Strength of Materials

Notes by-

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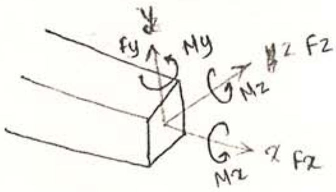
A

- Mechanics
  - Particles & Rigid bodies (Applied Mechanics)
  - Deformable bodies
    - Elastic [SOM]
    - Plastic
    - Fluid [Hydraulics, Fluid Mechanics Aerodynamics]

Particle : Dimensions neglected.
Rigid body: Deformation neglected; dimensions considered.

* Types of load ; according to effect they produce.



Fx : Axial force. (Normal to c/s ; passing th'. C.G)
: causes change in length.
: compressive ; Tensile.

Fy ; Fz : shear force: (Tangential to c/s)
: causes sliding of one c/s w.r.t. other.

Mx : Torsional mmt: (Normal to c/s)
[Torque / Torsion]
causes rotation of diff. c/s of member w.r.t. each other. (Twisting effect)

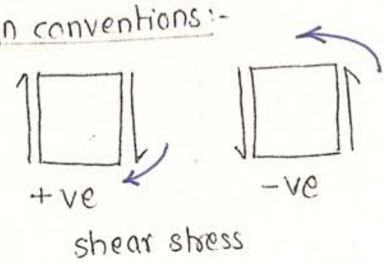
My, Mz : Bending mmt : (Tangential to c/s)
causes slope & deflection.

* Stress: Resistance to deformation per unit area. (N/mm^2)

Normal stress : Normal to c/s : $\sigma_n = \dfrac{\text{Resistive force} = \text{Applied force} = P}{\text{c/s area of member} = A}$

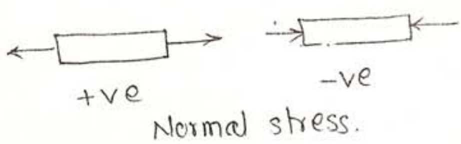
Shear stress : Tangential to c/s : $\tau = \dfrac{\text{shear force}}{\text{c/s area}}$

* Sign conventions :-



shear force ; which produces clockwise couple is +ve ; & that produces anticlockwise couple is -ve.

Tensile force (Normal) = +ve
compressive force = -ve



Unit's conversion : $1\,MPa = 1\,N/mm^2$
$1\,GPa = 1\,kN/mm^2$
$1\,GPa = 10^3\,MPa$

* strain : change in length per unit original length. (dl/l)

① Linear strain : $= \varepsilon = \dfrac{\text{change in dimension}}{\text{original dimension}}$.

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a) Longitudinal strain : $\varepsilon_L = \dfrac{\text{change in length}}{\text{original length}}$

(b) Lateral strain = $\varepsilon_{Lt} = \dfrac{\text{change in dia}}{\text{original dia.}}$

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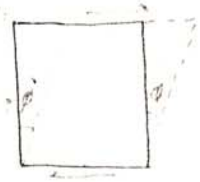
2) Shear strain:



shear strain = $\gamma = \tan\phi$

$\tan\phi \simeq \phi$.

* Sign convention:-

Linear strain: Tensile +ve

: Compressive -ve

* Poisson's Ratio: (μ)

$$\mu = \frac{\text{Lateral strain}}{\text{Linear strain}} = -\frac{\varepsilon_{Lt}}{\varepsilon_L}$$

$= 0.25$ to 0.35.

* Hookes Law: Within elastic limit; elongation is proportional to tensile force.

i.e. ~~strain~~ stress is proportional to strain

$\therefore dL \propto \frac{P \cdot L}{A}$ $\qquad \sigma \propto \varepsilon$

$\therefore \delta L = \frac{PL}{AE}$ $\qquad$ i.e. $\sigma = E\varepsilon$.

E = Modulus of elasticity. or Youngs Modulus.

