Notes by-

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[Plastic Theory]

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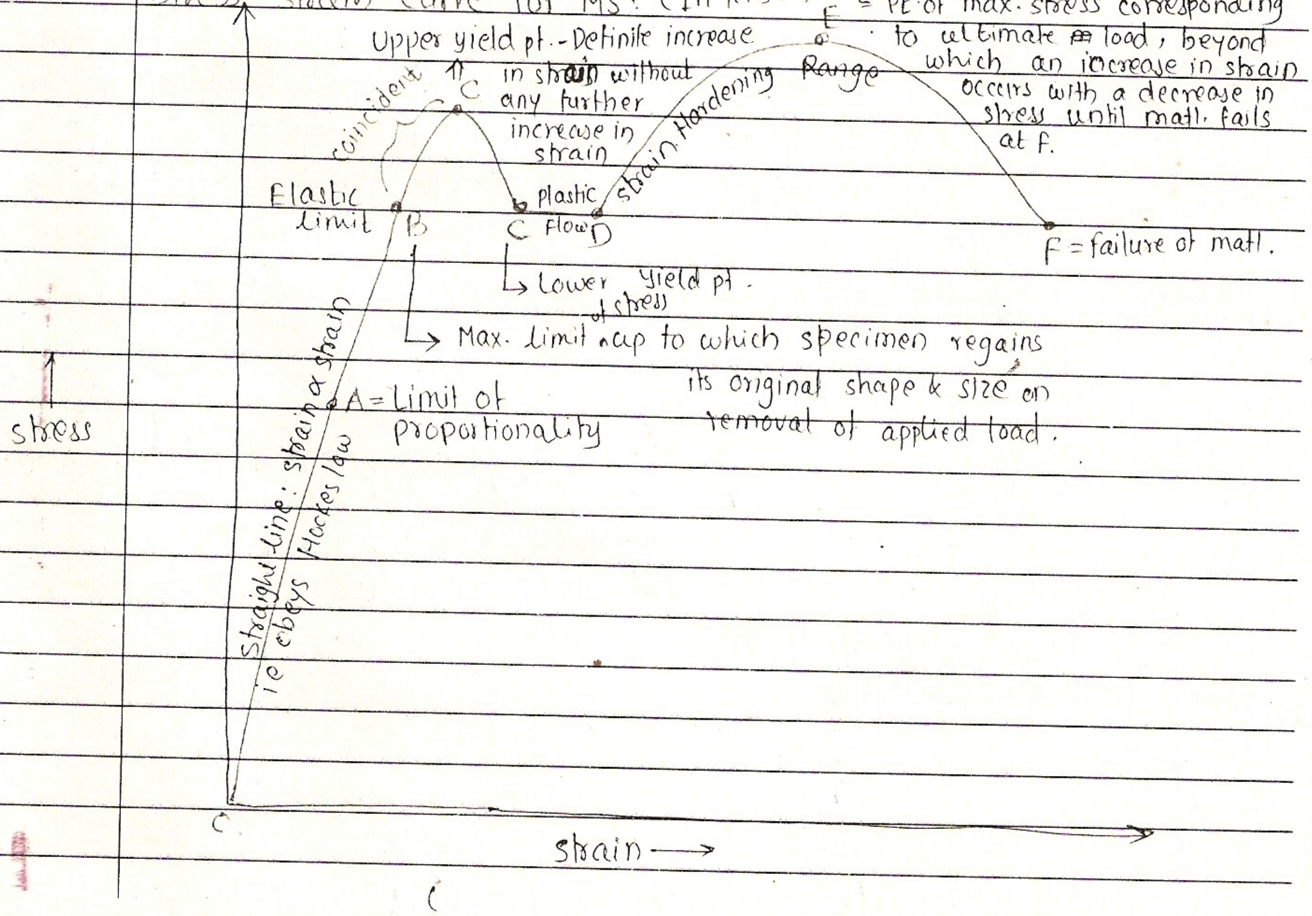
As steel is ductile material, it is able to absorb large deformations beyond the elastic limit without fracture. Due to this property, steel possesses a reserve of strength beyond yield which is tried to utilized in plastic method of design.

Thus plastic design is an aspect of limit design that extends the structural usefulness up to the plastic strength or ultimate load carrying capacity.

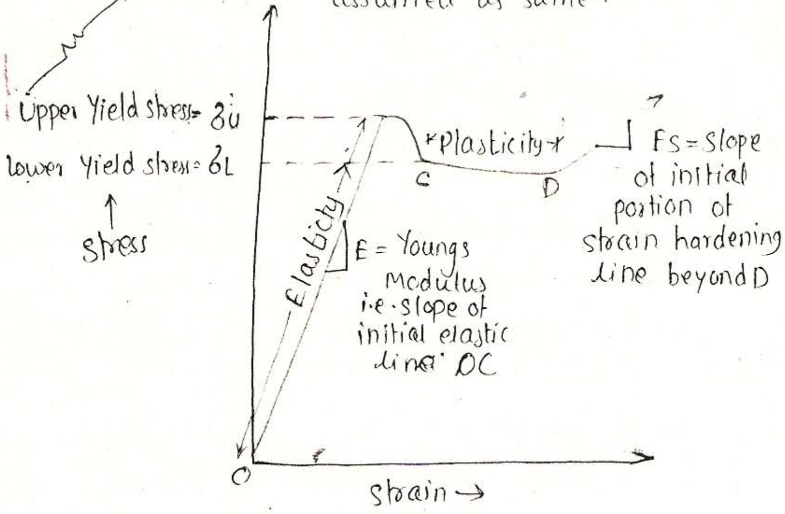
The limit design method is based on any chosen limit of structural usefulness- The limit can be attainment of the -

1) Yield Point stress (conventional Design)
2) Brittle fracture
3) Fatigue
4) Instability
5) Deflection.
6) Max. plastic strength /Limit Design/ Collape Design / Ultimate Design.

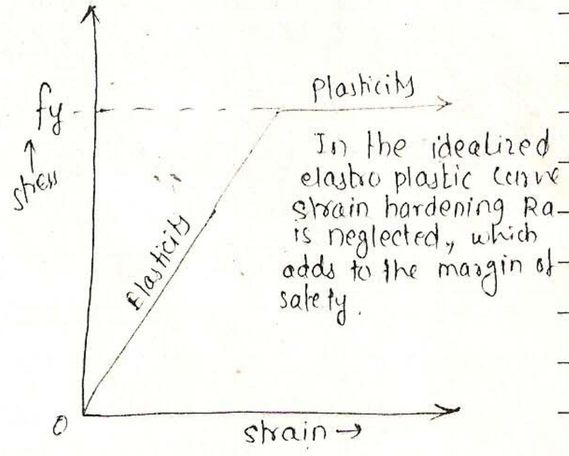
Stress - strain Curve for MS: (In tension)



As it is dependent on method (simply supported, Fixed) & rate of applicat^n of load.
In plastic Design of steel str. the elastic limit & lower yield pt may be assumed as same.



