## Notes by-

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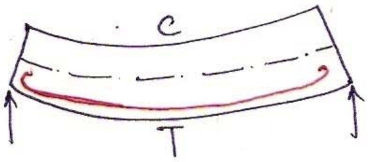
# Singly Reinforced Section

Basic formula for flexure : $\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}$

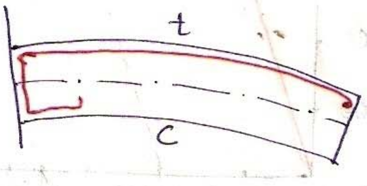
Torsion : $\frac{\tau}{R} = \frac{T}{J} = \frac{G\theta}{l}$

(See Notes) ~~of RoM~~




Stresses above NA = Comp.
Stresses below NA = Tensile

Simply supported beam




Stresses above NA = Tensile
Stresses below NA = Comp.

Cantilever beam.

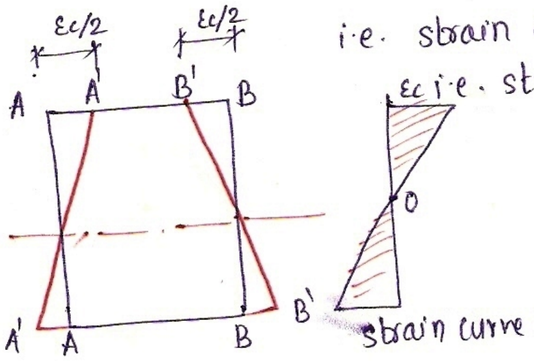
Concrete is weak in tension ∴ Area of concrete in tension zone is neglected. & Tension is entirely carried by bond bet$^n$ steel & concrete. Thus reinforcement is provided in tension zone only so it ~~H~~ is called as singly reinforced section.

## Basic assumptions (Cl.38/Pg-69)

* strength of a member at LS of collapse ⇒ Ultimate strength.

① Plane section normal to axis remains plane after bending.

i.e. strain at any point ∝ dist. from NA.
i.e. strain varies linearly with NA.




AA & BB are sections.

$\boxed{A'B - A'B' = \text{strain in concrete} = \varepsilon_c}$

② Ultimate state of collapse in flexure is said to have reached when max. comp. strain in concrete ($\varepsilon_{cu}$) in outermost comp. fibre reaches to 0.0035. (IS)

③ Distribution of compressive stress in concrete across the section is defined by idealized stress-strain curve.
The variation of comp. stress across the depth ($x_u$) of concrete in compression is referred as "stress Block".

max. comp. stress in concrete $= 0.67 \times \frac{f_{ck}}{\gamma_m = 1.5}$

0.67 ↓ "specimen effect" ; $\gamma_m = 1.5$ ⇓ FOS (Partial)

④ Tensile strength of concrete is neglected. i.e. Tension is entirely carried by bond betⁿ steel & concrete.

⑤ Bond betⁿ steel & concrete is rigid up to failure.

⑥ Stresses in reinforcement is derived from respective stress-strain curve depending upon grade of steel & partial FOS = 1.15 is applied.

⑦ Max. strain in tension reint. in a sectⁿ at failure $\not< 0.002 + \frac{f_y}{1.15 E_s}$.

i.e. failure is initiated by yielding of steel in tension.

Interview

Value of $\frac{0.87 f_y}{E_s}$ represents max. elastic strain + 0.002 (Inelastic strain)

This is due to ensuring sufficient yielding of steel to give large perceptible deflection providing a sort of warning of forecoming failure.

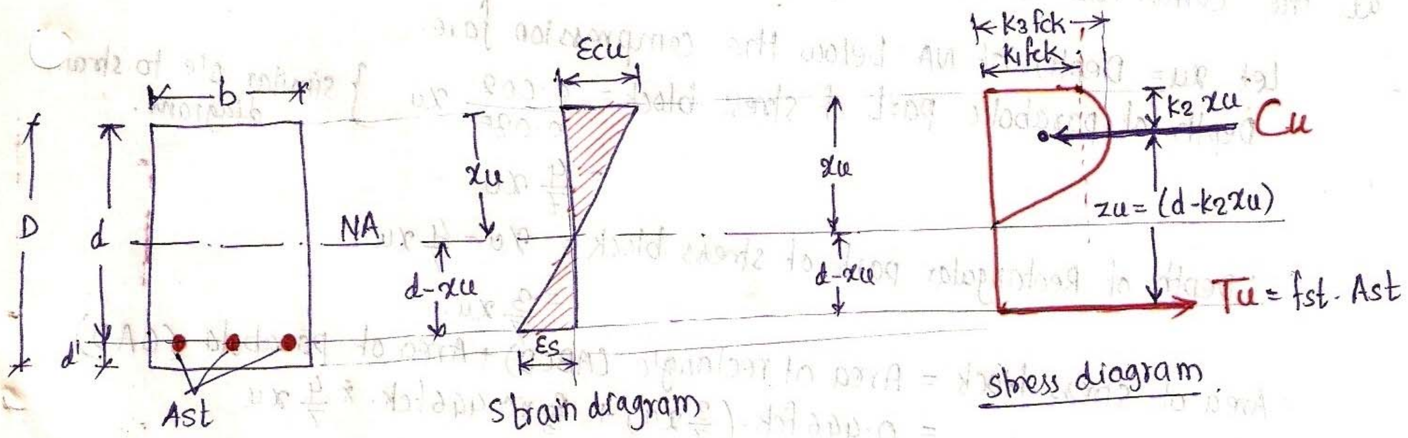
★ **Stress Block Parameters:-**

Shape of stress block is characterized by its max. ordinate ; area & position of its centroid which are defined as coeff. $k_1, k_2, k_3$ & known as stress block parameters.

∴ Max. ordinate of stress block $= k_3 \cdot f_{ck}$

Area of stress block $= k_1 \cdot f_{ck} \cdot x_u$

Dist. of centroid of stress block from extreme compression layer $= k_2 \cdot x_u$.




