Notes by-

Pravin S Kolhe,

BE(Civil), Gold Medal, MTech (IIT-K)

Assistant Executive Engineer,

Water Resources Department,

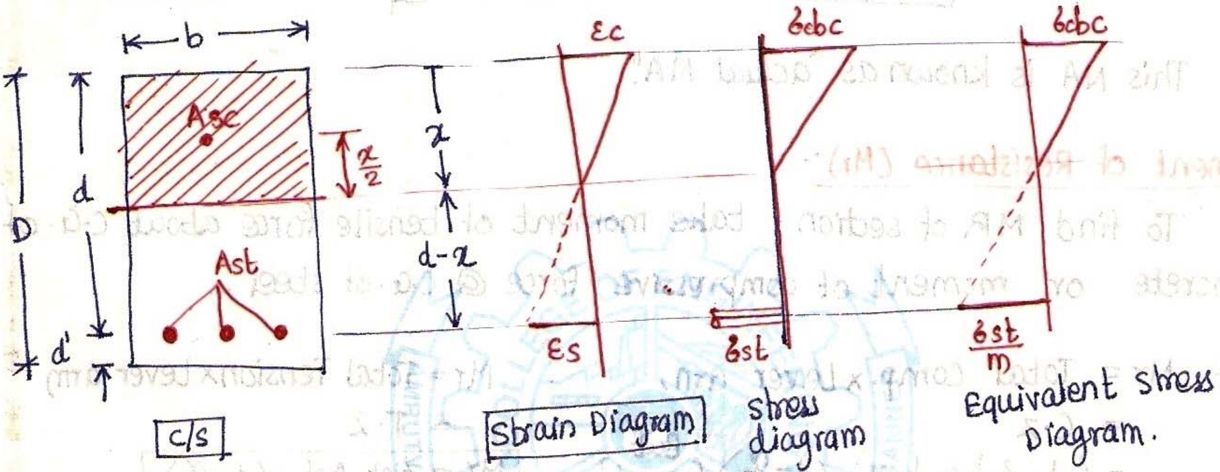
www.pravinkolhe.com

* Singly Reinforced Section :-

Assumptions :-

1. Plane sections before bending remains plane after bending.
2. concrete does not take tension.
3. Stress ∝ strain.
4. $m = \frac{280}{3\sigma cbc}$
5. There is perfect bond betⁿ steel & concrete.

* Analysis:-



Let b = Breadth of section
D = Total depth
d = effective depth = D-d'
x = Depth of NA from extreme compression fibre
= k·d
K = NA factor.
Ast = Cross sectional area of steel in tension
σcbc = Per. comp. stress due to bending in concrete (N/mm²)
σst = Per. tensile stress in steel (N/mm²)
m = Modular ratio.

Depth of NA :-

① $\frac{\sigma cbc}{\sigma st/m} = \frac{x}{d-x} \Rightarrow$ $x = \frac{m\sigma cbc}{m\sigma cbc + \sigma st} \times d = k \cdot d$

$$\therefore K = \frac{m\,\sigma cbc}{m\sigma cbc + \sigma st}$$

② As NA passes through C.G.
[Moment of Area of compression side = moment of Area in Tension zone .] from NA

$\therefore b \cdot x \cdot \frac{x}{2} = m \cdot Ast(d-x)$ → Equivalent area.

$$\therefore \frac{bx^2}{2} = m \cdot Ast\,(d-x)$$

$$\therefore x^2 = m \cdot 2 \cdot \frac{Ast}{b} \cdot (d-x) \qquad \text{But } pt = \frac{Ast}{b \cdot d}$$

$$\therefore pt \cdot d = \frac{Ast}{b}$$

~~$\therefore x^2 = 2m \cdot pt \cdot (d-x) \cdot d$~~

~~$\therefore x^2 = 2mpt \cdot d - 2mpt \cdot x \cdot d$~~

~~$\therefore x^2 + 2mpt \cdot x \cdot d = 2mpt \cdot d$~~

$$\Rightarrow \boxed{x = \left[\sqrt{mp(mp+2)} - mp\right] \cdot d}$$

This NA is known as "actual NA".

