

Prestressed Concrete

Notes by-

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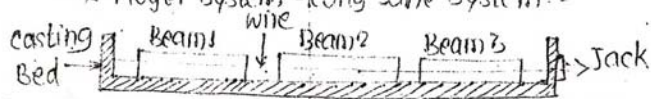
www.pravinkolhe.com

* Tensioning Devices:-

- ① Mechanical → Wedge/Direct bearing/loop.
- ② Hydraulic →
- ③ Electrical → change in length due to heating.
- ④ Thermal →
- ⑤ Chemical → Use of expanding cement

* Pretensioning System:-

* Hoyer System - Long Line System:-



* Post-tensioning System:-

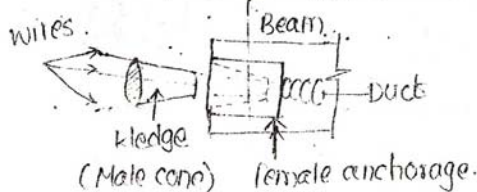
- * Principle:-
- ① Wedge action ⇒ frictional grip
 - ② Direct Bearing ⇒ Bolt action
 - ③ Looping of wire.

Wedge action ⇒ ① Freyssinet System → used in India

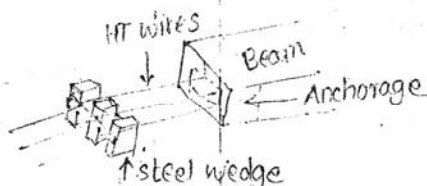
- ② Clifford-Udall
- ③ Anderson
- ④ Magnel-Blaton anchorage:

Main advantage of Freyssinet system:-

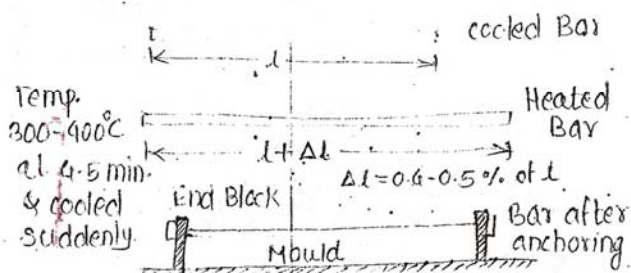
No. of cables are tensioned simultaneously.



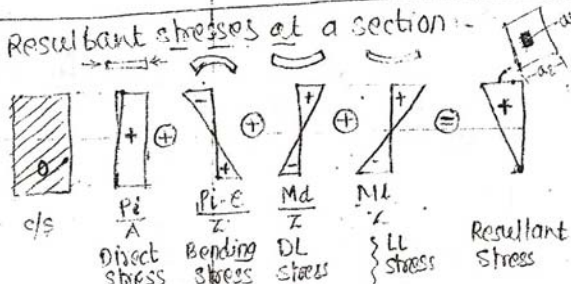
Magnel-Blaton system:-



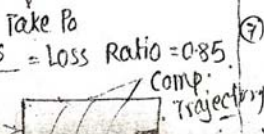
Electrical or Thermal prestressing.



* Resultant stresses at a section:-

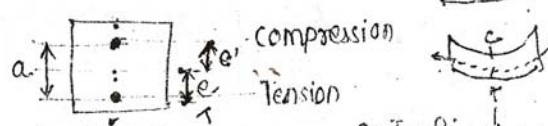
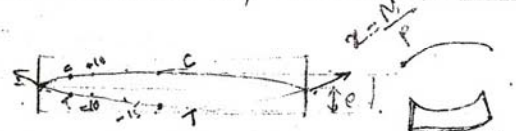


$$\frac{P_o}{P_i} = \frac{\text{Initial PS}}{\text{Final PS}} = \frac{\text{Final/eff. PS}}{\text{Initial PS}} = \text{Loss Ratio} = 0.85$$