$k_1 = \frac{\text{Avg. comp. stress in concrete } (f_{av})}{\text{Cube crushing strength of conc. } (f_{ck})}$

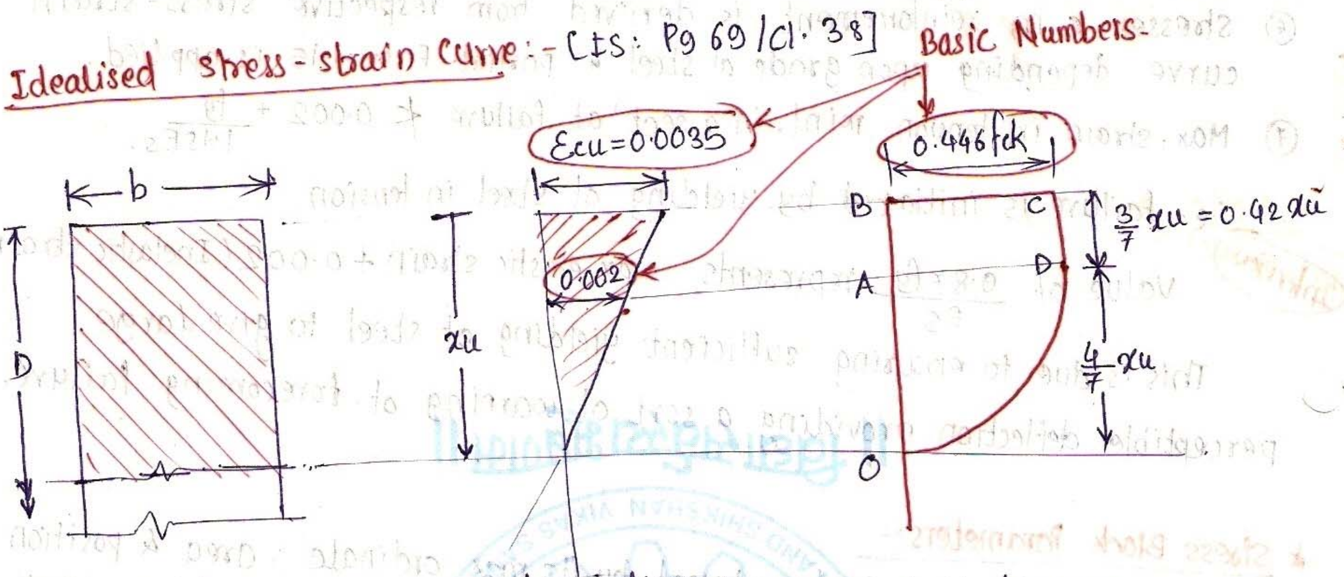
$k_2 = \frac{\text{Dist. of centroid of stress block from comp. fibre (extreme)}}{\text{Depth of stress block } (x_u)}$

$k_3 = \frac{\text{Max. ordinate of stress block } (f_{max})}{\text{cube crushing strength of concrete } (f_{ck})}$

$\therefore$ Total comp = $C_u$ = Area of stress block $\times$ b
$= k_1 \cdot f_{ck} \cdot b \cdot x_u$

Total Tension = $T_u = f_{st} \cdot A_{st}$
Lever arm = $z_u = (d - k_2 x_u)$
$\therefore M_{ur} = C_u \cdot z_u = k_1 \cdot f_{ck} \cdot b \cdot x_u (d - k_2 x_u)$
$M_{ur} = T_u \cdot z_u = f_{st} \cdot A_{st} \cdot (d - k_2 \cdot x_u)$

**Idealised stress-strain curve :-** [IS: Pg 69/Cl. 38]



Beam c/s.  Strain diagram  stress diagram

Idealized stress-strain curve prescribed in IS code is "Rectangular Parabolic". It consist of parabola emerging from NA with its apex lying at the point corresponding to 0.002 strain & rectangular beyond that pt. & terminating at the comp. face where strain is max. ~~is~~ i.e. 0.0035.

Let $x_u$ = Depth of NA below the compression face.
Depth of parabolic part of stress block = $\frac{0.002}{0.035} x_u$ } similar $\Delta$le to strain diagram.
$= \frac{4}{7} x_u$

$\therefore$ Depth of Rectangular part of stress block = $x_u - \frac{4}{7} x_u$
$= \frac{3}{7} x_u$.

$\therefore$ Area of stress block = Area of rectangle (ABCD) + Area of parabola (OAD)
$= 0.446 f_{ck} \cdot (\frac{3}{7} x_u) + \frac{2}{3} \times 0.446 f_{ck} \cdot \times \frac{4}{7} x_u$
$= 0.446 f_{ck} \cdot x_u (\frac{17}{21})$
$= 0.361 f_{ck} \cdot x_u$
$\approx 0.36 f_{ck} \cdot x_u$.

Dist. of centroid of stress block '$\bar{x}$' from comp. face is obtained by taking moments of area @ top comp. face.
i.e. Varignon's theorem.

$$\therefore \bar{x} = \frac{a\left(0.446\, fck \times \frac{3}{7} xu\right)\left(\frac{1}{2} \cdot x \cdot \frac{3}{7} xu\right) + \left[\frac{2}{3}\left(0.446\, fck \times \frac{4}{7} xu\right)\right]\left[\frac{3}{7} xu + \frac{3}{8} \cancel{x} \cdot \frac{4}{7} xu\right]}{0.361\, fck \cdot xu}$$

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$= 0.416\, xu$

$\therefore \boxed{\bar{x} \approx 0.42 xu}$

By using these parameters ; Total compression resisted by concrete ,

[i.e. $C = 0.36\, fck \cdot xu$] is calculated.

& Moment of resistance contributed by concrete in compression . is calculated .