*** Moment of Resistance (Mr):-**

To find MR of section; take moment of tensile force about C.G. of concrete or moment of compressive force @ C.G. of steel.

$$\therefore Mr = \text{Total comp.} \times \text{Lever arm} \qquad Mr = \text{Total Tension} \times \text{Lever arm}$$

$$= C \cdot Z \qquad\qquad = T \cdot Z$$

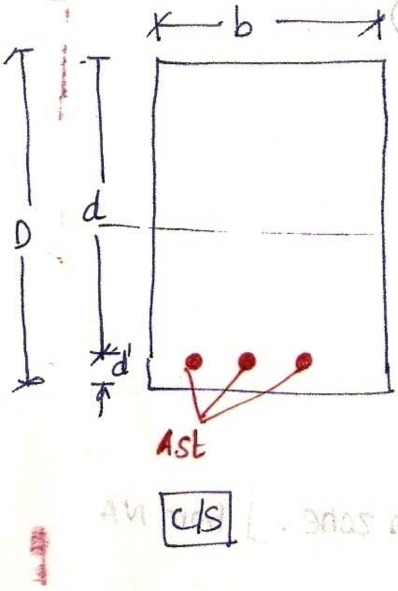
$$= \left(\tfrac{1}{2} \cdot \sigma_{cbc} \cdot b \cdot x\right) \cdot \left(d - \tfrac{x}{3}\right) \quad \text{But } [x = kd] \qquad \boxed{Mr = \sigma_{st} \cdot Ast \cdot \left(d - \tfrac{x}{3}\right)}$$

$$= \tfrac{1}{2}\sigma_{cbc} \cdot b \cdot k \cdot d \cdot \left(d - \tfrac{k \cdot d}{3}\right)$$

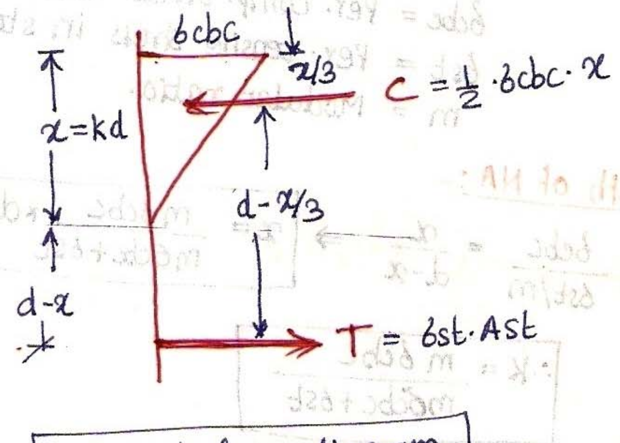
$$\boxed{Mr = \tfrac{1}{2} \cdot \sigma_{cbc} \cdot b \cdot k\left(1 - \tfrac{k}{3}\right) \cdot d^2}$$

$$\boxed{Mr = R \cdot b \cdot d^2}$$

where, $\boxed{R = \tfrac{1}{2}\sigma_{cbc} \cdot k \cdot \left(1 - \tfrac{k}{3}\right)}$



C/S



Internal force diagram

① Balanced or Critical Section :-

A section is said to be balanced when max. stresses in both (i.e. steel & concrete) reach their permissible value simultaneously. The NA of the section is known as "Critical or balanced NA" as x_{cr} –

$$\frac{\sigma_{cbc}}{\sigma_{st}/m} = \frac{x_{cr}}{d - x_{cr}} \Rightarrow \boxed{x_{cr} = \frac{m\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}} \cdot d}$$

$$\& \quad \boxed{M_{cr} = \frac{1}{2} \cdot \sigma_{cbc} \cdot b \cdot x_{cr} \left(d - \frac{x_{cr}}{3}\right)} \Rightarrow \text{comp. side}$$

$$\text{or} \quad \boxed{M_{cr} = \sigma_{st} \cdot A_{st} \cdot \left(d - \frac{x_{cr}}{3}\right)} \Rightarrow \text{Tension side.}$$

Pl. Note :- In LSM ; "max. strain" reachs their max. permissible limits while in WSM "Max. stresses" reaches their max. permissible limits.