P-line, c-line, Thrust line, internal Resisting couple, strength concept

Locus of pt. of application of this resultant force in any structure is 'P' line



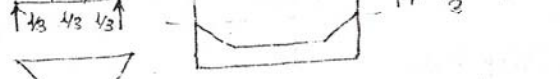
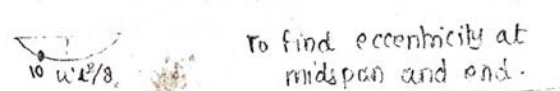
$$a = \frac{Md}{P_i} \quad \text{at DL} \quad \alpha = \frac{ML}{P_o} \quad \text{at LL}$$

$$e + e' = a \Rightarrow a = a - e$$

$$\text{Stress due to DL} = \frac{P_i}{A} + \frac{P_i e'}{Z} \rightarrow \text{Service Transfer stage}$$

$$\text{Stress due to LL} = \frac{P_o}{A} + \frac{P_o e}{Z} \rightarrow \text{Working stage}$$

Load Balancing concept:- (zero SF concept)



Design of End Block

- ① Area of one anchorage = $\pi/4 \times (\text{dia. of cone})^2$
- ② If it is Magnel-Blaton, convert circular section in rectan square as, side of square = $\sqrt{\text{Area}}$ or for Freyssinet system, side = Dia.
- ③ Bearing stress = $\frac{\text{force in each cable}}{\text{Area of anchorage}}$
- ④ Stress in concrete = $\frac{\text{force in each cable}}{\text{Area of concrete for each anchorage}}$
- ⑤ check that ③ & ④ $\neq$ Per stress = 0.8 fck = 0.8 x 0.8 fck.

$$\text{Bursting force} = \frac{0.3 \times P_o}{n} \left[1 - \frac{a_1}{a_2} \right]$$

P_o = force in each cable, n = no. of cable
 a_1 = side of anchorage = $\sqrt{A}$ or D
 a_2 = side of concrete block = width of section

⑦ Providing 8 mm dia. bar, $\rightarrow$ 0.87 f_y
 spacing = $\frac{\text{dist} \times \text{Per stress} \times \text{Depth of beam}}{\text{Bursting force}}$

friction

$$f = (u \alpha + k \cdot x) \times P_i$$

u = coeff. of friction bet concrete & duct

α = slope = $2 \times \frac{4e}{l} \Rightarrow$ one side prestress

= $2 \times \text{slope} = 2 \times \frac{4e}{l} \Rightarrow$ two side prestress

= 0 $\Rightarrow$ straight cable.

k = wave effect per m length

x = total span (m)

P_i = initial prestressing force

✓ X X ✓ X

slip of Anchorage

$$f = \frac{E_s \Delta}{L}$$

E_s = Mod. of Ela. of steel

Δ = slip of anchorage (mm)

L = span (mm)

$$\text{also } \Delta = \frac{P_i L}{A E_s}$$

✓ X X ✓ X

Elastic shortening of concrete

$$f = m \times \frac{P_i}{A_c}$$

m = Modular Ratio = $\frac{E_s}{E_c}$

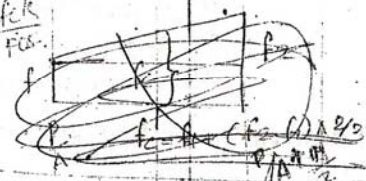
= $\frac{210}{36000} = \frac{2.8}{3600}$

P_i = Initial PS force

A_c = Area of concrete

$$\text{also, } f = m \times f_c, f_c = \text{Bending stress}$$

$$f_c = \frac{P_i}{A} + \frac{P_i \cdot e \cdot y}{I} = f_1 + \frac{P_i}{A} \left(\frac{h}{2} + e \right)$$



✓ X ✓ ✓ X

3% 1%

shrinkage of concrete

$$f = \text{shrinkage strain} \times E_s$$

ϵ for Pretension = $0.0003 = 30 \times 10^{-6}$

ϵ for post tension = $\frac{0.002}{\log_{10}(t+2)} = \frac{200 \times 10^{-6}}{\log_{10}(t+2)}$

t = age of concrete at transfer (days)