Enlarged Stress-strain Curve.

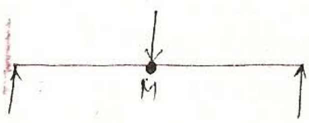


Idealized elasto-plastic Curve

Scope of Plastic Analysis

The plastic method of structural analysis is used in designing Redundant framed MS str., continuous or restrained beams & girders which carries load by virtue of flexural resistance. It is not suggested for statically determinate str. & pin jointed members. In case of simply supported beam, carrying a pt. load at midspan where BM is max., As load increases, moment increases until extreme fibre stresses reaches to yield point. on further increase in load, deformat^n & deflection increases rapidly. Thus load causing ~~critical~~ first yield is assumed to be critical load for s.s. beam, where large unacceptable deformat^n occurs.

Now, consider a fixed beam which have 3 peak mmts. - two at ends (equal) & one at centre. As load applied, the c/s at the greatest of 3 peak ~~gets~~ will reach the yield pt. As load increases, a zone of yielding developes there but remaining part of str. is elastic & controls deflections.

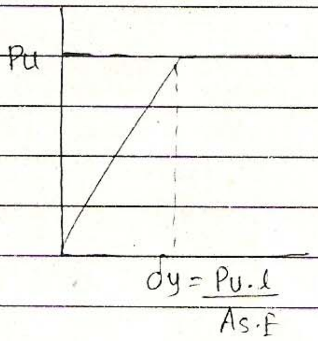


If $M_2 > M_1$, stress reachest to yield pt but remaining str. is in elastic limit as it has resistance ~~to~~ of flexure at both ends.

Thus there is considerable reserve of load carring capacity beyond the yield load Py. Thus redundant str. can be more economically designed due to plastic reserve of strength.

* Ultimate load carring capacity of tension member = $P_u = f_y \cdot A_s$.

$$\boxed{P_{at} = 0.85\, A_s \cdot f_y}$$



Where P_{at} = calculated max. load rapacity

A_s = Effective c/s area of member

f_y = Yield stress of steel.

* Ultimate load carring capacity of comp. member:

$P_u = f_y \cdot A_s$

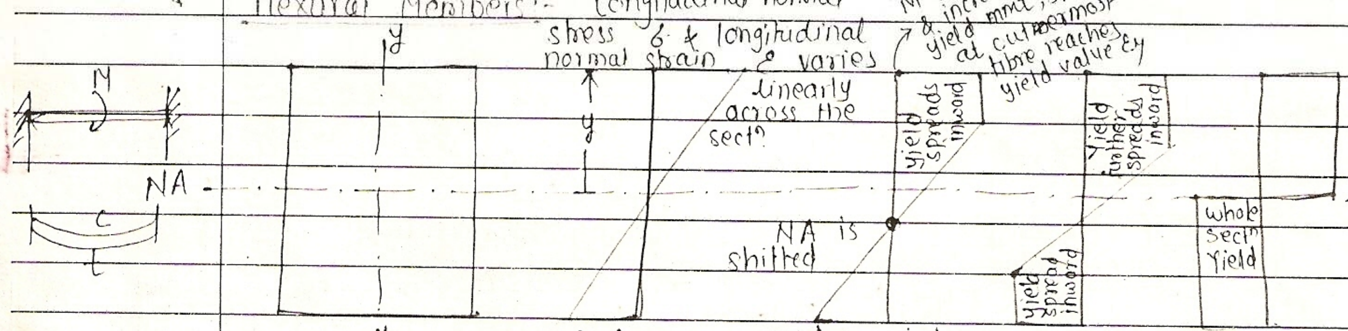
$$\boxed{P_{ac} = 1.7\, A_s\, \sigma_{ac}}$$

P_{ac} = Calculated max. load carring capacity

A_s = Effective c/s area of member

σ_{ac} = Max. per. stress in axial comp.

* Flexural Members:- Longitudinal normal stress σ & longitudinal normal strain ε varies linearly across the sectn.



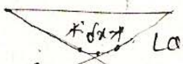
C/s of Beam subjected to steadily increasing moment 'M'	σ < fy Elastic limit stage BM = σ.Z	Yield stress just attained at lower fibre (NA) is shifted to maintain equilibrium. BM = fy.Z	Partial Plasticity	'M' is fully plastic mmt. any further increase in moment causes rotation since no resisting mmt can be developed until strain hardening occurs.

Assumtion in bending of beams:-

1) Stress-strain relationship is an ideal elasto plastic stress-strain relat'ship.
2) Beam is assumed as to bend by pure couple & SF & axial force is neglected.
3) The strains other than longitudinal strain are neglected.
4) Plane section before bending remains plane after bending.
5) C/s of member is symmetrical @ axis of bending.

Plastic Hinge :- Plastic hinge is a zone of yielding due to flexure in a Structural members. Although hinges are not actually formed, but it can be seen that large changes of slope occurs over small lengths.

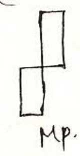
Friction Hinge or Rusty Hinge



Large change of slope at smaller length 'dx'

NOTE all the plastic Hinge concept is based in this sentense.

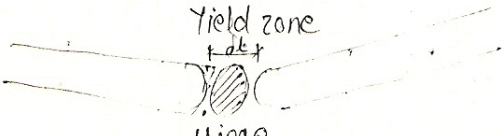
Let M_p = Max. moment of Resistance of a fully yielded c/s.



The member can remain within plastic limit until, M' reaches Mp.
applied

If 'M' is increased beyond 'Mp', rotation occurs at 'Mp'. The zone is acts as HINGE EXCEPT A CONSTANT RESTRAINING MOMENT 'Mp'.

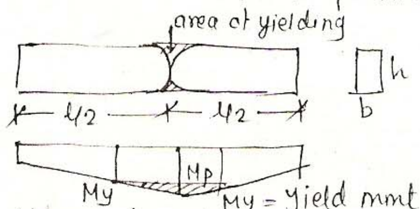
∴ Plastic Hinge can be defined as,. YIELDED ZONE DUE TO FLEXURE IN A STRUCTURE IN WHICH INFINITE ROTATION CAN TAKE PLACE AT A CONSTANT RESTRAINING MOMENT 'Mp' of the section.

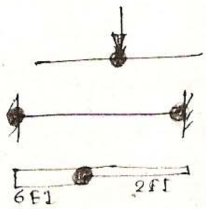


As the value of moment adjacent to yield zone is more than yield mmt, up to certain length depening upon loading & geometry of sectⁿ, but for simplicity it is neglected. But it can not be neglected for calculatⁿ of deflections & design of bracings.

