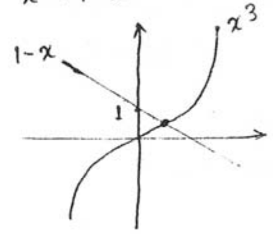
$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$ ⟶ General Working formula.

Solution is when x_{n+1} & x_n are "<u>reasonably Close</u>"
This is fastest converging numerical technique for root of eq^n.

θ + Q: cubic eq^n has either 1 sol^n or 3 sol^n. ②

12 olve: $(x^3+x-1)=0$

$x^3 = 1-x$



* Start from, $x = 0.5$ ... select by approximation.

n = No. of iteration.

n	$x_0=0$ x_n
1	0.~~7~~143
2	0.6832
3	0.6823
4	0.68232
5	
6	
7	
8	

} i.e. the solution.

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

Let $f(x_n) = x^3+x-1$

$f'(x_n) = 3x_n^2+1$

Let $x_1 = 0.5 - \frac{(-0.375)}{1.75} = 0.7143$

$x_2 =$ ~~0.7143 - (~~

hrases in Mathematics: 'Well behaved'
'Well conditioned'
'ill conditioned' $\rightarrow$

Convergance rate is extremely high.

lve: $2\sin x - x = 0$

Dont compare term $(2\sin x - x)$ with zero
compare $2\sin x = x$

in $f(x)$: $f(x_n) = 2\sin x_n - x_n$

or: $f'(x_n) = 2\cos x_n - 1$

starting from 1)

4.9296.

Ans+2 = 4
Ans+2 = 6
Ans+2 = 8

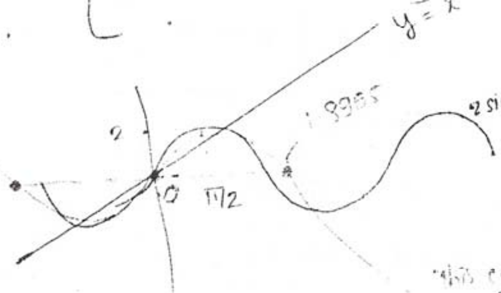
ach to

n	$x_0 = 1$
	-7.4727
1	14.4785
2	6.9351
3	16.6357
4	8.3439
5	4.9545
6	-5.2928
	-4.7884
7	3.2091
8	2.0927
9	1.9229
10	1.8957
11	1.8955
12	1.8955
13	1.8955

$$x_n - \frac{2\sin x_n - x_n}{2\cos x_n - 1}$$

$\boxed{\text{Most accurate sol}^n = 1.8955}$

$$\left[\text{Ans} - \frac{2\sin \text{Ans} - \text{Ans}}{2\cos \text{Ans} - 1}\right]$$



this case is shown for $(2\sin x - x)$

22/11/21

Pro: $2x - \cos x - 3 = 0$ by Newton Rapson Method.