✓ ✓ ✓ ✓

7% 6%

creep in concrete

$$f = \epsilon_{cr} \times E_s \times f_c$$

ϵ_{cr} = Elastic creep strength

$$f = \phi \times E_s \times f_c$$

$$f = \phi \times m \times E_c \times f_c$$

m = Modular Ratio

= $\frac{E_s}{E_c}$

where ϕ = creep strain

Elastic strain

f_c = same as (3)

X ✓ ✓ ✓ ✓

6% 5%

creep in steel or Relaxation in steel

0-90 MPa

(2-3%)

X ✓ ✓ ✓ ✓

2% 3%



$$f = f_{pi} (1 + \alpha + kL)$$

$$\text{Anchorage} = \frac{\Delta E_s}{L}$$

$$\text{concrete elastic shortening} = m \times f_c$$

$$\text{concrete shrinkage} = \epsilon \times E_s$$

$$\epsilon_{pre} = 30 \times 10^{-6}$$

$$\epsilon_{post} = \frac{200 \times 10^{-6}}{\log_{10}(t+2)}$$

$$\text{concrete creep} = \phi \times m \times E_c \times f_c$$

$$\text{creep in steel \& Relaxation} = \phi \times E_s \times f_c$$

$$f_1 = \frac{P}{A}$$

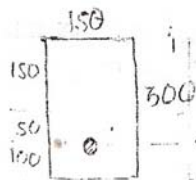
$$f_2 = \frac{P}{A} + \frac{P \cdot e \cdot y}{I}$$

t = days

Handwritten notes and diagrams on the right side of the page, including a vertical list of terms and a small diagram of a beam cross-section.

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4.1

Krishnaraju.



$$A_{st} = 4 \times \frac{\pi}{4} (5)^2 = 78.54 \text{ mm}^2$$

$$f_i = 1200 \text{ N/mm}^2$$

$$P_i = 94.25 \text{ kN}$$

$$e = 50 \text{ mm}$$

$$\text{stress at soffit} = 0$$

$$f_2 = \frac{P_i}{A} \pm \frac{M P_i e}{Z}$$

$$A = 150 \times 300 = 45 \times 10^3 \text{ mm}^2$$

$$P_i = 94.25 \text{ kN}$$

$$e = 50 \text{ mm}$$

$$Z = \frac{I}{y} = \frac{bd^3}{12} \times \frac{2}{D} = 2.25 \times 10^6 \text{ mm}^3$$

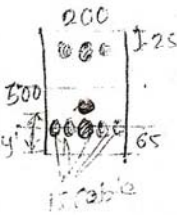
$$f_2 = \frac{94.25 \times 10^3}{45 \times 10^3} \pm \frac{94.25 \times 50 \times 10^3}{2.25 \times 10^6}$$

$$= 2.094 \pm 2.094$$

$$f_1 = 4.19 \text{ MPa} \rightarrow \text{at soffit}$$

$$f_2 = 0 \rightarrow \text{at Top}$$

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4.2



$$A_{st} = 15 \times \frac{\pi}{4} \times 15^2 = 294.5 \text{ mm}^2$$

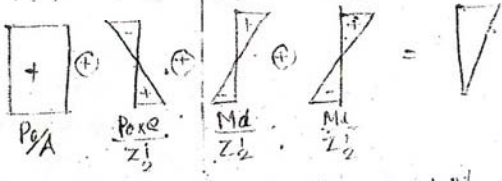
$$A_{st} = 3 \times \frac{\pi}{4} \times 15^2 = 58.9 \text{ mm}^2$$

$$f_i = 840 \text{ MPa}$$

$$\text{stress at extreme fibre at midspan}$$

$$l = 6 \text{ m}$$

$$w_l = 6 \text{ kN/m}$$



$$P_i = 840(294.5 + 58.9) = 296.86 \text{ kN}$$

$$\text{Assume Loss Ratio} = 0.85 = P_o / P_i$$

$$P_o = P_i \times 0.85 = 296.86 \times 0.85 = 252.32 \text{ kN}$$

$$A = 200 \times 500 = 100 \times 10^3 \text{ mm}^2$$

$$y = 150 \text{ mm}$$

$$I = \frac{bd^3}{12} = 450 \times 10^6 \text{ mm}^4$$

$$Z = 3 \times 10^6 \text{ mm}^3$$

$$e = 65 \text{ mm}$$

$$\bar{y} = \left[\frac{15 \times 65 + 3 \times 275}{18} \right] = 100 \text{ mm}$$