The plastic hinges are reached first at the sections subjected to greatest deformatⁿ (curvature).

Possible Places of plastic Hinges :- ① pt. of concentrated load
② At the ends of members
③ Change in geometry pt.
④ Point of zero SF under Udl.





Hinge length of plasticity zone is $= \frac{L}{3}$ L = span.

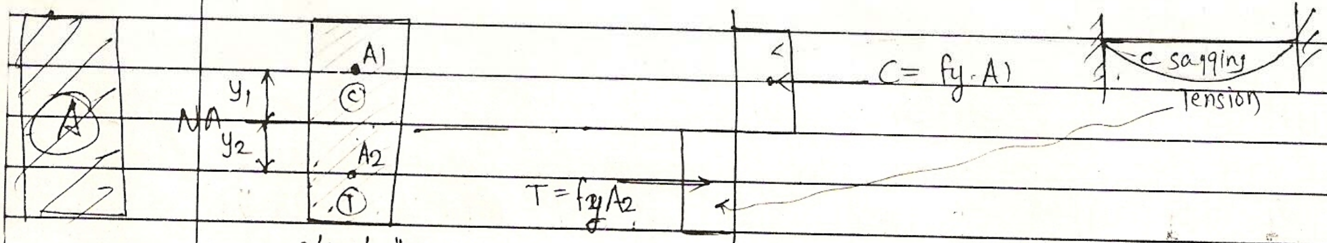
Fully plastic mmt of a section :-

Fully plastic mmt is the max. resistance of a fully yielded c/s. Following assumptions are maid to evaluate fully ~~mo~~ plastic mmt. in addition to the above :-

i) Instability (Deflectⁿ, deformatⁿ, yielding) of sh. will not occurs ~~pro~~ prior to attaintment of the ~~r~~ ultimate load.

2) Connections will provide full continuity ~~see~~ such that Mp can be transmitted.

3) Loading is proportional.



c/s of beam
(Fully Yielded) (Bending Stress Distribution)

For eq^m, Force of Comp. = Force of Tension

$\therefore fy \cdot A_1 = fy \cdot A_2$

$\therefore A_1 = A_2 = \frac{A}{2}$

Where A_1 = Area under comp. (Above NA)
A_2 = Area under Tension (below NA)
A = Total c/s area of beam

As $A_1 = A_2$, NA of plastified section is known as Equal Area Axis.

The couple produced by two forces (i.e. C & T) is resisted by plastic mmt.

$\therefore M_p = M_C + M_T = C y_1 + T y_2 = fy \cdot A_1 \cdot y_1 + fy \cdot A_2 \cdot y_2$

$= fy \cdot \frac{A}{2} y_1 + fy \cdot \frac{A}{2} y_2 = \frac{A}{2} [fy \cdot y_1 + fy A_2] = fy [\frac{A}{2}(y_1 + y_2)]$

V.imp $\therefore \boxed{M_p = fy \cdot Z_p}$

Where Z_p = Plastic modulus of section.

$= \frac{A}{2} \cdot (y_1 + y_2)$

Where y_1 & y_2 dist. of C.G. of Area under comp. & area under tension from NA. resp.

* Redistribution of Moment & Reserve of Strength :-

Under the gradually increasing load on a structure, the plastic moment is reached at a sect^n that is MOST HIGLY STRESSED. On the further increase in load, the value of plastic mmt is maintained & the section rotates.

"OTHER LESS HIGHLY STRESSED SECTIONS MAINTAIN IN EQUILIBRIUM

WITH THE INCREASED LOAD BY PROPORTIONATE INCREASE IN THE MOMENT & SUCCESSIVE FORULATION OF PLASTIC HINGES OCCURS AT OTHER SETIONS, THIS ~~CONINUES~~ CONTINUES TILL THE "ULTIMATE LOAD IS REACHED." This it can be said that flexural members can sustain the ultimate load due to redistribut^n of mmt.

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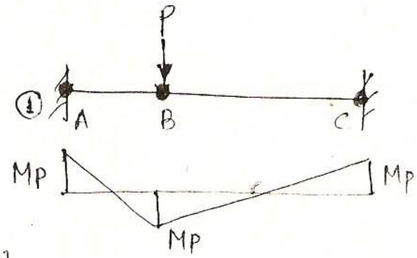

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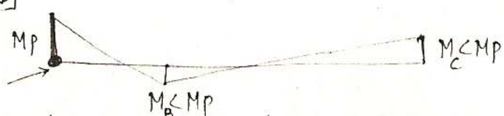
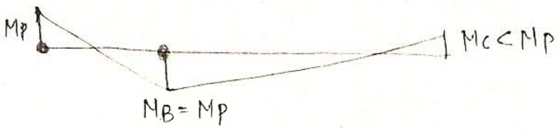


As load increases, the beam reaches in elastic limit at end 'A' & plastic hinge is formed. The mmt at B & C are less than plastic mmt.

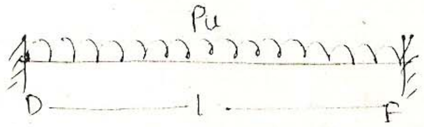


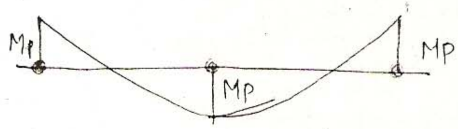
On further increase in load, plastic sect^n at A rotates without absorbing any more mmt. The MB increases till it reaches to Mp, & then plastic hinge is formed at B.

on further increase in load, plastic hinge is formed at 'C'. & "THE ULTIMATE LOAD IS REACHED".

∴ Format^n of plastic Hinges allows a subsequent redistribut^n of mmt until fully plastic mmt is reached at each critical section. "THUS REDISTRIBUTION OF MOMENT IS MAIN CONTRIBUITING FACTOR TO RESERVE OF STRENGTH."