Sol^n - $f(x_n) = 2x - \cos x - 3$

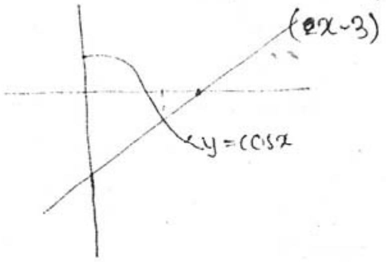
$f'(x_n) = 2 + \sin x$

n	x_n
	$x_0 = 1.5$
1	1.52359
2	1.52359

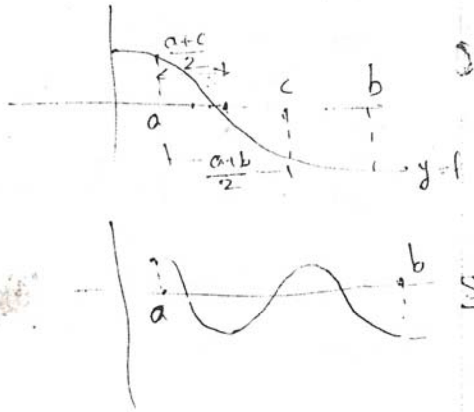
$\therefore \boxed{x = 1.5236}$

$$x_1 = (x_n) - \frac{f(x_n)}{f'(x_n)}$$

$$1.5 - \frac{2\times1.5 - \cos 1.5 - 3}{2+\sin(1.5)}$$



Root of the eq^n by Bisection method :-



Pro: Find Root of the eq^n

$f(x) = x^3 + x^2 - 1 = 0$

at $x = 0.5$, $f(x) = -0.625$

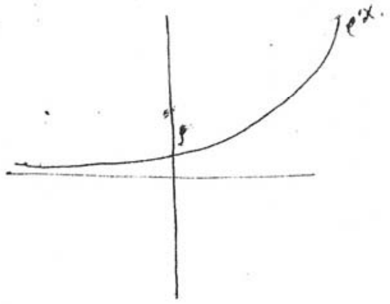
at $x = 1$, $f(x) = +1$ } o.k.

x	$f(x)$	
1	+	
−1	−	
$\frac{+1-1}{2} = 0$	−	∴ Root does not lie bet^n
$\frac{1+0}{2} = 0.5$	−	∴ Root lies bet^n 0.5 & 1.
$\frac{1+0.5}{2} = 0.75$	−	Root lies bet^n 0.75 & 1
$\frac{1+0.75}{2} = 0.875$	+	∴ Root lies bet^n 0.75 & 0.875
$\frac{0.75+0.875}{2} = 0.8125$	+ 0.1965	∴ Root lies bet^n 0.75 & 0.8125
$\frac{0.75+0.8125}{2} = 0.78125$	+ 0.0871	∴ Root lies bet^n ~~0.8125~~ 0.78125 & 0.75
$\frac{0.78125+0.75}{2} = 0.7656$	+ 0.03497	
$\frac{0.7656+0.75}{2} = 0.7578$	+ 0.0094	
$\frac{0.7578+0.75}{2} = 0.7539$	-3.13×10^{-3}	
$\frac{0.7578+0.7539}{2} = 0.7558$	+	
$\frac{0.7558+0.7539}{2} = 0.75475$	−	

0.7552	+
0.7549	+
0.7548	−

$\boxed{\text{Ans} = 0.75485}$

22/11/24 (3)



Q10: $3\sin x + e^{x} - 5x^2 + 3 = 0$ $\boxed{0.8768}$

Put $x = 0$	+
2	−
$\frac{0+2}{2} = 1$	+
$\frac{2+1}{2} = 1.5$	−
$\frac{1.5+1}{2} = 1.25$	+
$\frac{1.25+1.5}{2} = 1.375$	+
$\frac{1.5+1.375}{2} = 1.4375$	−
$\frac{1.4375+1.375}{2} = 1.40625$	+
$\frac{1.40625+1.4375}{2} = 1.4219$	+
$\boxed{1.4297}$	−

b

$y = f($

b

Sol^n of simultaneous eq^n

① Gauss Elimination mtd.
② Gauss Jordan Mtd.
③ ~~Elgration~~ Iteration

* Col^m vector $\begin{Bmatrix} x \\ x \\ x \\ x \\ x \end{Bmatrix}$, Row vector $\begin{bmatrix} x & x & x & x & x \end{bmatrix}$

* Square matrix $[\ \ ]_{m \times n}$, where $m = n$.

* Diagonal Matrix $\begin{bmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{bmatrix}$

* Upper triangular matrix = $\begin{bmatrix} x & x & x & x \\ & x & x & x \\ \text{zero} & & x & x \\ & & & x \end{bmatrix}$

* Lower triangular matrix $\begin{bmatrix} x & & \text{zero} & \\ x & x & & \\ x & x & x & \\ x & x & x & x \end{bmatrix}$

Identity matrix = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Praveen

Travins

R_3 $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$

Singular matrix if $|A| = 0$.