## Modes of failure :-

V. Imp. Aptitute

Max. strain criteria. (i.e. $\delta l / l$) is accepted as failure criteria for RCC, under flexure (i.e. bending)

$\therefore$ RC member is considered to fail when strain in concrete ($\varepsilon c$) in extreme comp. fibre reaches its ultimate value $\varepsilon cu$ (i.e. 0.0035)

When this max. strain in concrete is reached , the actual strain in steel can be

= its failure strain (Balanced section)
< its failure strain (Over reinforced section)
> its failure strain (Under reinforced section)

depending upon relative proportions of steel & concrete. (Grades.)

$\therefore$ Based on %age of steel in a section 3 failure modes occur –

① Balanced failure ⇒ Balanced section
② U-R failure ⇒ UR section
③ O-R failure. ⇒ OR section.

## ① Balanced section :-

When Ratio of steel to concrete (%age of steel) in a section is such that the strain in steel & strain in concrete reach their max. value simultaneously the section is referred to as a balanced section. or critical section.

~~i.e. $\varepsilon steel = \varepsilon concrete = \varepsilon failure$~~

The %age of steel in this section is known as "critical steel percentage".

· This is ideal condition,

Thus in balanced section,

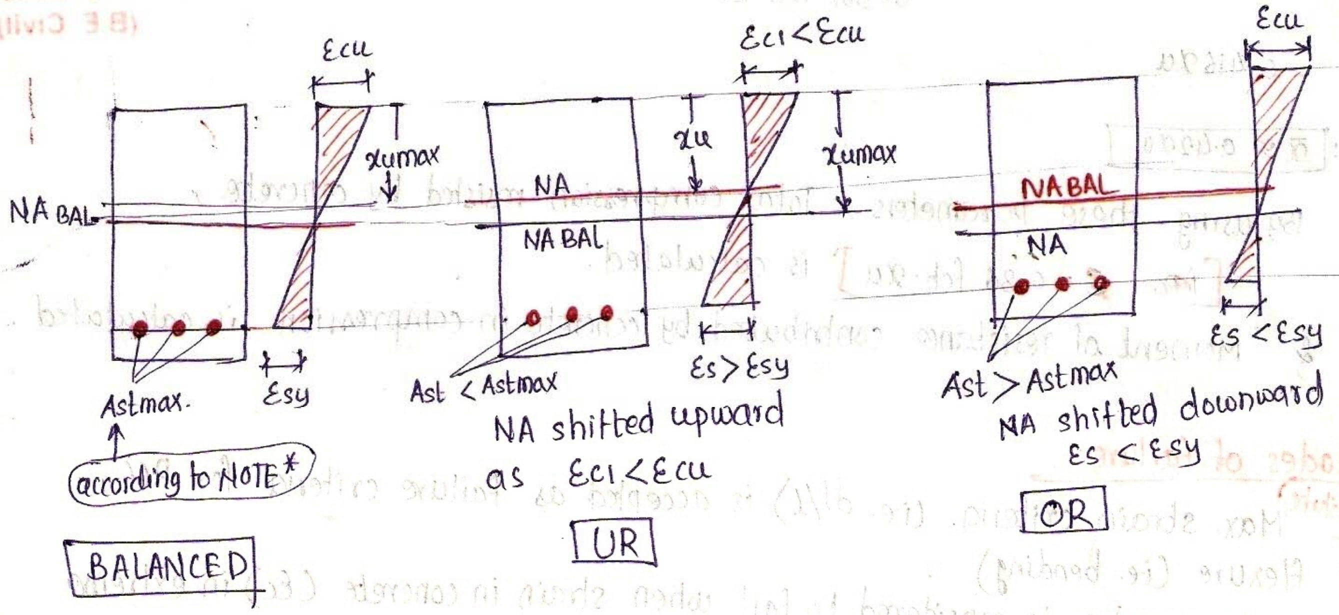
$$\left.\begin{aligned} \varepsilon steel &= \varepsilon steel(yield) \\ \varepsilon concrete &= \varepsilon concrete(ultimate) \end{aligned}\right\} \text{failure strain.}$$

Aptitute

NOTE* It can be observed that , Balance section, ⇒ min. concrete ⇒ Max. steel.
UR section ⇒ min. steel ⇒ max. concrete.
} for same section.

$\therefore$ UR is preferred ; as cost of steel > cost of concrete

② ~~UR Section~~ :-




**UR section**

① Percentage of steel in section < critical percentage

② Failure is due to yielding of steel

③ since Ast < Ast balanced
∴ NA shifts upward than NA bal to satisfy eqm i.e. C = T

④ Upward movement of NA results in increase in lever arm & thus increased MR till extreme comp. fibre reaches its ultimate strain Ecu.

⑤ In UR sectn, failure is initiated by yielding of steel but finally collapse will occurs due to crushing of concrete. (Interview)

⑥ Failure of UR section is characterised by substantial deflection & extensive cracking of concrete giving a sort of warning of impending failure.

⇑ Thats why "UR" is preferred to "OR"

**OR section**

① Percentage of steel in section > critical percentage

② Failure is due to crushing of concrete.

③ since Ast > Ast bal.
NA shifts down the NA BAL to satisfy eqm i.e. ~~Ct~~ C = T

(Interview)

④ ⇐ Thats why "UR" is preferred to Balanced section

⑤ In OR section concrete reaches its ~~max.~~ ultimate (failure) strain but strain in steel is less than yield (failure) strain. thus section will fails only by crushing of concrete.

⑥ Failure of OR section occurs by crushing of concrete without giving any warning. (Interview)

∴ Design code restrict %age of steel up to balanced ~~or seler~~ section only, thus OR are not allowed. Also xu is not increased beyound xubal.
∴ xuBAL = xumax & AstBal = Astmax

## Properties of SR section

Consider a singly reinforced rectangular beam of width 'b' & total depth 'D' subjected to an "Ultimate Bending moment M", under the ultimate load.

Steel is provided in tension side & total area of steel = Ast

These bars are provided with effective cover ~~d' = D-d~~ d'.