$\boxed{Pu = 1.493\,Py}$ → for a particular case





$$\frac{Pu \cdot L}{8} = 2Mp$$

$$\therefore Pu = 16\frac{Mp}{L} \quad \dots (a)$$

Pu: Ultimate load, Mp = Ultimate mmt : ~~Yield~~ m = plastic mmt.

at elastic limit,

Mmt at centre = $\frac{1}{2}$ End moment.

$$\therefore \frac{Py \cdot L}{8} = My + \frac{1}{2}My = \frac{3}{2}My$$

Py = Yield load, My = Yield mmt.

$$\therefore Py = \frac{24}{2}\frac{My}{L} \quad \dots (b)$$

eq^n a/b

$$\therefore \frac{Pu}{Py} = \frac{\frac{16\,Mp}{L}}{\frac{12\,My}{L}} = \frac{4}{3}\frac{Mp}{My}$$

$$\therefore \boxed{\frac{Pu}{Py} = \frac{4}{3}\frac{Mp}{My}}$$

If $\frac{Mp}{My}$ = Shape factor = 1.12 for I section

$$\therefore \frac{Pu}{Py} = \frac{4}{3} \times 1.12 = 1.493$$

$$\boxed{\therefore Pu = 1.493\,Py}$$

Thus for I section fixed beam carrying udl over its span, ULTIMATE LOAD IS ∠49.3 % GREATER THAN THE LOAD AT THE FIRST YIELD. THIS IS RESERVED STRENGTH WHICH IS DISREGARDED IN CONVENTIONAL DESIGN.

* Shape factor :- Shape factor is the ratio of plastic mmt to yield mmt. This depends on the shape of c/s & denoted by 'f'.

$$\therefore f = \frac{MP}{My} = \frac{fy \cdot Zp}{fy \cdot Z} = \frac{Zp}{Z}$$ Zp = Plastic Sectⁿ Modulus, Z = Elastic Sectⁿ Modulus.

* Load Factor :- Load factor is the ratio of collapse / ultimate load to working, yield load. 'F'.

$$\therefore F = \frac{Pu}{Pw} = \frac{MP}{Mw} = \frac{fy \cdot Zp}{fy \cdot Z} = \frac{fy \cdot Zp}{\sigma \cdot Z} = FOS \times f$$

(fy = yield stress, σ = Per. stress)

$$\boxed{F = FOS \times f}$$

$\therefore$ F load factor can be defined product of factor of safety & Shape factor.

If $f = 1.12$,

& $FOS = \frac{fy}{\sigma} = \frac{fy}{0.66 fy} = 1.515$

$\therefore$ load factor $= 1.515 \times 1.12 = 1.70$.

Load Factor F :-

DL	1.7
DL + LL	1.7
DL + WL	1.7
DL + LL + WL	1.3 (Reduced by 25% for wind / seismic)

The prime function of load factor is to ensure safty of str. under service condition.

* MECHANISM :-

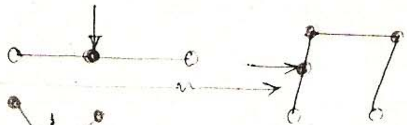
When a structure is subjected to load, sufficient No. of plastic higes are formed to transfer all the moment that is possible. The segment of beam betⁿ plastic hinges are able to move without increase in load. This system of member is called as mechanism.

If an indeterminate structure has redundancy 'r' ~~mak~~ becomes statically determinate after insertion of 'r' plastic hinges. Any further ~~Hig~~ hinge converts this statical determancy in to mechanism.

∴ at collapse, No. of plastic hinges reqd = $r+1$

* Types of Mechanism :-

① Beam Mechanism →

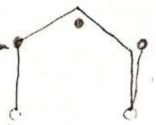


② Panel Mechanism →

③ Joint Mechanism →

No. of ~~joints~~ members at a joint $\not> 3$

④ Gable Mechanism →



⑤ Composite or combined mechanism

* No. of independent Mechanism :-

Let N = No. of possible plastic Hinges.

n = No. of possible independent Mechanism.

r = No. of redundancies,

Then, $\boxed{n = N - r}$

After finding out No. of independent mechanism, all the possible combinations are made, such that EXTERNAL WORK is MAX. or the INTERNAL WORK IS MIN. This is done to obtain lowest possible load.

* Plastic collapse :-

1) Partial Collapse :- No. of plastic hinges in collapse Mechanism $<(r+1)$

2) Complete collapse :- No. of plastic hinges in collapse mechanism $= r+1$

3) Overcomplete Collapse : No. of plastic hinges in collapse Mech $> r+1$

* Conditions in Plastic analysis :-

① Eq^m cond^n :- $\Sigma F = 0$, $\Sigma M = 0$

② Mechanism Cond^n :- Ultimate load is reached when mechanism forms.

③ Yield cond^n :- / Plastic Moment Cond^n :- M at any sect^n < M_p

* Principal of Virtual Work :-

If the system of forces in eq^m is subjected to virtual displacement, the workdone by external forces equal to work done by internal forces.

i.e. $\boxed{W_E = W_i}$

This principle is used to find collapse load. i.e. work done by external forces is equated to work absorbed by plastic hinges.

* Theorem of Plastic Analysis :-

Usually it is not possible to satisfy 3 eq condit^n of plastic analysis in one operation. ~~A solution got on the basis of MECHANISM will~~.

If load carring capacity is calculated from MECHANISM, it will either correct or TOO HIGH, whereas load carring capacity calculated from YIELD / PLASTIC MOMENT condition will be either correct or TOO LOW.

i.e. MECHANISM COND^n : CORRECT SOL^n OR TOO HIGH : as Plastic Hinge forms

YIELD / PLASTIC MOMENT COND^n : CORRECT OR TOO LOW : $M < M_p$

& Third cond^n i.e. EQUILIBRIUM COND^n has to be satisfied in both cases.

Thus, depending upon method choose, the sol^n (i.e. load carring capacity) obtained is either UPPER LIMIT (UPPER BOUND) below which correct sol^n lies ~~&~~ or LOWER LIMIT (LOWER BOUND) above which correct solution lies.

$(P \leq P_u)$ * Static or LOWER BOUND THEOREM :- (Kist / Gvozder / Greenberg / Prager / Horne)

For a given frame & loading, if there exists any distribution of BM throughout the frame which is safe & eq^m with set of load 'P', the value of load 'P' must be less than or equal to collapse load. (P_u)

$\therefore \boxed{P \leq P_u}$

$P \geq P_u)$ * Kinematic or UPPER BOUND THEOREM :- (Gvozder / Greenberg)

For a given frame subjected to set of loads 'W', the value of 'P' (~~cap~~ load carring capacity) is found & corresponding to any mechanism, must be either greater than or equal to collapse load.

$\boxed{P \geq P_u}$

$(P = P_u)$ * Uniqueness Theorem :- (Horne)

For a given frame & loading if it is safe & statically admissible, BM distribut^n is equal to fully plastic moment at sufficient c/s to cause failure of a frame as a mechanism due to rotat^n of plastic hinge, then corresponding load is equal to collapse load. (P_u)

$\therefore \boxed{P = P_u}$

∴ Analysis of str :- :- Static moment diagram :- Correct or low solⁿ : $P > P$
:- Mechanism :- Correct or High solⁿ. $P < P$

∴ If both the ~~set~~ theorem satisfies for a given problem then solⁿ is said to be correct one.