Symmetric matrix : $a_{ij} = a_{ji}$ $\begin{bmatrix} 2 & 5 & 6 \\ 5 & 3 & 1 \\ 6 & 1 & 4 \end{bmatrix}$

Skew symmetric $a_{ij} = -a_{ji}$ $\begin{bmatrix} 4 & 5 & 6 \\ -5 & 3 & 1 \\ -6 & -1 & 4 \end{bmatrix}$

$$\underset{m\times n}{[A]}\ \underset{n\times p}{[B]} = \underset{m\times p}{[C]}$$

$[A][B][C][D]\ldots$ start from any one — the order not to be changed.

Solution of Simultaneous eq^n :-

① Gaussian Elimination Method :-

$$\underbrace{[A]}_{\substack{\text{Square Matrix} \\ \text{Coefficients}}}\ \underbrace{\{X\}}_{\substack{\text{Unknowns} \\ \text{col}^m \text{ Vector}}} = \underbrace{\{B\}}_{\substack{\text{Non homogeneous} \\ \text{RHS}}}$$

The underlined phylosophy of Gaussian Elimination is in reducing the coefficient matrix to an upper triangular matrix by a series of row/column multiplications & additions.

For hand computations the sequence is to get zero's in the first col^m first, second col^m next & so on... The last unknown will be obtained first & subsequently in sequence the other unknowns by a series of back substitutions.

eg :-

$$2x + y + z = 10$$
$$3x + 2y + 3z = 18$$
$$x + 4y + 9z = 16$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} 10 \\ 18 \\ 16 \end{Bmatrix}$$

$R_2 - 1.5R_1$

$R_3 - 0.5R_1$

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 0.5 & 1.5 \\ 0 & 3.5 & 8.5 \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} 10 \\ 3 \\ 11 \end{Bmatrix}$$

③

xuvin, ~~3.5R1~~ $R_3 - ~~3.5R1~~ 3 \quad R_3 - 7R_2$

avins

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 0.5 & 1.5 \\ 0 & 0 & -2 \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} 10 \\ 3 \\ -10 \end{Bmatrix}$$

$-2z = -10 \quad \boxed{z = 5}$

$0.5y + 1.5z = 3$

$\therefore 0.5y = 3 - 1.5 \times 5$

$\therefore y = -9$

to be $\quad 2x + y + z = 10$

nged. $\quad \therefore 2x = 10 + 9 - 5$

$\therefore x = 7$

$\therefore x = 7$

$y = -9$

$z = 5$

---

so:- $\quad 3x + y + 2z = 4$

$x - 3y + z = -8$

$2x + 4y - 4z = 2$

cing

s of

$$\begin{bmatrix} 3 & 1 & 2 \\ 1 & -3 & 1 \\ 2 & 4 & -4 \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} 4 \\ -8 \\ 2 \end{Bmatrix}$$

first

be $\quad R_3 - \frac{2}{3} R_1 \quad ; \quad R_2 - \frac{1}{3} R_1$

wins

$$\begin{bmatrix} 3 & 1 & 2 \\ 0 & -10/3 & 1/3 \\ 0 & 10/3 & -16/3 \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} 4 \\ -9.33 \\ 2/3 \end{Bmatrix}$$

$R_3 + R_2$

$$\begin{bmatrix} 3 & 1 & 2 \\ 0 & -10/3 & 1/3 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -9.33 \\ 1.33 \end{bmatrix}$$

$-3z = 1.33$

$z = -0.443$

$x = -1$

$y = 3$

$z = 2$

-10

Pro:-
$$3x_1 + x_2 - 2x_3 + 0.5x_4 = -9$$
$$2x_1 + 3x_3 - 2x_4 = 15$$
$$x_1 - 3x_2 + x_3 - x_4 = 0$$
$$-x_1 + 2x_2 + 2x_3 + 2x_4 = 3$$

