Notes by-

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Doubly Reinforced Section – LSM

Doubly reinforced sections are reinforced in both, i.e. compression & tension regions.

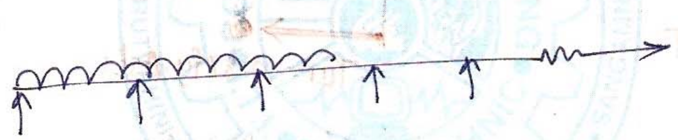
(Aptitude)

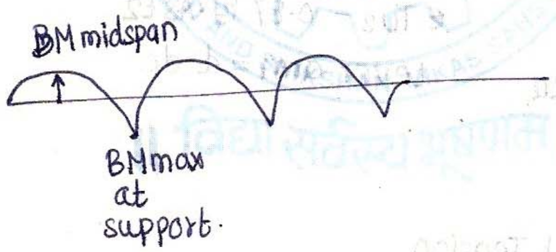
Why to adopt DR section?

① Dimensions of sections are restricted due to –
- i) Head room consideration.
- ii) Space / Floor to floor ht.
- iii) Appearance etc → [Most imp. in practice]

& applied moment ($\frac{wl^2}{8}$ – udl & $\frac{wl}{4}$ – pt. load) exceeding MR capacity of SR section.

② High BM exist over relatively smaller length of beam.
eg: at support of continuous beam,
Then instead of providing reqd. cross section throughout the span, provide c/s according to midspan BM & at support adopt DR section.





③ To reduce chances of deflection i.e. to increase stiffness by providing steel on comp. side.

④ When load is eccentric [col^m]

⑤ chances of "Reversal of stresses" → Note.

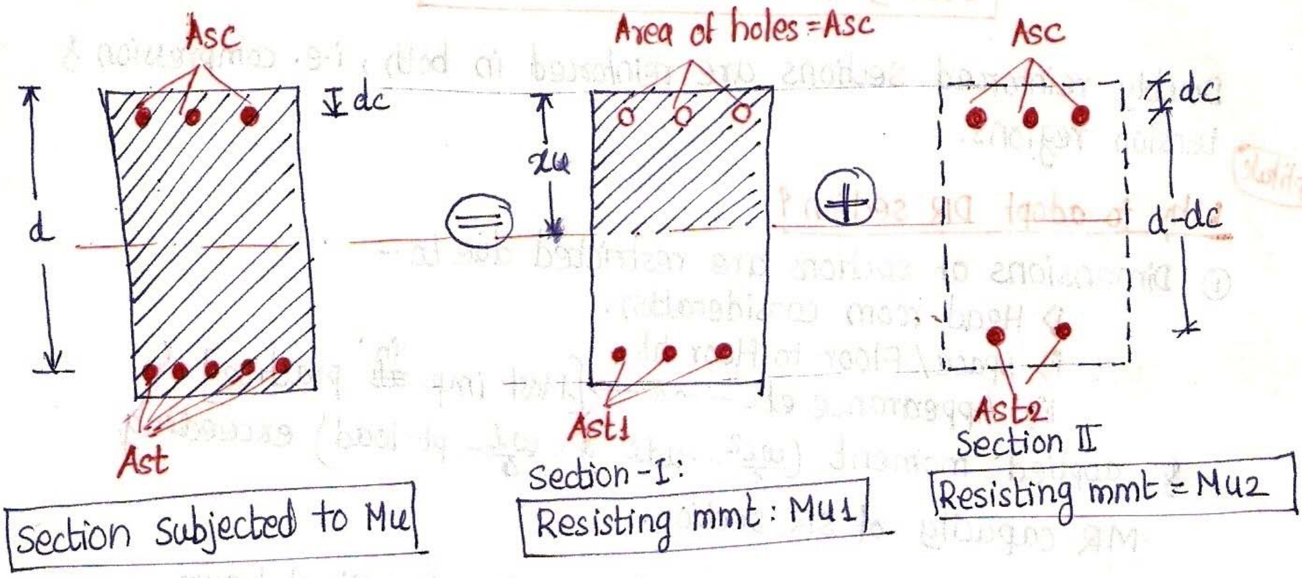
*** Properties of DR rectangular section :-**