* Method of analysis :-

① Static Method. / ~~Lower~~ ~~Upper~~ Lower Bound *

② Kinematic Method. / ~~Lower~~ Upper Bound *

① Static method of plastic analysis :- The free & redundant BMD are drawn for a given str. A combined BMD is drawn in such a way that a mechanism is formed. Collapse load is foundⁿ by eqᵐ eqⁿ. It is checked that BM $\not>$ Mp at any section.

② Kinematic Method :-

It consist of locating the possible places of plastic hinges. The possible independent & combined mechanisms are ascertained. The P_u can be found by applying principle of Virtual work.

BMD corresponding to collapse mechanism is drawn & it is checked that BM $\not>$ Mp at any sectⁿ.

* Note :- When beam & sway mechanism are combined one hinge is cancelled. For both mechanism has 1 degree of freedom. For the eqᵐ, hinge rotation in ~~each~~ both mechanisms, α, β should be equal, which results in cancellation of one hinge.

* Design :-

① Find working load,
② Factored / Ultimate / Collapse load = (load factor) × (Working load)
③ Find Mp .... from Virtual work principle
④ Find Plastic modulus of sectⁿ ⇒ $Z_p = M_P/f_y$.
⑤ Section is selected which furnishes the reqd. elastic modulus $(Z_e = Z_P/f_y)$

Note: In case of continuous beams, Mp for various spans may be different, to keep uniform c/s, largest Mp is selected & beam is designed. But to achieve economy, lowest Mp is selected & section is designed & cover plates are provided for "excess Mp."

$P > f$

$P < P$

* Limitations of plastic Analysis:-

Simple plastic Theory assumes that -

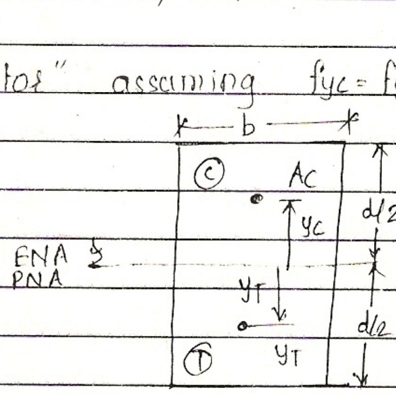
1) Fabricat^n is done in ductile steel, i.e. we can not use brittle matl.
2) Effects of SF & axial forces are neglected, thus in case of truss, members are subjected to axial force rather than bending, so plastic analysis is not recommanded for "Trusse"
3) Strength is assumed to be main criteria & deflections are checked only when its value is larger.

* Comparism of plastic analysis & elastic analysis:-

1) Plastic design gives more economical sect^n than elastic design for simillar load conditions, as they uses reserve strength of matl. beyond elastic limit. (Matl. saving = 10 - 15 %)
2) Design procedure of plastic analysis is much simpler & rational, ~~fo~~ especially for indeterminate & complicated structures.
3) In elastic design, design procedure is repeated several time to obtain "optimum section" which consumes more time, the plastic analysis gives "Balanced Section" in single attempt.
4) FOS is same for both the designs.
5) Stress produced by settlement, erection are more correct.

* Finding out "shape factor" assuming $f_{yc} = f_{yt}$

① $f = \dfrac{Z_P}{Z_e}$



$Z_e = I / y_{max}$

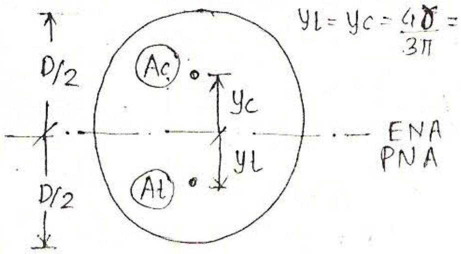
$Z_P = A_C \cdot y_C + A_T \cdot y_T$

$\therefore f = \dfrac{Z_P}{Z_e} = \dfrac{(A_C \cdot y_C + A_T \cdot y_T)}{~~Z_P~~\ I/y_{max}}$

Rectangular Section

$= \dfrac{[\,b \cdot d/2 \times d/4 + b \cdot d/2 \cdot d/4\,]}{\frac{bd^3}{12} / d/2} = \dfrac{2 \cdot b \cdot d^2/8}{bd^2/6} = \dfrac{bd^2}{4} \times \dfrac{6}{bd^2} = \dfrac{6}{4} = 1.5.$

② Solid circular Section :-



$$y_t = y_c = \frac{4r}{3\pi} = \frac{2D}{3\pi}$$

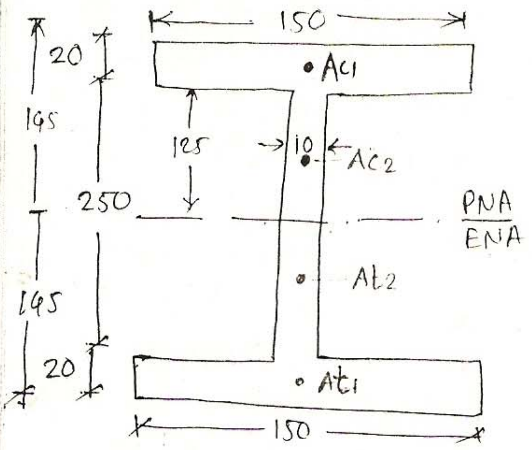
$$f = \frac{Z_p}{Z_e = I/y} = \frac{[A_c \cdot y_c + A_t \cdot y_t]}{\frac{\pi}{64} \cdot D^4 / y_{max}} = \frac{\left[\frac{\pi}{8} D^2 \times \frac{2D}{3\pi} + \frac{\pi}{8} D^2 \times \frac{2}{3}\right.}{\frac{\frac{\pi}{64} \cdot D^4}{D/2}}$$

$$\therefore f = \frac{2\left(\frac{\pi D^2}{8} \cdot \frac{2D}{3\pi}\right)}{\frac{\pi}{64} \cdot D^3 \cdot 2} = \frac{D^3/6}{\frac{\pi D^3}{32}} = \frac{D^3}{6} \times \frac{32}{D^3 \pi}$$

$$\therefore f = \frac{32}{6\pi}$$

$$\therefore \boxed{f = 1.6976} \approx 1.7$$

③ I section :-



$$f = \frac{Z_p}{Z_e}$$

$$Z_p = [A_c \cdot y_c + A_t \cdot y_t] = [A_{c1} \cdot y_{c1} + A_{c2} \cdot y_{c2}] + [A_{t1} \cdot y_{t1} + A_{t2} \cdot$$

$$= \left\{[(150 \times 20) \times (125 + 10)] + \left[(10 \times 125) \times \left(\frac{125}{2}\right)\right]\right\} \times 2$$