$[4\times4]$ $R_1^2 =$

$R_2^2 =$

$R_3^2 =$

R_1^3

$R_2^3 =$

R_3^3

as machine / program does the Row operati
we can do Row or column operation

& I

* Gauss Jordan method:-

In this mtd; back substitution is avoided & the solutions evolves directly by reducing the coefficient matrix to the identity matrix. first

&

Ca
the

Pro:
$$\begin{array}{c} R_1^0 \\ R_2^0 \\ R_3^0 \end{array}\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} 10 \\ 18 \\ 16 \end{Bmatrix}$$

$$\begin{array}{cc} R_1^0 & R_1' \\ R_2^0 & R_2' \\ R_3^0 & R_3' \end{array}$$

$\therefore R_1' = R_1^0/2$
$R_2' = -3R_1' + R_2^0$
$R_3' = -R_1' + R_3^0$

$$\begin{array}{l} R_1' = R_1^0/2 \\ R_2' = -3R_1' + R_2^0 \\ R_3' = -R_1' + R_3^0 \end{array}\begin{bmatrix} 1 & 0.5 & 0.5 \\ 0 & 0.5 & 1.5 \\ 0 & 3.5 & 8.5 \end{bmatrix}\begin{Bmatrix} \\ \\ \end{Bmatrix} = \begin{Bmatrix} 3 \\ 5 \\ 11 \end{Bmatrix}.$$

i.e.

$$R_1^2 = R_1' - R_2' \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0.5 & 1.5 \\ 0 & 3.5 & 8.5 \end{bmatrix}\begin{Bmatrix} \\ \\ \end{Bmatrix}\begin{Bmatrix} 2 \\ 5 \\ 11 \end{Bmatrix}$$

$[4\times4]$

$$R_1^2 = R_1^1 - R_2^1, \quad R_2^2 = R_2^1/0.5, \quad R_3^2 = -3.5R_2^2 + R_3^1 \qquad \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & -2 \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} 2 \\ 6 \\ -10 \end{Bmatrix}$$

$$R_1^3 = R_1^2 - R_3^2/2, \quad R_2^3 = R_2^2 - 1.5R_3^2, \quad R_3^3 = R_3^2/-2 \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} 7 \\ -9 \\ 5 \end{Bmatrix} \checkmark$$

agreement operation row or operation

$\therefore$ $x = 7$, $y = -9$, $z = 5$

Simpson's Rule: Simpson $\frac{1}{3}$rd Rule

$\frac{3}{8}$ less accurate: Not used

Iteration Method:- (Guass Seidel) mtd:- (m/c friendly).

used in programming

evolved matrix.

- Each of the unknowns is expressed in terms of the others.

first. A set of initial values are arbitarily assumed.

- Based on these initial values the unknowns are calculated. In subsequence steps, the values of the unknowns are revised based on the previous step.

$$10x + 2y + z = 9$$
$$2x + 10y - 2z = -44$$
$$-2x + 3y + 10z = 22$$

i.e. express each unknown in terms of other

i.e. $x = 0.9 - 0.2y - 0.1z$

$y = -4.4 - 0.2x - 0.2z$

$z = 2.2 + 0.2x - 0.3y$

Take ~~any value~~ set of initial values.

Select $x = y = z = 1$. (Take any thing).