$$e = D/2 - \bar{y} = 150 - 100 = 50 \text{ mm}$$

$$M_d = w_l \times l = 24 \times 6 \times 10^4 \times 10^6$$

$$= 1.44 \text{ kN/m}$$

$$M_u = \frac{1.44 \times 6^2}{8} = 6.48 \text{ kNm}$$

$$w_l = 6 \text{ kN/m}$$

$$M_u = 27 \text{ kNm}$$

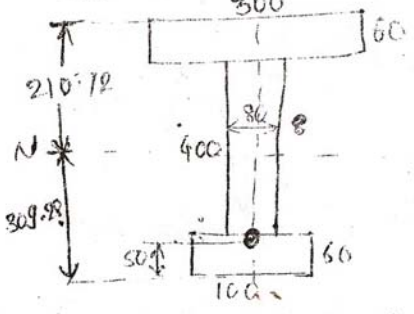
$$\text{stress at Top} = \frac{252.32 \times 10^3}{6 \times 10^4} - \frac{252.32 \times 50}{3 \times 10^6} + \frac{6.48 \times 10^6}{3 \times 10^6}$$

$$+ \frac{27 \times 10^6}{3 \times 10^6} = 11.16 \text{ MPa}$$

$$\text{stress at bottom} = -2.75 \text{ MPa}$$

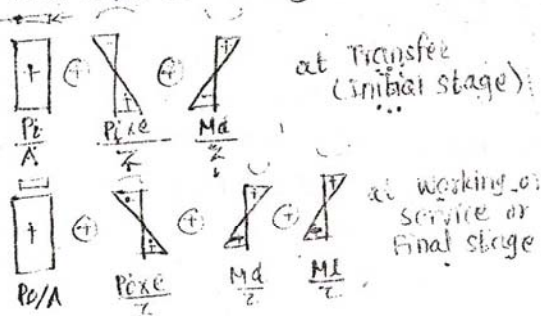
$$w_l = 2 \text{ kN/m}$$

$$l = 8 \text{ m}$$



$$P_o = 100 \text{ kN} - \text{effective Ps force}$$

$$\text{Find stress at working & at transfer}$$



$$P_i = 100 \text{ kN}$$

$$\text{Loss Ratio} = 85\% = \frac{P_o}{P_i} \Rightarrow P_o = \frac{100}{0.85} = 117.65 \text{ kN}$$

$$A = (300 \times 60) + (400 \times 100) + (60 \times 100)$$

$$= [18 + 32 + 6] \times 10^3$$

$$= 56 \times 10^3 \text{ mm}^2$$

$$\bar{y} = \frac{y_1 A_1 + y_2 A_2 + y_3 A_3}{A}$$

$$= \frac{[60 + 400 + 30] \times 18 \times 10^3 + [60 + 200 \times 2 \times 10^3] + [80 \times 6 \times 10^3]}{56 \times 10^3}$$

$$= \bar{y} = 309.28 \text{ mm from bottom}$$

$$y_t = 210.72 \text{ mm}$$

$$y_b = 309.28 \text{ mm}$$

$$e = 309.28 - 50 = 259.28$$

$$I = \frac{bd^3}{12} + ah^2$$

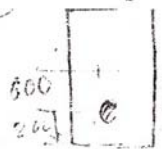
$$= \frac{300 \times 60^3}{12} + 18 \times 10^3 [210.72 - 30]^2 +$$

$$+ \frac{80 \times 400^3}{12} + 32 \times 10^3 [210.72 - 260]^2 +$$

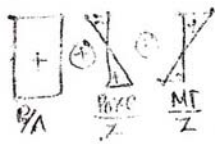
$$+ \frac{100 \times 50^3}{12} + 6 \times 10^3 [309.28 - 30]^2$$

$$I = 598.3 \times 10^6 + 504.4 \times 10^6 + 409 \times 10^6$$

$$= 1566.7 \times 10^6 \text{ mm}^4$$



$\phi = 14 \text{ mm}$ H1
 $y' = 200 \text{ mm}$
 $f_0 = 700 \text{ N/mm}^2$
 $M_{\max} = ?$
 $z = \frac{b d^3}{12} \times \frac{2}{d} = 18 \times 10^6 \text{ mm}^3$