$$= 966.25 \times 10^3 \ mm^3$$

$$Z_e = \left(\frac{BD^3}{12} - \frac{bd^3}{12}\right) / y_{max}$$

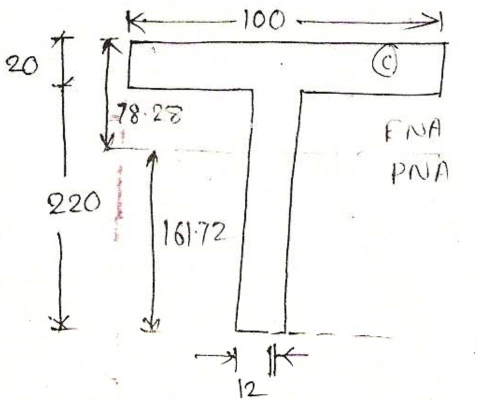
$$= \left[\frac{150 \times 290^3}{12} - 2\left(\frac{70 \times 250^3}{12}\right)\right] \times \frac{1}{145}$$

$$= \frac{122.571 \times 10^6}{145} = 845.316 \times 10^3 \ mm^3$$

$$\therefore f = \frac{Z_p}{Z_e} = \frac{966.25 \times 10^3}{845.316 \times 10^3}$$

$$\boxed{f = 1.143}$$

④ T section :-



$$f = \frac{Z_p}{Z_e}$$

To find Position of ENA or PNA.

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{[100 \times 20 \times (220 + 10)] + [(220 \times 12)(220/2)]}{[100 \times 20 + 220 \times 12]}$$

$$= 161.72$$

$$Z_e = \frac{I}{y}$$

$$\therefore I = \frac{bd^3}{12} - A \cdot h^2 = \left[\left(\frac{100 \times 20^3}{12} + 100 \times 20(78.28 - 10)^2\right) + \left(\frac{12 \times 220^3}{12} + 12 \times 220(161.72 - 110)^2\right)\right]$$

$$\therefore I = 27.101 \times 10^6 \ mm^4$$

SS/3

$\therefore Z_e = \dfrac{I}{y_{max}} = \dfrac{27.101 \times 10^6}{161.72} = 161.584 \times 10^3 \text{ mm}^3$

Now, $Z_p = A_c \cdot y_c + A_t \cdot y_t$

$= [100 \times 20 \times (26.7 + 10) + 26.7 \times 12 \times 26.7/2] + [12 \times 193.3 \times 193.3/2]$

$= 301.87 \times 10^3 \text{ mm}^3$

$\therefore f = \dfrac{Z_p}{Z_e} = \dfrac{301.87 \times 10^3}{161.584 \times 10^3} = 1.78$

In case of 'I' sect^n if $f_{yc} = 200$ MPa, $f_{yt} = 250$ MPa.

Then, $M_e = Z_e$ [least of f_{yc} and f_{yt}]

$= 845.316 \times 10^3 [200] \times 10^{-6}$ · · ~~self~~ See second last pro.

$= 169.062$ kNm

$M_p = \cancel{A_c y_c} \; A_c \cdot y_c \cdot f_{yc} + A_t \cdot y_t \cdot \cancel{f_t} \; f_{yt}$

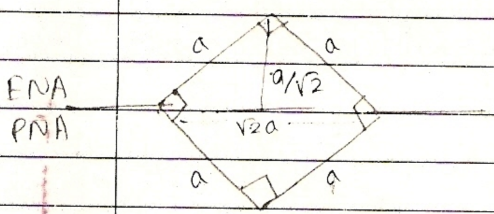
$= (150 \times 20 \times 135 + 10 \times 125 \times 125/2) \times 200 + (150 \times 20 \times 135 + 10 \times 125 \times 125/2) \times 250$

$= 217.4 \times 10^6$ Nmm

$= 217.4$ kNm

$\therefore f = \dfrac{M_p}{M_e} = \dfrac{217.4}{169.062} = 1.28$

* Shape factor for diamond section ($f_{yc} = f_{yt}$)



$I = \dfrac{a^4}{12}$; $y_{max} = a/\sqrt{2}$

$\therefore Z_e = \dfrac{a^3}{6\sqrt{2}}$

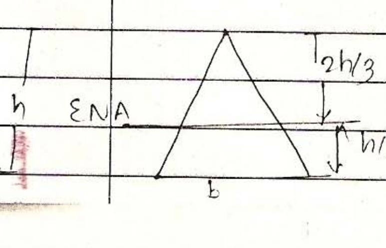
$Z_p = A_c y_c + A_t \cdot y_t$

$= \left[\frac{1}{2} \sqrt{2}a \cdot \frac{a}{\sqrt{2}} \times \frac{1}{3}\left(\frac{a}{\sqrt{2}}\right)\right] \times 2$

$\therefore Z_p = a^3 / 3\sqrt{2}$

$\boxed{\therefore f = 2}$

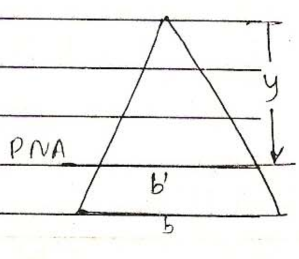
* Shape factor for triangular section :-



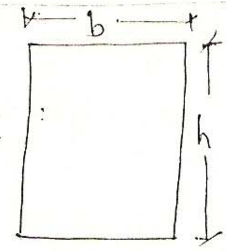
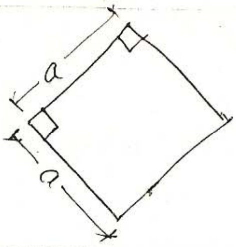
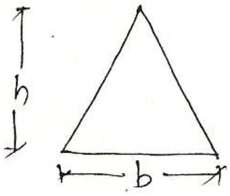
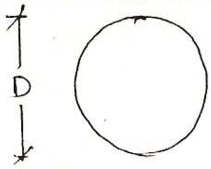
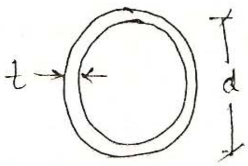
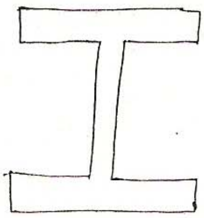
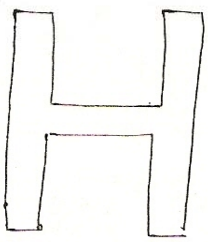
$Z_e = \dfrac{bh^3/36}{2h/3} = \dfrac{bh^2}{24}$

$Z_p =$

$\boxed{f = 2.34}$



Shape factor for various sections:

	Rectangle	$f = 1.5$
	Diamond	$f = 2$
	Triangle	$f = 2.34$
	Solid circular	$f = 1.7$
	Ring	$f = 1.27$
	Wide flange I sect^n (strong axis)	$f = 1.14$
	Wide flange (weak axis)	$f = 1.5$

To find collapse load :-

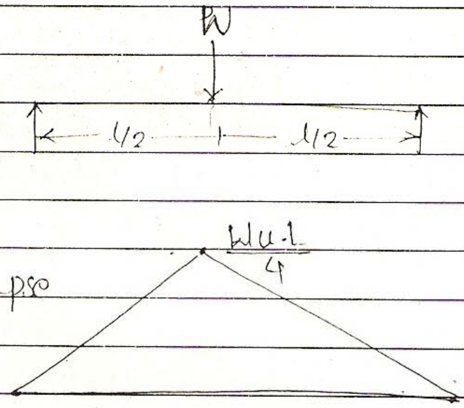
NOTE

* External Work done = W_e = Force x Displacement (Pt. load)
= Intensity of load x area under mechanism diagram under udl.