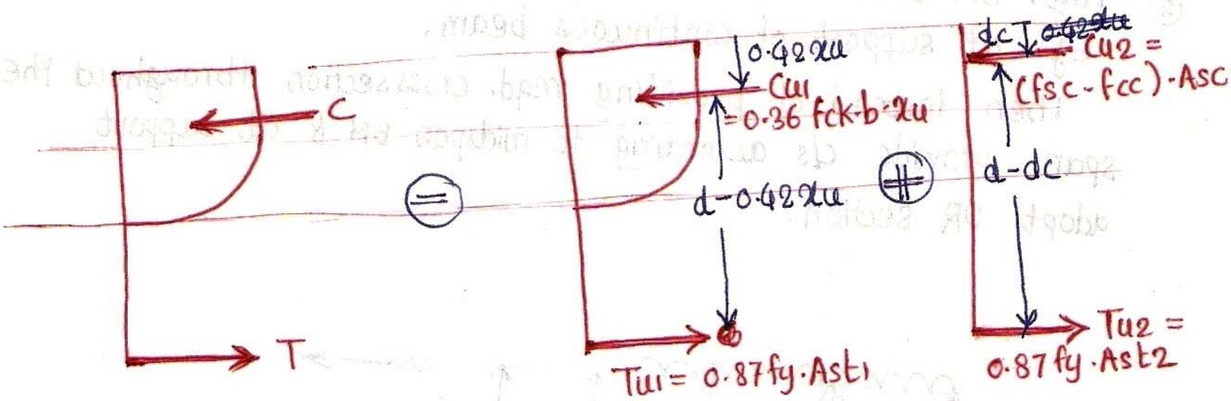
(Phylosophy)

The DR section is considered as it is composed of : sect^n I & sect^n II.

Sect. I : A SR sect^n in which comp. in concrete C_{u1} is balanced by tension in steel A_{st1}. This section resist part of moment M_{u1} out of total moment M_u.

Sect. II : A section with comp. steel A_{sc} & additional tension steel A_{st2} resisting balance mmt $M_{u2} = (M_u - M_{u1})$





$Cu_1 = 0.36 \cdot fck \cdot b \cdot xu$
$Tu_1 = 0.87 \cdot fy \cdot Ast_1$
Lever arm = $d - 0.42xu$

$Cu_2 = (fsc - fcc) \cdot Asc$
& $Tu_2 = 0.87\, fy \cdot Ast_2$
Lever arm = $d - dc$

* Depth of NA

For eqm ;

Total comp. = Total Tension

$\therefore Cu = Tu$

$$Cu_1 + Cu_2 = Tu_1 + Tu_2$$

Where, Cu_1 = comp. force provided by concrete
Cu_2 = comp. force provided by steel in comp. zone
Tu_1 = Tensile force provided by tensile steel to balance Cu_1
Tu_2 = Tensile force provided by tensile steel to balance Cu_2.

∴ Expressing forces in terms of stresses in matl, we have,

$$0.36\, fck \cdot b \cdot xu + (fsc - fcc) \cdot Asc = 0.87\, fy \cdot Ast_1 + 0.87 \cdot fy \cdot Ast_2$$

$$[0.36\, fck \cdot b \cdot xu - fcc\, Asc] + [fsc \cdot Asc] = 0.87 \cdot fy \cdot Ast$$

$$\therefore xu = \frac{0.87 \cdot fy \cdot Ast - (fsc - fcc) \cdot Asc}{0.36\, fck \cdot b} \Rightarrow \boxed{xu = \frac{0.87\, fy\, Ast - fsc \cdot Asc}{0.36\, fck \cdot b}}$$

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Where f_{sc} = Stress in steel in comp.

f_{cc} = stress in concrete in comp. at the level of Asc
$= 0.45\ f_{ck} \cdot A_{sc}$

A_{sc} = Area of steel in comp.

$f_{cc} \cdot A_{sc}$ = Deduction for comp. in concrete which is displaced by comp. steel.

A_{st} = Area of steel in tension $= A_{st1} + A_{st2}$.

* Ultimate MR:-

MR is calculated by taking moments of C_{u1} & C_{u2} @ centroid of tension steel as,

$$M_u = M_{u1} + M_{u2}$$

Where; M_{u1} = Ultimate MR offored by section I which consist of a couple formed by C_{u1} & T_{u1} & lever arm $(d - 0.42 x_u)$

M_{u2} = Ultimate MR offored by section II, which consist of a couple formed by C_{u2} & T_{u2} & lever arm $(d - d_c)$

Where d_c = effective cover to comp. steel.

$\therefore M_{u1} = 0.36 \cdot f_{ck} \cdot b \cdot x_u \cdot (d - 0.42 \cdot x_u)$

$M_{u2} =$ ~~0.87 fy. As~~ $(f_{sc} - f_{cc}) \cdot A_{sc} \cdot (d - d_c) \simeq f_{sc} \cdot A_{sc} \cdot (d - d_c)$

$$\therefore M_u = 0.36 f_{ck} \cdot b \cdot x_u (d - 0.42 x_u) + (f_{sc} - f_{cc}) A_{sc} \cdot (d - d_c)$$

$$\therefore M_u \simeq 0.36 \cdot f_{ck} \cdot b \cdot x_u (d - 0.42 x_u) + f_{sc} \cdot A_{sc} \cdot (d - d_c)$$

Neglecting f_{cc}

(Aptitute)

* Area of tension as well as comp. steel is limited up to 4 % Ag. Why?