Interview ∴ Effective depth, i.e. depth from extreme ~~fibre~~ compression fibre to the centroid of the tension reinforcement, $d = D - d'$

### ① UR section :-

Fig. shows strain & stress distribution diagram for UR section.

Let $b$ = width of section.

$d$ = effective depth of section = Total depth − eff. cover = $D - d'$

$x_u$ = Depth of NA from extreme comp. fibre.

$k_u$ = NA factor $= \frac{x_u}{d}$

$A_{st}$ = Area of tension steel.

$p_t$ = percentage of steel $= \frac{100 A_{st}}{b \cdot d}$ → Note that 'd' NOT 'D'

$M_{ur}$ = Ultimate moment of Resistance of a section.

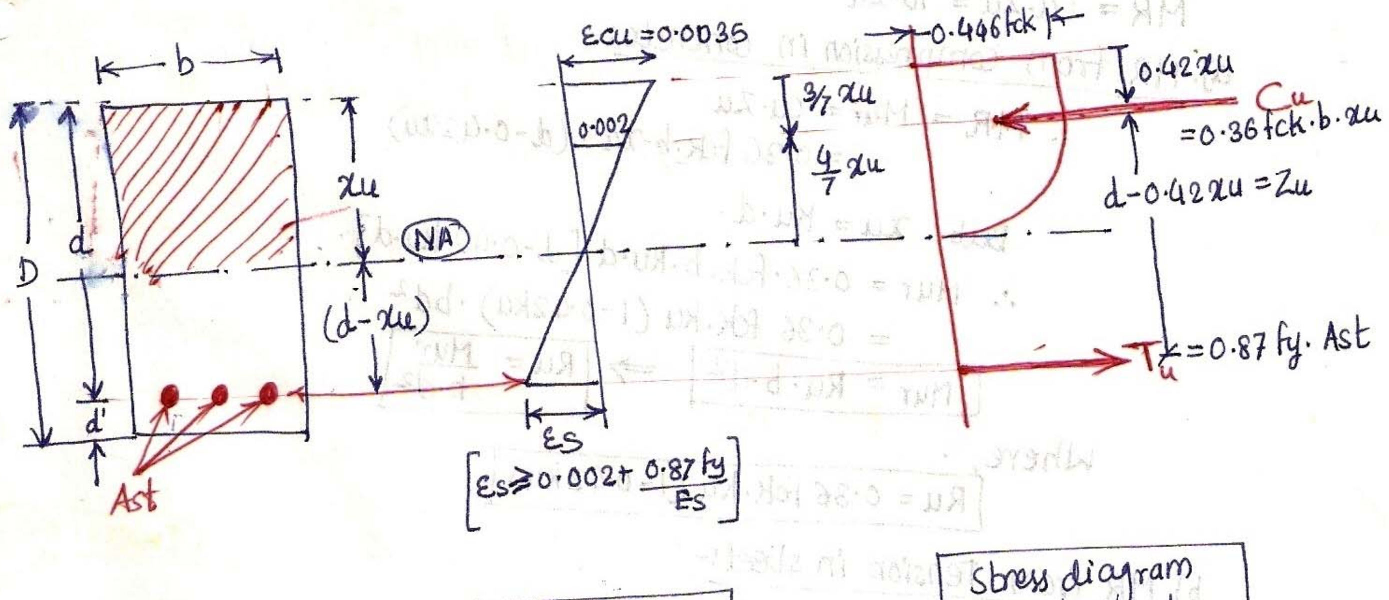
$R_u$ = Moment of Resistance factor $= \frac{M_u}{b \cdot d^2}$

$M_u$ = Ultimate design moment.

$f_{yd}$ = Design yield stress

$= \frac{f_y}{1.15} = 0.87 f_y$

$f_y$ = ~~character~~ Yield stress (failure) stress. stress at upper yield pt




C/s of Beam

Strain diagram

Stress diagram (Idealized according to code)

For UR Section;

$$\varepsilon_s \geq 0.002 + \frac{0.87 f_y}{E_s}$$

$$f_{yd} = 0.87 f_y$$

## a] Depth of NA :-

Considering eq$^m$ of internal forces.

i.e. Total Compression = Total Tension.

i.e. $C_u = T_u$

i.e. $0.36 f_{ck} \cdot b \cdot x_u = 0.87 f_y \cdot A_{st}$

$$\therefore \boxed{x_u = \frac{0.87 \cdot f_y \cdot A_{st}}{0.36 \cdot f_{ck} \cdot b}}$$

NA depth factor :-

$$K_u = \frac{x_u}{d}$$

$$= \frac{0.87 f_y}{0.36 f_{ck}} \left(\frac{A_{st}}{b \cdot d}\right)$$

$$\boxed{K_u = \frac{0.87 f_y}{0.36 f_{ck}} \cdot p_t}$$

Where $p_t$ = steel factor = $\frac{A_{st}}{b \cdot d}$

## b) Ultimate Moment of Resistance :-

$$MR = C_u \cdot Z_u = T_u \cdot Z_u$$

### a] MR from compression in concrete :-

$$\therefore MR = M_{ur} = C_u \cdot Z_u$$

$$= 0.36 f_{ck} \cdot b \cdot x_u \cdot (d - 0.42 x_u)$$

But $x_u = K_u \cdot d$.

$$\therefore M_{ur} = 0.36 \cdot f_{ck} \cdot b \cdot K_u \cdot d \, [d - 0.42 K_u \cdot d]$$

$$= 0.36 f_{ck} \cdot K_u (1 - 0.42 K_u) \cdot bd^2$$

$$\boxed{M_{ur} = R_u \cdot b \cdot d^2} \Rightarrow \boxed{R_u = \frac{M_{ur}}{b \cdot d^2}}$$