② Under- Reinforced section :-

When percentage of steel (i.e. $p_t = \frac{100 A_{st}}{b.d}$) in the section is less than that reqd. for balanced section, then ~~stres~~ steel reaches its permissible stress but concrete does not goes its permissible stress & section will fail due to yielding of steel initially & thereafter due to crushing of concrete. But before collapse it will show deflection and cracks on section which is a sort of warning to the occupant to leave the structure.

Such a section is known as UR section.

$\therefore$ $p_t < A_{st\,cr}$

$M_r < M_{r\,cr}$.

$x < x_{cr}$

Since steel reaches its permissible stress $\therefore$ M_r is calculated from tension side as,

$$\boxed{M_r = \sigma_{st} \cdot A_{st} \cdot (d - x/3)}$$

③ Over reinforced Section :-

When percentage of steel in a section is more than that reqd. in balanced section, then stresses in concrete reach its max. permissible value & section will fail due to crushing of concrete such a section is known as OR section.

As concrete reaches its max. permissible stresses, M_r is calculated from compression side as,

$$\boxed{M_r = \frac{1}{2} \sigma_{cbc} \cdot b \cdot x \left(d - x/3\right)}$$

Pro : 2] A RC beam 250x700 mm is reinforced with 4 ϕ 16 mm bars placed at an eff. cover of 40mm. Find the central point load the beam can carry over a s.s. span of 6 m.

Er. Pravin Kolhe
(B.E Civil)

Sol^n :- $b = 250 mm$

$D = 700 mm$

$d = 700 - 40 = 660 mm$

$Ast = 4 \times \frac{\pi}{4} \times 16^2 = 804.3 \ mm^2$

$l = 6 m$

$w = ?$

* $x_{cr} = \frac{m\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}} \cdot d$

But $m = \frac{280}{3\sigma_{cbc}} = 13.33$; $\sigma_{st} = 140 N/mm^2$

$\therefore x_{cr} = \frac{(280/3) \times 660}{(280/3) + 140} = 264 \ mm$

* $x_{act} = (\sqrt{(mp(mp+2)} - mp) \cdot d$

But $p = \frac{Ast}{b \cdot d} = \frac{804.3}{250 \times 660} = 4.87 \times 10^{-3}$

$\therefore x_{act} = (\sqrt{4.87 \times 10^{-3} \times 13.33 (4.87 \times 10^{-3} \times 13.33 + 2)} - 4.87 \times 10^{-3} \times 13.33) \times 660$

$= 199 \ mm.$

as $x_{act} < x_{cr}$

$\therefore$ Sect^n is UR

$\therefore$ ~~Mr = σs~~ $\therefore$ ~~Cor~~ steel will reach its max. per. stresses.

$\therefore$ Calculate MR from tension side.

$\therefore Mr = \sigma_{st} \cdot Ast \cdot (d - x/3)$ $\quad d(1 - \frac{x}{3d})$

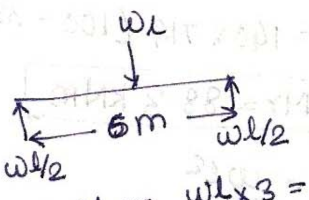
$= 140 \times 804.3 (660 - 199/3)$

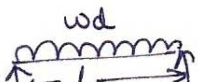
$Mr = 66.85 \ kNm$

~~$\therefore Mr = M$~~

~~$\therefore 66.85 = w \times \frac{3}{2}$~~

~~$\therefore w = 44.56 \ kN$~~





$M = \frac{wd \cdot l^2}{8}$

$\therefore M = \frac{wl}{2} \times 3 = \frac{3w_L}{2}$

$\therefore Mr =$ BM due to SW + BM due to LL

$\therefore$ BM due to SW $= \frac{wl^2}{8} = \frac{[0.25 \times 0.7 \times 25] \times 6^2}{8} = 19.68 \ kNm$