$x_1 = 0.9 - 0.2 - 0.1 = 0.6$ — Latest available value

$y_1 = -4.4 - 0.2(0.6) + 0.2(1)$

$= -4.32$.

$$z_1 = 2.2 + \underbrace{0.2(0.6) - 0.3(-4.32)}_{\text{Latest available values.}}$$

$\therefore z_1 = 3.616$

n	x	y	z
1	1	1	1
1	0.6	−4.32	3.616
2	1.402	−3.957	3.667

$$\therefore x_2 = 0.9 - 0.2(-4.32) - 0.1(3.616) = 1.402$$

$$y_2 = -4.4 - 0.2(1.402) + 0.2(3.616) = -3.957$$

$$z_3 = 2.2 + 0.2(1.402) - 0.3(-3.957) = 3.667$$

$$x_3 = 0.9 - 0.2(-3.957) - 0.1(3.667) =$$

$$y_3 = -4.4 - 0.2(1.402) + 0.2(-3.957) =$$

$$z_3 = 2.2 + 0.2(1.402) + 0.3(-3.957) =$$

Ans: 1 = x

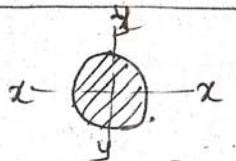
−2 = y

3 = z

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L} \Rightarrow \frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

T = Torsion
J = Polar MI
τ = shear stress
R = Radius of shaft (i.e. NA = y)
G = shear modulus
θ = Angle of Twists
L = Length of shaft

M = Moment
I = MI
σ = Normal stress
y = Dist. of extreme fibre from NA
E = Youngs Mod.
R = Radius of curvature.



$J = I_{xx} + I_{yy}$
$= \frac{2\pi}{64} D^4$
$= \frac{\pi}{32} D^4$

R = D/2

$Z_T = \frac{J}{R} = \frac{\pi}{16} D^3$

$J = \frac{\pi}{32}(D^4 - d^4)$

R = D/2

$Z_T = \frac{J}{R} = \frac{\pi}{16D}(D^4 - d^4)$

For Rectangular section,
$I = \frac{bD^3}{12}$, $Z = \frac{bD^3}{6}$

Power Transmitted by shaft

$P = \left(\frac{2\pi N}{60}\right) \times \underbrace{T}_{\text{Torque}}$ (P = Force × Dist)

Rotation
$\theta = \frac{2\pi N}{60}$

* Trapezoidal Formula:-

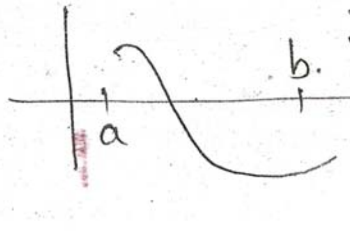
$$I = \frac{h}{2}\left[y_0 + y_n + 2(y_1 + y_2 + \cdots + y_{n-1})\right]$$

* Simpson's 1/3rd Rule

$$I = \frac{h}{3}\left[y_0 + y_n + 2(y_2 + y_4 + \cdots + y_{2n-2}) + 4(y_1 + y_3 + \cdots + y_{2n-1})\right]$$

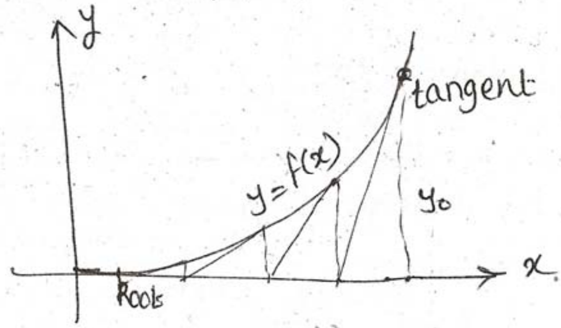
$\boxed{n = \text{ODD}}$

* Bisection Method:-



Select + −
Take avg. of "VALUES" of + & −
& continue.