* Internal work done = W_i = $M_p \cdot \theta$
= Plastic Moment x Rotation

Governing M_p :- Largest value obtained from possible mechanism
Governing P_u :- least value — " —

1) Simply Supported Beam, pt. load at centre.



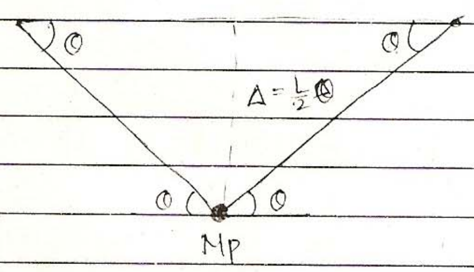
Static Method,

BMD at Collapse

$$M_p = \frac{W_u L}{4}$$

$$\therefore \boxed{W_u = \frac{4M_p}{L}}$$

Beam Mechanism



Kinematic Method,

$W_e = W_i$

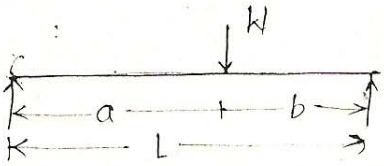
$\therefore W_u \times \Delta = M_p \cdot (\theta + \theta)$

$\therefore W_u \times \frac{L}{2} \cdot \theta = 2M_p \theta$

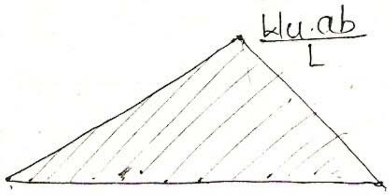
$$\boxed{W_u = \frac{4M_p}{L}}$$

$\therefore Z_e = \frac{I}{y_{max}} = \frac{27.101 \times 10^6}{161.72} = 161.589 \times 10^3 \, mm^3$

II] S.S. beam carrying pt. load not at centre.



BMD at collapse

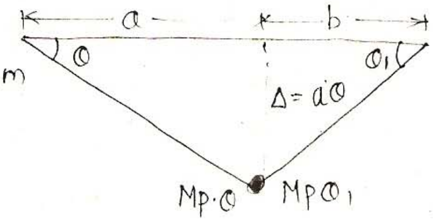


Static Method

$\therefore M_p = \frac{W_u \cdot ab}{L}$

$\therefore \boxed{W_u = \cancel{\frac{M_p \cdot ab}{L}} \; \frac{M_p \cdot L}{ab}}$

Beam Mechanism



$\cancel{\frac{\Delta}{a} \; \frac{\theta}{a} = \frac{\theta_1}{b} \Rightarrow \theta_1 = \theta \frac{b}{a}}$

$\theta_1 = \frac{\Delta}{a}, \quad \theta_2 = \frac{\Delta}{b}$

Kinematic Method :-

$W_e = W_i$

$\therefore \cancel{Pu} \; W_u \cdot \Delta = M_p\theta + M_p\theta_1$

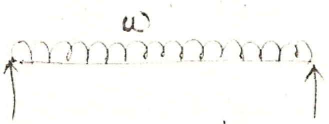
$\therefore \cancel{W_u \cdot \theta \cdot a = M_p(\theta +}$

$\therefore W_u \cdot \Delta = M_p\left(\frac{\Delta}{a} + \frac{\Delta}{b}\right)$

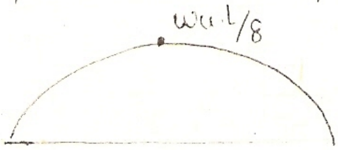
$\therefore W_u = M_p\left(\frac{b+a}{ab}\right)$

$\therefore \boxed{W_u = \frac{M_p \cdot L}{ab}}$

III) S.S. beam carrying udl



BMD at collapse

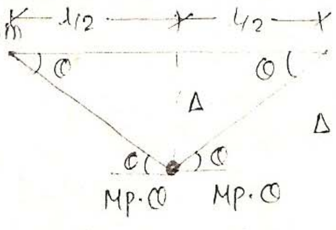


Static Method,

$\therefore M_p = \frac{W_u \cdot L}{8}$

$\therefore \boxed{W_u = \frac{8M_p}{L}}$

Beam Mechanism



$\Delta = \frac{L}{2}\theta$

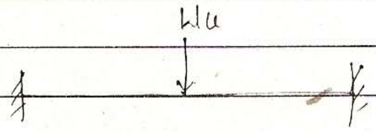
Kinematic Method,

$W_u \times \frac{1}{2}\left(\cancel{\Delta}\right)\left[\frac{L}{2}\theta\right] = M_p\theta + M_p\theta$

$W_u \times \cancel{\theta} \times \frac{L}{2}\left(\right. \quad W_u \cdot \frac{L}{4} = 2M_p$

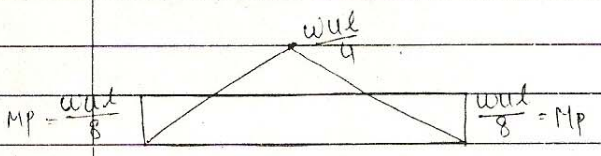
$\therefore \boxed{W_u = 8M_p/L}$

* Fixed beam: Pt. load at centre



Static Method.

$$\frac{Wu\,l}{8} = M_p + M_p$$

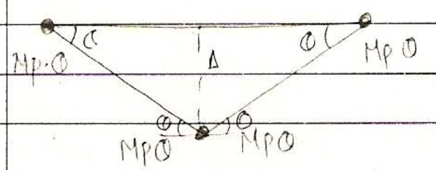


$\therefore$ ~~$M_p = \frac{l}{8}Wu$~~ $\boxed{Wu = \frac{8M_p}{L}}$

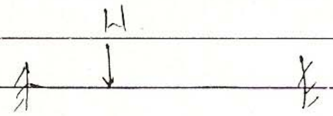
Kinematic Method.