$\therefore$ BM due to LL $= Mr - 19.68 = 66.85 - 19.68 = 47.178 \ kNm.$

$\therefore \frac{wl}{4} = 47.17 \Rightarrow w_L = \frac{47.17 \times 4}{6}$

$w_L = 31.45 \ kN$

Pro: A RC slab S.S. over a span of 3m is reinforced with ϕ10 mm bars at 110 mm c/c. The total depth of the slab is 120 mm. Assuming M20 & MS calculate safe load that can be carried by slab. Assume $d' = 20$ mm & $m = 13.33$.

Soln:- Assume 1m width of slab.

$\therefore b = 1000$ mm

$D = 120$ mm

$d = 120 - 20 = 100$mm

$$Ast = \frac{1000 \cdot ast}{spacing} = \frac{1000 \times \frac{\pi}{4}(10)^2}{110} = 714 \text{ mm}^2$$

$m = 13.33$

$\Rightarrow \sigma cbc = 7$ N/mm^2

$\sigma st = 140$ N/mm^2 ... MS.

$$\therefore x_{cr} = \frac{m\sigma cbc \cdot d}{m\sigma cbc + \sigma st} = \frac{13.33 \times 7 \times 100}{13.33 \times 7 + 140} = 40 \text{ mm}$$

$$x_{actual} = (\sqrt{mp(mp+2)} - mp) \cdot d$$

$$\therefore p = \frac{Ast}{b \cdot d} = \frac{714}{1000 \times 100} = 7.14 \times 10^{-3}$$

$\therefore mp = 7.14 \times 10^{-3} \times 13.33 = 0.0952$

$\therefore x_{act} = 35$ mm.

$\therefore x_{cr} = 40$mm $< x_{act} = 35$ mm.

$\therefore$ Section is UR

$\therefore$ Steel will reach its max. per. value.

$\therefore$ Calculate Mr from Tension side

$$\therefore Mr = \sigma st \cdot Ast \cdot (d - x/3)$$

$$= 140 \times 714 (100 - 35/3)$$

$\therefore$ **Mr = 88.3 KNm**

$$\therefore Mr = \frac{wl^2}{8}$$

$\therefore w = 7.85$ KN/m

$\therefore w = wd + wl$

$\therefore wl = w - 0.25 \times b \cdot D$

$= 7.85 - 25 \times 1 \times 0.12$

wl = 4.85 KN

To find actual stresses in steel & concrete:

Pro: A rectangular beam 230x540 mm having eff. cover of 40mm is reinforced with 3 ϕ 16 mm bars. Calculate stresses developed in the matls. when bending moment of 50 kN.m is applied. Assume m = 13.33.

Soln:-

$b = 230$ mm

$d = 500$ mm

$A_{st} = 603\ mm^2$

$M = 50$ kNm

$m = 13.33$

Let $x_{cr} = \frac{m\sigma_{cbc} \times d}{m\sigma_{cbc} + \sigma_{st}} = \frac{13.33 \times 7 \times 603}{13.33 \times 7 + 140\ 275} = ~~241\ mm~~\ 152.7\ mm.$

$x_{act} = \left(\sqrt{2mp(mp+2)} - mp\right) \times d$

$mp = \frac{13.33 \times 603}{230 \times 500} = 0.07$

$\therefore x_{act} = 155$ mm.