* Newton Rapson Method



$$x = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Assume any value of x & use above formula

* Solⁿ of linear algebraic Eqⁿ:-

1. Gauss Elimination ⇒ Upper Triangular Matrix
2. Gauss Jordon ⇒ Unit Matrix
3. Gauss Seidal ⇒ Iteratⁿ ⇒ Indirect

Three moment Eqⁿ

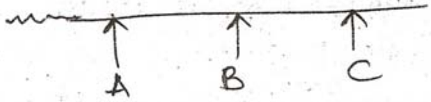
$$M_A\left(\frac{l_1}{I_1E_1}\right) + 2M_B\left(\frac{l_1}{I_1E_1} + \frac{l_2}{I_2E_2}\right) + M_C\left(\frac{l_2}{I_2E_2}\right) = -\frac{6A_1\bar{x}_1}{l_1E_1I_1} - \frac{6A_2\bar{x}_2}{l_2E_2I_2} + \frac{6Eh_a}{l_1} + \frac{6Eh_c}{l_2}$$

Modified SD eqⁿ for S.S. End of continuous Beam:-

$$M_{ba} = \overline{M_{ba}} - \frac{1}{2}\overline{M_{ab}} + \frac{3EI\theta_B}{l} - \frac{3EI\Delta}{l^2}$$

Modified SD eqⁿ for overhang

$$M_{ba} = \overline{M_{ba}} - \frac{1}{2}\overline{M_{ab}} + \frac{3EI\theta_B}{l} - \frac{3EI\Delta}{l^2}$$



MDM: $DF = \frac{\text{Relative stiffness}}{\text{Total stiffness}}$

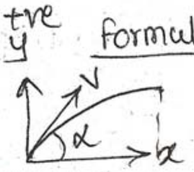
Relative stiffness $= \frac{I}{l}$ ⇒ Fixed End
$= \frac{3}{4}\frac{I}{l}$ ⇒ S.S. end

Redundant Analysis.

Member	F'	unit	l	A	E	$\frac{\Sigma F'ul}{AE}$	$\frac{\Sigma ul}{AE}$
		F = F' + uR				$R = -\left(\frac{\Sigma F'ul}{\Sigma ul}\right)$	

Formulae

① $y = x\tan\alpha - \dfrac{gx^2}{2V^2\cos^2\alpha}$



② Poissions Ratio $= \mu = -\dfrac{\text{Lateral strain}}{\text{Linear strain}}$

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y — Deflection

↓

$EI\dfrac{dy}{dx}$ — Slope

↓

$EI\dfrac{d^2y}{dx^2}$ — Moment

↓

$EI\dfrac{d^3y}{dx^3} = SF$

↓

$EI\dfrac{d^4y}{dx^4} = -\omega$ loading intensity

$\dfrac{T}{J} = \dfrac{\tau}{r} = \dfrac{G\theta}{L} \Rightarrow \dfrac{M}{I} = \dfrac{\sigma}{y} = \dfrac{E}{R}$

Mod. of Ela.	Mod. of Rigidity	Bulk Modulus
E	G	K
Youngs Mod.	Shear Mod.	-
Normal stress $\propto$ Longi. strain	Shear stress $\propto$ Shear strain	Vol. stress $\propto$ Vol. strain
i.e. $\sigma_n \propto \varepsilon_L$	$\tau \propto \gamma$	$\sigma_v \propto \varepsilon_v$
$\sigma_n = E\cdot\varepsilon_L$	$\tau = G\cdot\gamma$	$\sigma_v = K\cdot\varepsilon_v$
$E = \dfrac{\sigma_n}{\varepsilon_L}$	$G = \dfrac{\tau}{\gamma}$	$K = \dfrac{\sigma_v}{\varepsilon_v}$
$E = 2G(1+\mu)$	$=$	$3K(1-2\mu)$

* Stress due to gradually applied load $= \dfrac{P}{A}$
* Stress due to suddenly applied load $= \dfrac{2P}{A}$

$\sigma_x = \left(\dfrac{\sigma_x+\sigma_y}{2}\right) + \left(\dfrac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$

$\tau_{xy} = \left(\dfrac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$

Principal stresses:-

$\sigma_{1,3} = \left(\dfrac{\sigma_x+\sigma_y}{2}\right) \pm \sqrt{\left(\dfrac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$\tan 2\theta_n = \dfrac{-2\tau_{xy}}{(\sigma_x-\sigma_y)}$ [Principal Plane]