$$Wu\left(\frac{l}{2}\theta\right) = 4M_p\theta$$

$$\boxed{Wu = \frac{8M_p}{L}}$$



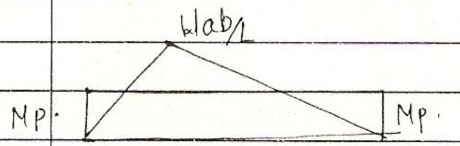
* Fixed Beam: Pt load not at centre

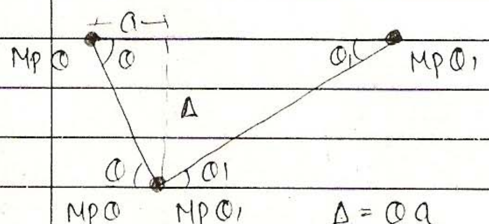


Static Mtd.

$$\frac{Wab}{l} = 2M_p.$$

$$\therefore \boxed{W = \frac{2M_p l}{ab}}$$





$\Delta = \theta a$

$\theta = \Delta/a \quad \theta_1 = \Delta/b$

$\therefore Wu\cdot\Delta = 2M_p\theta + 2M_p\theta_1$

$\therefore$ ~~$Wu\cdot(\theta\cdot a) = 2M_p\theta + 2M_p$~~

$\therefore Wu\cdot\Delta = 2M_p\left(\frac{\Delta}{a}\right) + 2M_p\left(\frac{\Delta}{b}\right)$

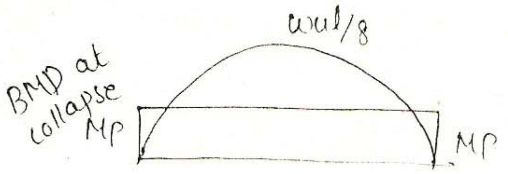
$\therefore Wu = 2M_p\left(\frac{a+b}{ab}\right)$

$$\therefore \boxed{Wu = \frac{2M_p l}{ab}}$$

Collapse BM = $\frac{W_u L}{4}$ ... S.S.

$= \frac{W_u L}{8}$ ... Fixed

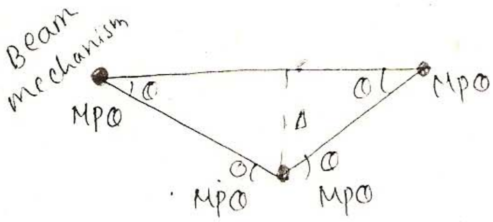
* Fixed Beam: Carrying udl:



Static Mtd:

$W_u \cdot \frac{L}{8} = 2M_p$

$\boxed{\therefore W_u = \frac{16 M_p}{L}}$



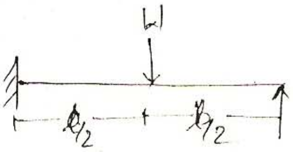
∴ Kinematic Mtd.

$W_u \cdot \Delta = \cancel{4 W_u \Theta}\ 4M_p \cdot \Theta$

$\therefore W_u \frac{L}{2 \cdot 2} \Theta = 4 M_p \Theta$

$\therefore \boxed{W_u = \frac{16 M_p}{L}}$

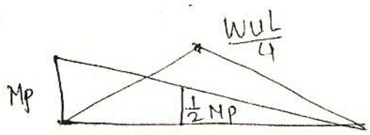
Propped Cantilever:



Static Mtd.

$\frac{W_u L}{4} = M_p + \frac{1}{2} M_p = \frac{3}{2} M_p$

$\therefore \boxed{W_u = \frac{6 M_p}{L}}$



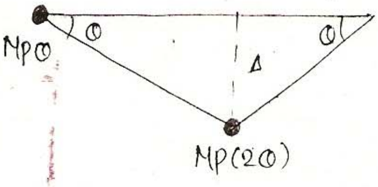
Kinematic Mtd

$\therefore W_e = W_i$

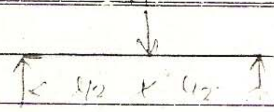
$\therefore W_u \Delta = 3 M_p \Theta$

$\therefore W_u \frac{L}{2} \Theta = 3 M_p \Theta$

$\therefore \boxed{W_u = \frac{6 M_p}{L}}$

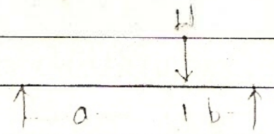


Scummary

(1) 

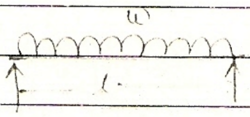
$W_u = \frac{4M_P}{L}$

S.S. load at Midspan

(2) 

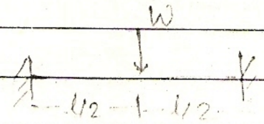
$W_u = \frac{M_p \cdot L}{ab}$

S.S. load not at midspan

(3) 

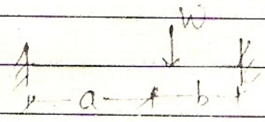
S.S., udl

$W_u = \frac{8M_P}{L}$

(4) 

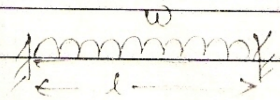
fixed, load at midspan

$W_u = \frac{8M_p}{L}$

(5) 

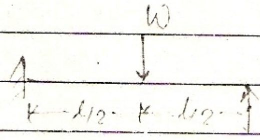
fixed load at not midspan

$W_u = \frac{2M_pL}{a \cdot b}$

(6) 

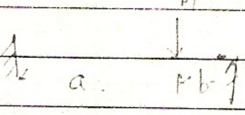
fixed, udl

$W_u = \frac{16M_p}{L}$

(7) 

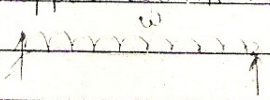
Propped cantilever, load at midspan

$W_u = \frac{6M_P}{L}$

(8) 

$W_u = \frac{L+b}{ab} \cdot M_p$

Propped cantilever load not at midspan

(9) 

$W_u = \frac{11.655\,M_p}{l^2}$

The great poet Dr. Oliver Wendell Holmes in his "The Deacon's Masterpiece" wrote of such a structure.

"HAVE YOU HEARD OF THE WONDERFUL ONE HOSS SHAY,
THAT WAS BUILT IN SUCH A LOGICAL WAY
IT ~~CAN~~ RAN FOR A HUNDRED YEARS TO A DAY
AND THEN, OF A SUDDEN, it ..
.. WENT TO PIECES ALL AT ONCE, –
ALL AT ONCE AND NOTHING FIRST, –
JUST AS BUBBLE DO WHEN THEY BURST."