No Tension Condition

$\frac{P_0}{A} + \frac{P_0 e}{Z} = \frac{M_T}{Z} \Rightarrow P_0 = \frac{700 \times 4 \times \pi (10)^2}{4} = 431 \text{ kN}$

① $\frac{P_0}{A} = \frac{431 \times 10^3}{600 \times 250} = 2.87 \text{ MPa}$

② $\frac{P_0 e}{Z} = \frac{431 \times 10^3 \times 100}{18 \times 10^6} = 2.87 \text{ MPa}$

③ For No Tension Condition,

$\frac{P_0}{A} + \frac{P_0 e}{Z} - \frac{M_T}{Z} = 0$

$2.87 + 2.87 = \frac{M_T}{Z}$

$M_T = 5.75 \times Z$

$= 5.75 \times 18 \times 10^6$

$M_T = 86.2 \text{ kNm}$

$M_T = M_d + M_l$

$M_d = \frac{w_d \times l^2}{8}, w_d = 25 \times 230 \times 600 \times 10^{-6} = 3.75 \text{ kN/m}$

$M_d = \frac{3.75 \times 6^2}{8} = 16.875$

$M_l = 86.2 - 16.875 = 69.325 \text{ kNm}$

$M_l = \frac{w_l \times l^2}{8} \Rightarrow 69.325$

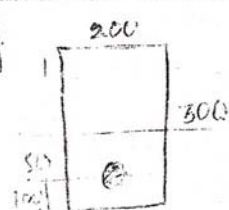
$w_l = \frac{69.325 \times 8}{36} = 15.4$

$w_l = w_d + w_s$

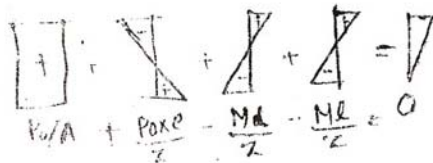
$w_s = \frac{15.4}{1.5} = 10.27 \text{ kN/m}$

Allowable WORKING load = 10.27 kN/m

106/4.5



$l = 6 \text{ m}$
 $w_d = 4 \text{ kN/m}$
 $V = 24 \text{ kN/m}$
 $P = ?$
 Zero fibre stress at bottom at service
 $e = 50 \text{ mm}$



$\frac{P_0}{A} + \frac{P_0 e}{Z} - \frac{M_d}{Z} - \frac{M_l}{Z} = 0$

$A = 200 \times 300 = 6 \times 10^4 \text{ mm}^2$

$e = 50 \text{ mm}$

$w_d = 24 \times 6 \times 10^4 \times 10^{-6} = 1.44 \text{ kN/m}$

$M_d = \frac{1.44 \times 6^2}{8} = 6.48 \text{ kNm}$

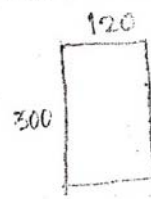
$w_s = 4 \text{ kN/m}$

$w_d + M_d = 18 \text{ kNm}$

$P_0 \left(\frac{10^3}{6 \times 10^4} + \frac{50 \times 10^3}{3 \times 10^6} \right) - \frac{(6.48 + 18) \times 10^3}{3 \times 10^6} = 0$

$P_0 = 244.8 \text{ kN}$

111/4.6



$w_d = 4 \text{ kN/m}$

$l = 6 \text{ m}$

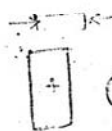
$P_0 = 180 \text{ kN}$

$e = 0$

Locate P line.