$\therefore$ let $M_r = M = \sigma_{st} \cdot A_{st} \cdot (d - x/3)$

$\therefore 50 = \sigma_{st} \times 603\ (500 - 155/3)$

$\therefore \sigma_{st} = 1.849 \times 10^{-4}\ kN/mm^2$

$\boxed{\sigma_{st} = 184.9\ N/mm^2}$

concrete stress:

$\frac{\sigma_{cbc}}{\sigma_{st}/m} = \frac{x}{d-x}$ $\Rightarrow \sigma_{cbc} = \frac{x\,\sigma_{st}}{m(d-x)} = \frac{155 \times 184.9}{13.33(500-155)}$

$\therefore$ ~~$\frac{\sigma_{st}}{m} = \sigma_{cbc}\left(\frac{d-x}{x}\right) \cdot m$~~

~~$= 184.9$~~

$\therefore \boxed{\sigma_{cbc} = 6.23\ N/mm^2}$

NOTE:- To differ the actual stresses in material from max. per. stresses adopt following symbols.

$\boxed{f_{st} = 184.9\ N/mm^2} < 275\ N/mm^2$

$\boxed{f_{\sigma cbc} = 6.23\ N/mm^2} < 7\ N/mm^2$

$\therefore$ safe.

& as $x_{cr} = 152.7 < x_{act} = 155$ mm

Section is OR

& concrete will reach to its max. per. value, which can be logically seen from results.

Type-III : Design of section

Pro: Design a S.S. beam of span 6m carrying a udl of 30 kN/m inclusive of SW. The width of beam is 300 mm. The matl. used are M25 grade of concrete & Fe415 grade of steel.
Assume eff. cover = 40mm.

$$\therefore M = \frac{wl^2}{8} = \frac{30 \times 6^2}{8} = 135 \text{ kNm}$$

$b = 300\text{mm}$

$M25 \Rightarrow \sigma_{cbc} = 8.5 \text{ N/mm}^2$ } IS : 456/2000 Pg. 81/82 / Table 21, 22

$Fe415 \Rightarrow \sigma_{st} = 230 \text{ N/mm}^2$

$$m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 8.5} = 10.98$$

Let we design for balanced section.

$$\therefore x_{cr} = \frac{m\sigma_{cbc} \cdot d}{m\sigma_{cbc} + \sigma_{st}} = \frac{10.98 \times 8.5 \times d}{10.98 \times 8.5 + 230}$$

$$= 0.29d$$

$$\therefore M_r = \frac{1}{2} \sigma_{cbc} \cdot b \cdot x_{cr} \left(d - \frac{x_{cr}}{3}\right)$$

$$\therefore 135 \times 10^6 = \frac{1}{2} \times 8.5 \times 300 \times 0.29d \left(d - \frac{0.29d}{3}\right)$$

$$\therefore 365.111 \times 10^3 = d^2\left(1 - \frac{0.29}{3}\right)$$

$\therefore d_{reqd} = 635 \text{ mm}$

$\therefore D_{reqd} = 635 + 40$
$= 675 \text{ mm}$

$\therefore D_{pro} = 680 \text{mm}$

$\therefore d_{pro} = 640 \text{ mm}$.

$\therefore$ Ast calculation :-

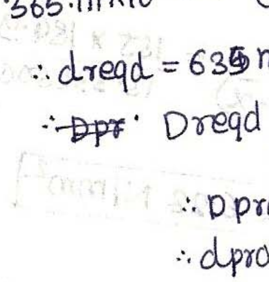
$$M = \sigma_{st} \cdot A_{st} \cdot (d - x/3)$$

$$\therefore 135 \times 10^6 = 230 \times A_{st} \left(640 - \frac{0.29 \times 640}{3}\right)$$

$\therefore A_{st\,reqd} = 1015 \text{ mm}^2$

Assuming 16 mm dia. bar,

$$\text{No. of bars reqd} = \frac{A_{st}}{a_{st}} = \frac{1015}{201} = 5.05 \simeq 5 \text{ No.}$$



5 𝛟 16 ≈ 5 # 16 mm ≈ 1005 mm² pro.