Where,

$$\boxed{R_u = 0.36 f_{ck} \cdot K_u (1 - 0.42 K_u)}$$

### b] MR from Tension in steel :-

$$M_{ur} = T_u \cdot Z_u$$

$$= 0.87 f_y \cdot A_{st} (d - 0.42 x_u)$$

But $x_u = \frac{0.87 f_y \cdot A_{st}}{0.36 f_{ck} \cdot b}$

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$$\therefore \; M_{ur} = 0.87\, f_y . A_{st} \left[ d - 0.42 \times \frac{0.87 \cdot f_y \cdot A_{st}}{0.36 \cdot f_{ck} \cdot b} \right]$$

$$\boxed{\therefore M_u = M_{ur} = 0.87\, f_y . A_{st} \left( d - \frac{f_y . A_{st}}{f_{ck} \cdot b} \right)} \longrightarrow \text{V. Imp.}$$

steel yielded first

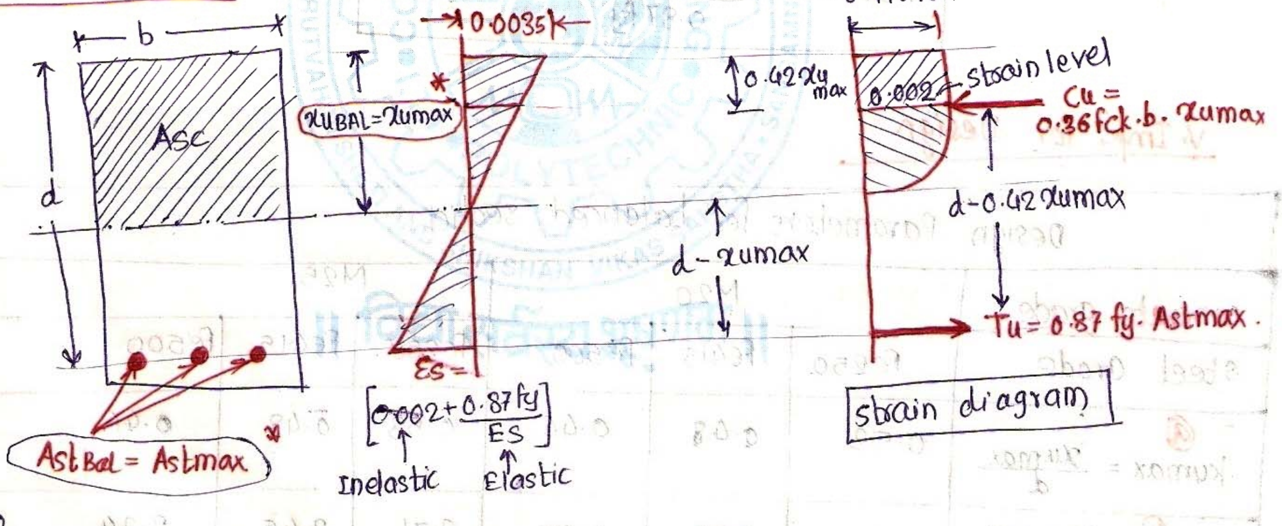
In many problems; we know Mu i.e. ultimate Resisting moment & we have to calculate "Ast".

∴ By solving above eqⁿ for Ast; we get

$$\boxed{A_{st} = \frac{0.5\, f_{ck}}{f_y} \left[ 1 - \sqrt{1 - \frac{4.6\, M_u}{f_{ck} \cdot b \cdot d^2}} \right] b \cdot d}$$

also, $$\boxed{P_t = \frac{A_{st}}{bd} = \frac{0.5\, f_{ck}}{f_y} \left[ 1 - \sqrt{1 - \frac{4.6\, R_u}{f_{ck}}} \right]}$$

② Balanced Section :-




Interview

* Note :- IS code does not permit OR section i.e. $\boxed{A_{st\,bal} = A_{stmax} = A_{st\,critical}}$
i.e. Ast is restricted up to Astbal.
& $x_u$ is restricted up to ~~Ast~~ $x_{ubal}$ i.e. $\boxed{x_{ubal} = x_{umax} = x_{u\,critical}}$

According to assumption, max. comp. strain in concrete = 0.0035
max. tensile strain in steel = $0.002 + \frac{0.87 f_y}{E_s}$.

The design factors $k_u$ - ( NA depth factor = $\frac{x_u}{d}$ ) ; $R_u$ ( MR factor = $\frac{M_{ur}}{b.d^2}$ ) & $P_t$ (Steel factor = $\frac{A_{st}}{b.d}$) for balanced section will be maximum values as,

$$k_{umax} = \frac{x_{umax}}{d} \;;\; R_{umax} = \frac{M_{urmax}}{b.d^2} \;\&\; P_{tmax} = \frac{A_{stmax}}{b.d}$$

∴ Value of $x_{umax}$; [from strain diagram];

$$\frac{x_{umax}}{d-x_{umax}} = \frac{0.035}{0.002+\frac{0.87 f_y}{E_S}} \Rightarrow \frac{x_{umax}}{d} = \frac{0.0035}{0.0035+\left(0.002+\frac{0.87 f_y}{E_S}\right)}$$

But $E = 2\times10^5\ N/mm^2$.

$$\boxed{\therefore x_{umax} = \frac{700}{1100+0.87 f_y}\cdot d} \longrightarrow (a)$$

$$\boxed{\therefore k_{umax} = \frac{x_{umax}}{d} = \frac{700}{1100+0.87 f_y}} \longrightarrow (b)$$

$$\boxed{MR = \therefore M_{ur\,max} = 0.36\cdot f_{ck}\cdot b\cdot x_{umax}\,(d-0.42\,x_{umax})} \cdots \text{Comp. side.}$$

$$\boxed{M_{ur\,max} = M_u = R_{umax}\cdot b\cdot d^2}$$

where
$$\boxed{R_{umax} = 0.32\, f_{ck}\cdot k_{umax}\,(1-0.42\,x_{umax})}$$

$$\boxed{P_{t\,max} = \frac{0.36\, f_{ck}}{0.87\, f_y}\cdot k_{umax}} \longrightarrow (c)$$

$$\boxed{\therefore A_{st\,max} = \frac{0.36\, f_{ck}\cdot b\cdot x_u}{0.87\, f_y}\cdot k_{umax}}$$