$M_T = w_d \times l^2 = 18 \text{ kNm}$

$M = \frac{4 \times 6^2}{8} = 18 \text{ kNm}$



$\frac{P_0}{A} = \frac{180 \times 10^3}{(120 \times 300)}$

$\frac{M}{Z} = \frac{18 \times 10^6}{(120 \times 300)^2}$

Stress at Top = 5 MPa

Stress at Top = 5 MPa

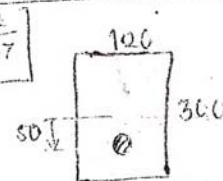
Stress at bottom = 5 MPa

Eccentricity at ends = 0

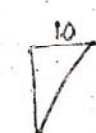
Eccentricity at midspan = $\frac{M}{P} = \frac{18 \times 10^6}{180} = 100 \text{ mm}$



112/4.7



$l = 6 \text{ m}$
 $w_d = 4 \text{ kN}$
 $P = 180 \text{ kN}$
 $e = 50 \text{ mm}$
 $A = 36 \times 10^3 \text{ mm}^2$



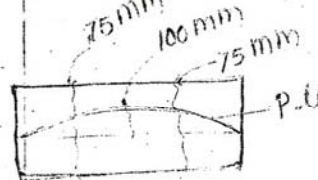
$Z = \frac{b \times d^2}{6} = 18 \times 10^5 \text{ mm}^3$

$M_T = \frac{4 \times 6^2}{8} = 18 \text{ kNm}$

Stress at Top bottom = $\frac{180 \times 10^3}{36 \times 10^3} + \frac{180 \times 10^3 \times 50}{18 \times 10^5} + \frac{18 \times 10^6}{18 \times 10^5}$

$= 10$

Position of P line:

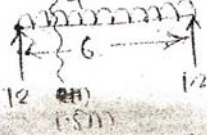


Mat midspan = 18 kNm

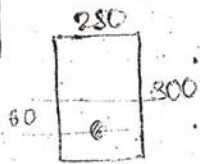
$a = \frac{18 \times 10^6}{P} = \frac{18 \times 10^6}{180 \times 10^3} = 100 \text{ mm}$

Mat quarter span = 13.5

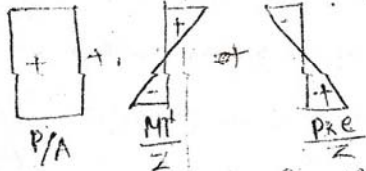
$a = \frac{M}{P} = \frac{13.5 \times 10^6}{180 \times 10^3} = 75 \text{ mm}$



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4.8



$P_0 = 540 \text{ kN}$
 $W = 68 \text{ kN}$

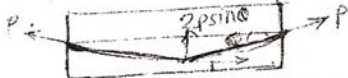
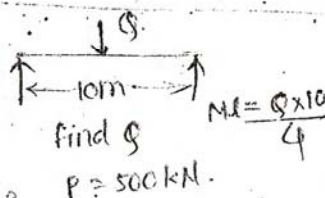
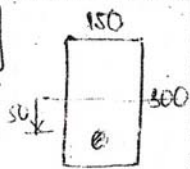


$A = 250 \times 300 = 75 \times 10^3 \text{ mm}^2$
 $Z = \frac{250 \times 300^2}{6} = 3.75 \times 10^6 \text{ mm}^3$

$M_I = \frac{68 \times 15}{4} = 51 \text{ kNm}$

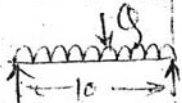
$\therefore$ stress at top = $\frac{51 \times 10^3}{75 \times 10^3} = 0.68 \text{ N/mm}^2$
 stress at bottom = $\frac{51 \times 10^3}{3.75 \times 10^6} = 1.36 \text{ N/mm}^2$

4.11
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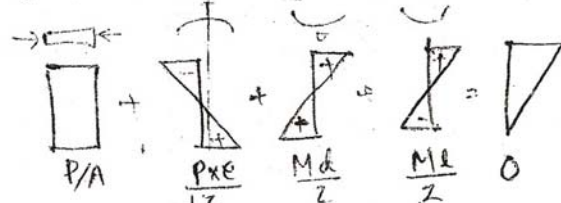
$2P \sin \theta = Q$
 $2P \tan \theta = Q$
 $2 \times 500 \times 0.01 = Q$
 $Q = 10 \text{ kN}$

b) considering SW of beam.