* Young's Modulus:-

$$E = \frac{\sigma_n}{\varepsilon_L} = \frac{(P/A)}{(\delta L/L)} = \frac{PL}{A \cdot \delta L}$$

* Modulus of Rigidity or shear Modulus:-

shear stress $\propto$ shear strain; within a limit

$\therefore \tau \propto \gamma$

$\tau = G\gamma$

$G = \frac{\tau}{\gamma}$

G = Modulus of Rigidity or shear modulus.

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Modulus of Elasticity	Modulus of Rigidity shear Modulus.
Young's Modulus	shear stress $\propto$ strain
Normal ~~strain~~ stress $\propto$ strain	$\tau \propto \gamma$
$\therefore \sigma \propto \varepsilon_L$	$\tau = G \cdot \gamma$
$\sigma = E \cdot \varepsilon_L$	$G = \frac{\tau}{\gamma}$
$E = \frac{\sigma}{\varepsilon_L}$	

* Volumetric stress / hydrostatic stress:- (σ_v) : change in volume
* Volumetric strain: Ratio of change in vol. (dv) to original vol. (V)

$\therefore \varepsilon_V = \frac{dV}{V}$

$dV = l \cdot b \cdot d$

$dV = \cancel{(l+\delta l)(b+\delta b)(d+\delta d)}$ $\delta l \cdot b \cdot d + \delta b \cdot L \cdot d + \delta d \cdot L \cdot b$

$$\boxed{\varepsilon_V = \frac{\delta V}{V} = \frac{\delta L}{L} + \frac{\delta b}{b} + \frac{\delta d}{d}}$$

volumetric strain = Algebraic sum of linear strain of all three sides.

$\therefore \varepsilon_V = \varepsilon_L + \varepsilon_{Lt} + \varepsilon_{Lt}$ $\left[\text{as } \mu = -\frac{\varepsilon_{Lt}}{\varepsilon_L} \Rightarrow \varepsilon_{Lt} = -\mu \cdot \varepsilon_L \right]$

$= \varepsilon_L - \mu \varepsilon_L - \mu \varepsilon_L$

$= \varepsilon_L (1 - 2\mu)$

* <u>Bulk Modulus</u> :- Ratio of volumetric stress to volumetric strain

$$\therefore K = \frac{\sigma_V}{\varepsilon_V}$$

* <u>Axial Flexibility (f)</u> :-

f = change in length produced by unit axial force.

$$\therefore \delta L = \frac{P \cdot L}{A \cdot E}$$

$$\therefore \frac{\delta L}{P} = \frac{L}{AE}$$

$$\therefore \boxed{f = \frac{L}{AE}} \quad (\text{mm/N})$$

AE = Axial Rigidity.

* <u>Axial stiffness (S)</u> :- Axial force reqd to cause unit change in length.

$$\therefore \delta L = \frac{P \cdot L}{A \cdot E}$$

$$\therefore \frac{AE}{L} = \frac{P}{\delta L}$$

$$\therefore \boxed{S = \frac{AE}{L}} \quad \text{N/mm}$$

Summary

① Axial force : $P = \sigma \times A$: Normal stress : $\sigma = P/A$

② shear force : $P = \tau \times A$: shear stress : $\tau = P/A$

③ Linear strain - $\varepsilon = \frac{\delta \text{dim}}{\text{dim}}$

Longitudinal strain = $\varepsilon_L = \frac{\delta L}{L}$; Lateral strain = $\varepsilon_{Lt} = \frac{\delta d}{d}$ (d = dia)

④ shear strain = $\gamma = \tan\phi$

⑤ Poisson's Ratio = $\mu = -\varepsilon_{Lt}/\varepsilon_L$

⑥ volumetric strain stress (σ_V) = P/A

⑦ volumetric strain = $\varepsilon_V = \frac{\delta V}{V}$

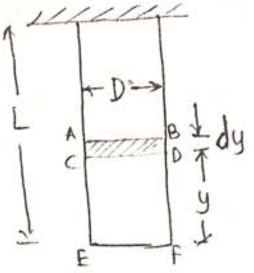
$= \frac{\delta L}{L} + \frac{\delta b}{b} + \frac{\delta d}{d}$