## V. Imp. for Design

| Design Parameters for balanced section. | | | | | | |
|---|---|---|---|---|---|---|
| concrete grade | M20 | | | M25 | | |
| steel Grade | Fe250 | Fe415 | Fe500 | Fe250 | Fe415 | Fe500 |
| (a) $k_{umax} = \frac{x_{umax}}{d}$ | 0.53 | 0.48 | 0.46 | 0.53 | 0.48 | 0.46 |
| (b) $R_{umax} = \frac{M_{ur\,max}}{b\cdot d^2}$ | 2.97 | 2.76 | 2.67 | 3.71 | 3.45 | 3.34 |
| or $R_{umax}$ | 0.149 fck | 0.138 fck | 0.133 fck | 0.149 fck | 0.138 fck | 0.133 fck |
| $P_{t\,max} = \frac{A_{st\,max}}{b\cdot d^2}$ (c) | 1.76 | 0.96 | 0.76 | 2.20 | 1.20 | 0.93 |

**Note :-** $k_{umax}$ is dependent of grade of steel. only but independent on grade of concrete. since $\varepsilon_{cu} = 0.0035$ is taken as constant. irrespective of grade of concrete.

# Types of problems on UR section.

**I - Section Analysis**

Given : Material (fck, fy)
: Section (b, d, Ast)

↓

To find Mur or max. LL carring capacity for given span or end cond$^n$.

**II - Section Design**

Given : (fck, fy)
: Mu

↓

- To design Balanced sect$^n$ for given b or b/d ratio
- To find Ast for given section
- To design UR section for given section.

**III - Mis.**

## Type - I : Section Analysis

Given : fck, fy, b, d, Ast
To find : Mur or allowable load on given span.
Procedure : ① calculate $x_u = \dfrac{0.87\, f_y \cdot A_{st}}{0.36\, f_{ck} \cdot b \cdot \cancel{d}}$

② Determine depth of balanced section NA i.e. $x_{ubal} = x_{umax}$.

$$\therefore x_{ubal} = x_{umax} = \frac{700}{1100 + 0.87 f_y} \cdot d$$

③ Compare $x_u$ with $x_{umax}$.

If $x_u > x_{umax}$ ⇒ sect$^n$ is ~~UR~~ OR ⇒ Not permited ∴ take $x_u = x_{umax}$.
If $x_u = x_{umax}$ ⇒ sect$^n$ is Balanced ∴ O.K.
If $x_u < x_{umax}$ ⇒ sect$^n$ is UR ⇒ ∴ O.K

④ Calculate $M_{ur} = 0.87\, f_y \cdot A_{st}\,(d - 0.42\, x_u)$

⑤ ~~B~~ obtain Mu in terms of wu & L & equate it with Mur to know 'wu' & hence calculate working load.

**Pro:** A RC beam of rectangular section 230×400 mm ~~is~~ is reinforced with 4# ~~ɸ~~12 mm, provided with effective cover of 31 mm. Calculate the ultimate MR of the section & max LL ~~load~~, ~~on~~ this beam can carry if it is S.S. over span of 3.5m. Use M20, Fe415.

~~Sol~~ Analysis : Given : b = 230 mm
d = 400 mm
Ast = 4 # 12 = 452.39 mm$^2$
d' = 31 mm
Steel Grade : Fe415
concrete grade : M20

Find : Mur = ?
w = ?

Let ① $x_u = \cancel{0.36 f_{ck}} \; \frac{0.87 \cdot f_y \cdot A_{st}}{0.36 \cdot f_{ck} \cdot b}$

$= \frac{0.87 \times 415 \times 452.39}{0.36 \times 20 \times 230}$

$= \cancel{304.136.56 mm} \; 98.63$ mm.

② Cheek; $x_{umax} = x_{ubal} = \frac{700}{1100 + 0.87 f_y} \times d$

$= \frac{700}{1100 + 0.87 \times 415} \times \cancel{230} \; \cancel{400} \; [400-31]$

$= \cancel{191.64 mm} \; 176.79 mm < x_u$

∴ Section is "UR."

∴ O.K.

③ $Mur = \cancel{0.36 \cdot f_{ck} \cdot b \; (d - 0.42 x_u)}$

$= 0.87 \, f_y \cdot A_{st} \cdot (d - 0.42 x_u)$

$= 0.87 \times 415 \times \cancel{4x} \; 452.39 \times (369 - 0.42 \times 98.63)$

∴ $Mur = \cancel{58.568 \times 10^6 \; Nmm} \; 53.50 \times 10^6$ Nmm

$= \cancel{58.568 \; kNm} = \cancel{56} \; 53.5$ kNm.

④ or $\boxed{Mur = 0.36 \cdot f_{ck} \cdot b \, (d - 0.42 x_u)}$ ← Comp. side

$= 0.36 \times 20 \times 230 \, (369 - 0.42 \times 98.63)$

$= 53.5$ kNm.

④ calculation of 'wl':-

for S.S. beam ; $M_u = \frac{w_u \cdot L^2}{8}$

∴ for eq$^m$, $Mur = M_u$

∴ $53.5 = \frac{w_u \times 3.5^2}{8}$

∴ $w_u = 34.94$ kN/m

But $w_u = 1.5 w$

∴ $w = \frac{34.94}{1.5} = 23.29$ kN/m

But $w = w_d + w_l$

∴ $23.29 = 25 \times bD + w_l$

∴ $w_l = 23.29 - 25 \times 0.23 \times 0.4$

$\boxed{w_l = 20.99 \text{ kN/m}}$

## II- Design Problem:-

Given: fck, fy, b, or b/d or Mu

To find: b, D, Ast for a balanced section.