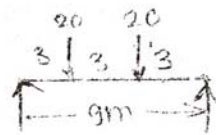
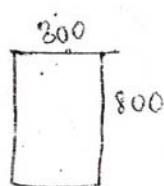
$W_d = 2.5 \times 150 \times 300 \times 10^{-6} = 1.125$
 $M_d = \frac{1.125 \times 10^2}{8} = 14.0625 \text{ kNm}$

If Q is concentrated load at centre.

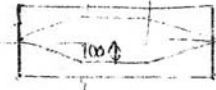
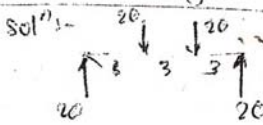


$\frac{P}{A} + \frac{Pxe}{Z} - \frac{M_d}{Z} - \frac{M}{Z} = 0$
 $\frac{500 \times 10^3}{45 \times 10^3} + \frac{500 \times 150 \times 10^{-3}}{2.25 \times 10^6} - \frac{14.0625}{2.25 \times 10^6} - \frac{2.25}{2.25 \times 10^6} = 0$

4.12
121



cable profile. $e = 100 \text{ mm}$ at middle.
 $P = ?$
 Neglect SW.

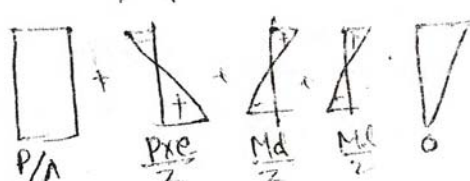


$M = Pxe$
 $\frac{W_d}{3} = Pxe$
 $P = \frac{W_d}{3e}$

$W_d = 24 \times 0.3 \times 0.8 = 5.76 \text{ kN/m}$
 $M_d = 58.32 \text{ kNm}$

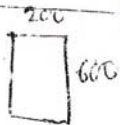
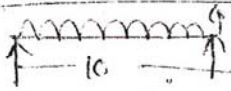
$P = \frac{W_d}{3e} = \frac{20 \times 10^3}{3 \times 100} = 666.67 \text{ kN}$
 SW not consid. fixed.

Now, considering SW,



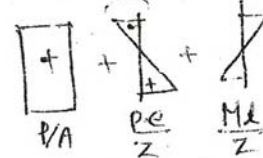
$\frac{P}{A} + \frac{Pxe}{Z} - \frac{M_d}{Z} - \frac{M}{Z} = 0$
 $\frac{P}{24 \times 10^4} + \frac{P \times 100}{32 \times 10^6} - \frac{58.32}{32} - \frac{60}{32} = 0$

$P = 507 \text{ kN}$



$P = ?$
 $e = 100 \text{ mm}$ at midspan,

Neglecting SW:



$M = \frac{W_d e^2}{8} = Pxe$
 $P = ?$

$A = 12 \times 10^4 \text{ mm}^2$
 $Z = 12 \times 10^6 \text{ mm}^3$
 $M_d = \frac{W_d e^2}{8} = \frac{4 \times 10^2}{8} = 50 \text{ kNm}$

$\frac{P}{12 \times 10^4} + \frac{P \times 100}{12 \times 10^6} - \frac{50 \times 10^3}{12 \times 10^6} = 0$

$P = 250 \text{ kN}$

$P = \frac{W_d e^2}{8} = \frac{4 \times 10^2}{8} = 50 \text{ kNm}$

at T	at W	Pre	Post	Fin
✓	X	X	✓	X
✓	X	X	✓	X
✓	✓	✓	✓	✓
✓	X	✓	✓	X
X	✓	✓	✓	✓
X	✓	✓	✓	✓