$= \varepsilon_L (1 - 2\mu)$

* Modulus of Elasticity (E)	Modulus of Rigidity (G)	Bulk Modulus (K)
Young's Modulus	shear Modulus	
Normal stress ∝ Longi. strain	shear stress ∝ shear strain	volumetric stress ∝ volumetric strain
$\sigma_n \propto \varepsilon_L$	$\tau \propto \gamma$	$\sigma_V \propto \varepsilon_V$
$\sigma_n = E \cdot \varepsilon_L$	$\tau = G \cdot \gamma$	$\sigma_V = K \cdot \varepsilon_V$
$E = \frac{\sigma_n}{\varepsilon_L} = \frac{\text{Normal stress}}{\text{Longi strain}}$	$G = \frac{\tau}{\gamma} = \frac{\text{shear stress}}{\text{shear strain}}$	$K = \frac{\sigma_V}{\varepsilon_V} = \frac{\text{Vol. stress}}{\text{Vol. strain}}$

$$\boxed{E = 2G(1+\mu) = 3K(1-2\mu)}$$

* stress & elongation in bar due to SW:-



Length of bar $= L$
Dia. of bar $= D$
wt. per unit vol. of bar $= \omega$
Consider a small ~~stree~~ strip of thk. dy at dist. 'y' from free end as shown.

$\therefore$ Downward force at $CD =$ wt. of bar CDEF

$$= \left[\frac{\pi}{4} \cdot D^2 \times y\right] \times \omega$$

$\therefore$ stress at section $CD = \sigma = \frac{\text{Force}}{\text{c/s area}}$

$$= \frac{\pi/4 \cdot D^2 \cdot y\omega}{\pi/4 \cdot D^2}$$

$$\therefore \sigma = y \cdot \omega$$

$$\therefore \sigma_{max} = L \cdot \omega$$

Elongat^n of length $dy = \frac{\sigma}{E} \cdot dy$

$$= \frac{y \cdot \omega}{E} \cdot dy$$

$$\therefore \text{Total elongation} = \int_0^L \cdot \frac{w}{E} \left(\frac{y^2}{2}\right) dy \quad (dy)$$

$$= \frac{w}{E}\left[\frac{y^2}{2}\right]_0^L$$

$$\boxed{\delta L = \frac{wL^2}{2E}}$$

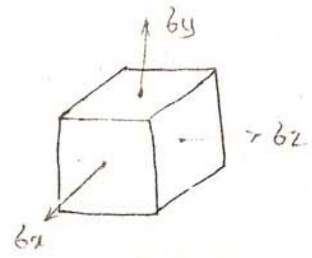
$$\boxed{\text{Maximum elongation} = \frac{P \cdot L}{A \cdot E} + \frac{\omega L^2}{2E}}$$

* Generalised Hookes Law:-

$$\varepsilon x = \frac{1}{E}(\sigma x - \mu \sigma y - \mu \sigma z)$$

$$\varepsilon y = \frac{1}{E}(\sigma y - \mu \sigma x - \mu \sigma z)$$

$$\varepsilon z = \frac{1}{E}(\sigma z - \mu \sigma x - \mu \sigma y)$$



* Temperature stresses:-

$$\delta L = \alpha \cdot t \cdot L$$

$$\therefore \frac{dL}{L} = \alpha \cdot t$$

$$\therefore \varepsilon L = \frac{dL}{L} = \alpha \cdot t$$

$dL =$ Expansion or contraction of the member (m)
$\alpha =$ coeff. of thermal expansion (/°c)
$t =$ change in temp. (°c)
$L =$ Original length of Member (m)

$$\therefore \sigma = E \cdot \varepsilon L = E \cdot \frac{dL}{L} = E \cdot \alpha \cdot t$$

$$\boxed{\sigma = E \cdot \alpha \cdot t}$$