M_{u2} = Additional MR due to couple of Asc & Ast2.

If more steel is provided in concrete section i.e. Asc ; to balance this more Ast2 will require.

But in tension zone, $A_{st} = A_{st1} + A_{st2}$.

This creates practical difficulties in compaction of concrete & which results in forming air pockets & voids which reduces the effective bond betn steel & concrete & also weakening the section.

$\therefore$ IS code does not permit to use comp. as well as tensile reinforcement more than 4 % of gross c/s area.

Area of ~~tesion steel~~:- Tension & comp. steel :-

$Mu = Mur = Mu1 + Mu2$

Taking moment of Tu1 @ Cu1 & Tu2 @ Cu2

$\therefore Mu1 = 0.87\, fy \cdot Ast1\, (d - 0.42\, xu)$

$$\therefore Ast1 = \frac{Mu1}{0.87\, fy\, (d - 0.42\, xu)}$$ → Tension steel for Mu1

For balanced section, $xu = xumax$

$Mu1 = Mu\, max$

$Ast1 = Ast1\, max$

$= \frac{Ptmax}{bd} \times b \cdot d$

UR: $xu < xumax$, $Mu1 < Mu1max$, $Ast1 < Ast1max$

OR: $xu > xumax$

$\therefore$ & $Mu2 = Mu - Mu1$

$= 0.87\, fy \cdot Ast2\, (d - dc)$

$$Ast2 = \frac{Mu2}{0.87\, fy \cdot (d - dc)}$$ → Tension steel for Mu2

For eqm, $Cu2 = Tu2$

$(fsc - fcc) \cdot Asc = 0.87\, fy \cdot Ast2$

$$Asc = \frac{0.87\, fy \cdot Ast2}{(fsc - fcc)} \approx \frac{0.87\, fy \cdot Ast2}{fsc}$$ → comp. steel for Mu2

Pro: Calculate MR of DR-RC beam of rectangular section of 300 × ~~500~~ 450 mm reinforced with 6-ϕ20mm bars on tension side &

a) 4-ϕ20 mm bars on comp. side

b) 5-ϕ20 mm bars on comp. side.

Use M20, Fe250, assume eff. cover = 35 mm on both sides.

$MR = Mur = Mu1 + Mu2$

~~Mur=~~

Given: $fck = 20\, N/mm^2$

$fy = 250\, N/mm^2$

$b = 300\, mm$

$D = 450\, mm$

$d = 450 - 35 = 415\, mm$

$dc = 35\, mm$

$Ast = 6\phi 20 = 1885\, mm^2$

$Asc = 4\phi 20 = 1256.6\, mm^2$

~~Check~~

a] $\therefore Mur = 0.36\, fck \cdot b \cdot xu\, (d - 0.42\, xu) + fsc \cdot Asc\, (d - dc)$

= ~~0.36 × 20 × 300 ×~~

$$x_u = \frac{0.87\, f_y \cdot A_{st} - 0.87\, f_y \cdot A_{sc}}{0.36\, f_{ck} \cdot b}$$

$$= \frac{0.87 \times 250 \times 1885 - 0.87 \times 250 \times 1256.6}{0.36 \times 20 \times 300}$$

$$= 63.28 \text{ mm}$$

$$x_{ucr} = K_u \cdot d$$
$$= 0.53 \times 415$$
$$= 220 \text{ mm} < x_u = 63.28 \text{ mm}.$$

$$\& \quad f_{sc} = 700\left(1 - \frac{d_c}{x_u}\right)$$
$$= 700\left(1 - \frac{35}{63.28}\right)$$
$$= 312.83 \text{ N/mm}^2 > 0.87 f_y = 217.5 \text{ N/mm}^2$$

$$\therefore M_{ur} = [0.36 \times 20 \times 300 \times 63.28\,(415 - 0.42 \times 63.28) + \sout{312}\, 217.5 \times 1256.6 \times (415 - 35)] \times 10^{-6}$$

$$\boxed{M_{ur} = 156.95 \text{ kNm}}$$

b) $M_{ur} = 0.36\, f_{ck} \cdot b \cdot x_u\,(d - 0.42 x_u) + 0.87\, f_y \cdot A_{sc} \cdot (d - d_c)$

$A_{sc} = 5\phi 20 = 1571 \text{ mm}^2$