Procedure:- ① Calculate design parameters [Take from Table]

a) $k_{umax} = \frac{x_{umax}}{d} = \frac{700}{1100+0.87f_y}$ ⇒ NA depth factor

b) $R_{umax} = \frac{Mu}{b \cdot d^2} = \frac{0.36\, fck \cdot \cancel{b} \cdot k_{umax}\,(1-0.42k_{umax})}{\cancel{0.87}}$

c) $\frac{pt_{max}}{b \cdot d} = Ast_{max} = \cancel{0.8}\ \frac{0.36\, fck \cdot b \cdot x_{umax}}{0.87 f_y}$

② Calculate effective depth required as

$$d = \sqrt{\frac{Mu}{R_{umax} \times b}} \longrightarrow \text{from eq}^n \text{ (b)}$$

$$\text{or} \quad d = \left[\frac{Mu}{R_{umax}(b/d)}\right]^{1/3} \longrightarrow \text{when } b/d \text{ is given.}$$

③ Calculate $Ast = \frac{0.36\, fck \cdot b \cdot x_{umax}}{0.87 f_y}$.

④ Assume appropriate cover to determine practical total depth;

$= D = d + d'$

Pro: A S.S. beam of 4.5m span carries a udl of 30 kN/m inclusive of SW. The width of the beam is 230 mm & is reinforced on tension side only. Design smallest concrete section. Material used are M20 & MS. Assume load factor = L.F. = 1.5;

Given:- b = 230 mm.
L = 4.5 m
w = 30 kN/m
concrete : M20
steel : Fe250
L.F. = 1.5.

Required : Smallest S.R. section.

Design:- Let ultimate moment $= L.F \times \frac{w \cdot l^2}{8}$

$= \frac{1.5 \times 30 \times 4.5^2}{8}$

$= 113.91$ kNm.

② $k_{umax} = \cancel{0.36 f_{ck} \cdot b} \dfrac{700}{1100 + 0.87 f_y} = \dfrac{700}{1100 + 0.87 \times 250}$

$= 0.53 \longrightarrow$ Directly taken from taken.

③ $R_{umax} = 0.36\, f_{ck} \cdot b \cdot k_{umax} (1 - 0.42\, k_{umax})$

$= 0.36 \times 20 \times 0.53 (1 - 0.42 \times 0.53)$

$= 2.97\ N/mm^2$ as $R_{umax} = \dfrac{M_u}{b \cdot d^2}$

④ $d_{reqd} = \sqrt{\dfrac{M_u}{R_{umax} \cdot b}}$

$= \sqrt{\dfrac{113.81 \times 10^6}{2.97 \times 230}}$

$= 408.35\ mm$

$\therefore x_{umax} = k_{umax} \cdot d = 0.53 \times 408.35 = 216\ mm$

⑤ $\therefore Ast_{max} = \dfrac{0.36\, f_{ck} \cdot b \cdot x_{umax}}{0.87 \cdot f_y}$

$= \dfrac{0.36 \times 20 \times 230 \times 216}{0.87 \times 250}$

$= 1647.8\ mm^2$

$\therefore$ Provide 6 # 20

Assuming effective cover = 55 mm

$\therefore$ Total depth of the beam $= 408 + 55 = 463\ mm \simeq 470\ mm$.

$\therefore$ size of beam : $230 \times 470$ mm.

Pro: Calculate the Ast reqd. for S.R. concrete beam 200 mm wide & 400 mm deep to resist an ultimate moment of 60 kNm. Assume M20 & Fe500. & eff. cover = 40 mm.

Solⁿ:- $M_u = 60\ kNm$.

$b = 200\ mm$

$D = 400\ mm$

$d = 400 - 40 = 360\ mm$

M20, Fe500.

check for UR:-

$\therefore x_{umax} = \dfrac{700 \times d}{1100 + 0.87 f_y} = \dfrac{700 \times 360}{1100 + 0.87 \times 500} = 164.2\ mm$

$Mu_{rmax} = 0.36\, f_{ck} \cdot b \cdot x_{umax} (d - 0.42\, x_{umax})$

$= 0.36 \times 20 \times 200 \times 164.2\ (360 - 0.42 \times 164.2)$

$= 68.805\ kNm < M_u = 60\ kNm$

$\therefore$ Sect$^n$ is UR

$$\therefore A_{st} = \frac{0.5 f_{ck}}{f_y}\left(1-\sqrt{1-\frac{4.6\,M_u}{f_{ck}\cdot b\cdot d^2}}\right)\cdot b\cdot d$$

$$= \frac{0.5\times 20}{500}\left(1-\sqrt{1-\frac{4.6\times 60\times 10^6}{20\times 200\times 360^2}}\right)\times 200\times 360$$

$$= 455.31\ mm^2$$

Pro:- A SR beam of concrete grade M25 has to resist an ultimate moment of 90 kNm. Design the section using 0.9 % steel of grade Fe415. Assume width of beam = 250mm

Sol$^n$:- Given: $M_u = 90$ kNm

M25

Fe 415

$p_t = \frac{A_{st}}{bd} = 0.9\%$

Find D, Ast

let $x_{umax} = \frac{700\times d}{1100 + 0.87 f_y} =$

$\therefore k_{umax} = \frac{x_{umax}}{d} = \frac{700}{1100+0.87\times 415} = 0.479$

$\therefore R_{umax} = 0.36\cdot f_{ck}\cdot k_{umax}(1-0.42k_{umax}) = \frac{M_{umax}}{b\cdot d^2}$

$= 0.36\times 25\times 0.479\,(1-0.42\times 0.479)$

$= 3.444$

$\therefore d_{reqd} = \sqrt{\frac{M_{umax}}{R_{umax}\cdot b}} = \sqrt{\frac{90\times 10^6}{3.444\times 250}}$

$= 323.3$ mm

$\therefore$ Assume $d' = 40$ mm.

$\therefore D_{reqd} = 363$ mm

$\therefore D_{pro} = 380$ mm.

$\therefore d_{pro} = 340$ mm.