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* Max. shear stress

$\tau_{\max/\min} = \pm\sqrt{\left(\dfrac{\sigma_x+\sigma_y}{2}\right)^2 + \tau_{xy}^2}$

Plane of max. shear stress

$\tan 2\theta_s = \dfrac{\sigma_x-\sigma_y}{2\tau_{xy}}$

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* Moment Area Theorem / Mohrs Theorem

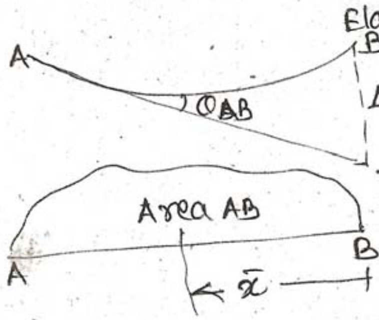
$\theta_{AB} = \displaystyle\int_{\theta_A}^{\theta_B} d\theta = \frac{1}{EI}\int_{x_A}^{x_B} M\cdot dx$

① First Moment-Area Theorem,

$\Delta_{BA} = \dfrac{1}{EI}\displaystyle\int_{x_A}^{x_B} M\cdot x\cdot dx$

② Second Moment area theorem.

$\Delta_{BA} = (\text{Area})_{AB}\cdot\bar{x}_B$



i.e. slope = Area of $\dfrac{M}{EI}$ dia. betn A & B

Deflection = mmt of Area of $\dfrac{M}{EI}$ dia. from B.

Eulers formula, $\boxed{P_{cr} = \dfrac{\pi^2 EI}{L_e^2}}$ $I = Ar^2$, $\dfrac{L_e}{r} = \lambda$

$L_e = L \Rightarrow$ Both end Hinged

$= 2L \Rightarrow$ one end fixed, other free

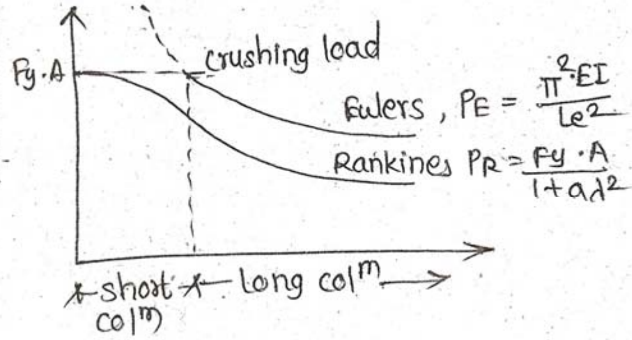
$= L/2 \Rightarrow$ Both end fixed

$= L/\sqrt{2} \Rightarrow$ one fixed other hinged.

$f_{cc} = \dfrac{P_E}{A} = \dfrac{P_{cr}}{A} = \dfrac{\pi^2 E}{\lambda^2} \Rightarrow$ Long colm $\dfrac{L}{D} > 12$

Rankines formula,

$\boxed{P_R = \dfrac{f_y\cdot A}{1+a\lambda^2}}$ $a = \dfrac{f_y}{\pi^2 E}$



stiffness method

$$[S] = \begin{bmatrix} 4EI/l & 2EI/l & \cdots \\ 2EI/l & 4EI/l & \cdots \\ \cdots & \cdots & \cdots \end{bmatrix}$$

$$[A] = \begin{bmatrix} F_1 & F_2 & F_3 & F_4 \\ - & - & - & - \end{bmatrix}$$

$$[S][A]^T$$

$$[A][S][A]^T = [K]$$

$$\{x\} = [K]^{-1} \cdot \{P\}$$

$$[S] \cdot [A]^T \{x\}$$

$$P^* = [FEM] + [S][A]^T \{x\}$$

ILD For Arch (3H)