$\therefore \frac{A_{st}}{b\cdot d} = p_t = 0.9\%$

$\therefore A_{st} = \frac{0.9\times 250\times 340}{100}$

$= 765\ mm^2$

OR

$A_{st} = p_t\times b\cdot d$

$= 0.009\times 250\times d$

$= 2.25d$

$$A_{st} = \frac{0.5 f_{ck}}{f_y}\left(1-\sqrt{1-\frac{M_u\times 4.6}{f_{ck}\cdot b\cdot d^2}}\right)\times b\cdot d$$

$$\therefore 2.25\cancel{d} = \frac{0.5\times 25}{415}\left(1-\sqrt{1-\frac{4.6\times 90\times 10^6}{25\times 250\times d^2}}\right)\times 250\times \cancel{d}$$

$\therefore 2.25 = 7.53\left(1-\sqrt{1-\frac{66240}{d^2}}\right)$

~~$\therefore 0.0893 = \left(1-\sqrt{1-662}\right)$~~

$\therefore 0.3 = 1-\sqrt{1-\frac{66240}{d^2}} \Rightarrow$

$1-0.3 = \sqrt{1-\frac{66240}{d^2}} = 0.7$

$\therefore 1-\frac{66240}{d^2} = 0.49$

$\frac{66240}{d^2} = 1-0.49$

$\therefore d^2 = 0.51$

$\therefore 0.51d^2 = 66240$

Ans. $\Rightarrow$ $\boxed{\therefore d = 360.39}$

Pro: A RCC rectangular beam 300x600 mm. Calculate MR of balanced section & amount of steel if characteristic strength of concrete is 22 $N/mm^2$ & yield strength of steel is 450 $N/mm^2$. Assume effective cover of 50 mm.

Sol$^n$:- Given :- $b = 300$ mm

$D = 600$ mm

$d = 600 - 50 = 550$ mm

Fe450, M22.

MR bal = ?

Ast bal = ?

Sol$^n$:- $x_{umax} = \dfrac{700 \times d}{1100 + 0.87 f_y} = \dfrac{700 \times 550}{1100 + 0.87 \times 450} = 258$ mm.

$\therefore MR_{bal} = 0.36 \cdot f_{ck} \cdot b \cdot \cancel{d^2} \cdot x_{umax} \, (d - 0.42 x_{umax})$

$= 0.36 \times 22 \times 300 \times 258 \, (550 - 0.42 \times 258)$

$= 270.72$ kNm ✓

$\therefore Ast_{max} = \dfrac{0.5 \, f_{ck}}{f_y} \left(1 - \sqrt{1 - \dfrac{4.6 \cdot Mu_{bal}}{f_{ck} \cdot b \cdot d^2}}\right) \cdot b \, d$

$= \dfrac{0.5 \times 22}{450} \left(1 - \sqrt{1 - \dfrac{4.6 \times 270.72 \times 10^6}{22 \times 300 \times 550^2}}\right) \times 300 \times 550$

$= 1559.3 \; mm^2$

Pro: A RCC beam of rectangular section 230x300 mm is reinforced on tension side with 3 # 12 mm.

calculate ① MRu ; if M18, Fe290

② Wallowable for S.S. span of 3m.

Assume eff. cover = 31 mm

Sol$^n$:- $b = 230$ mm

$D = 300$ mm

$\therefore d = 300 - 31 = 269$ mm

$Ast_{pro} = 3 \times \frac{\pi}{4} \times 12^2 = 339.3 \; mm^2$

M18,

Fe290.

$\therefore x_{umax} = \dfrac{700 \times d}{1100 + 0.87 f_y} = \dfrac{700 \times 269}{1100 + 0.87 \times 290} = 139.24$ mm

$\therefore Mu_{max} = 0.36 \times f_{ck} \cdot \cancel{b} \cdot x_{umax} \, (d - 0.42 x_{umax})$

$= 0.36 \times 18 \times 230 \times 139.24 \, (\cancel{d}\, 269 - 0.42 \times 139.24)$

$= 43.69$ kNm

$\therefore$ $Mu_{max} = \cancel{\frac{w_u l^2}{8}} = 43.69$

$\therefore$ $w_u = \cancel{\frac{43.69 \times 8}{3.2}} = 38.83$ kN/m

But ; $w_u = 1.5 w$

$\therefore w = \frac{38.83}{1.5} = 25.89$ kN/m

But $w = wd + wl$

$\therefore wl = w - wd = 25.89 - 25 \times b \times D$ $= 25.89 - 25 \times [(230 \times 300) \times 10^{-6}]$

$= \cancel{25 \times b \times D - 25.89}$

$= \cancel{25 \times 230}$ 24.165 kN/m.

But $x_u = \frac{0.87 \times fy \times Ast}{0.36 \times fck \cdot b}$

$= \frac{0.87 \times 290 \times 339.3}{0.36 \times 18 \times 230}$

$= 57.43$ mm

as $x_u = 57.43 < x_{umax} = 139.24$ mm

$\therefore$ Section is UR.

$\therefore Mu = 0.36 \times fck \times b \times x_u (d - 0.42 x_u)$

$= 0.36 \times 18 \times 230 \times 57.43 (269 - 0.42 \times 57.43)$

$\boxed{Mu = 20.96 \text{ kNm}}$

or from Tension side ;

$Mu = 0.87 fy \ Ast \ (d - 0.42 x_u)$

$= 0.87 \times 290 \times 339 (269 - 0.42 \times 57.43)$

$\boxed{Mu = 20.96 \text{ kNm}}$

$\therefore$ To calculate load carring capacity :-

$\therefore Mu = \frac{wu \cdot l^2}{8} \Rightarrow wu = \frac{20.96 \times 8}{32} = 18.63$ kN/m

But $w_u = 1.5 w$

$\therefore w = \frac{18.63}{1.5} = 12.42$ kN/m

But $w = wd + wl$

$\therefore wl = w - wd = 25 \times b \times D - wd = 25 \times 0.23 \times 0.3 - 12.42$

$\boxed{\therefore wl = 10.69 \text{ kN/m}}$