Strength of Materials

Notes by-

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Strain Energy

Axial force : Tensile or compressive : contraction or Elongation ($\delta l, \delta b, \delta d$)
Torque : Angular deformation (Twist) ($\delta\theta$)
Bending moment : Angular deformation. ($\delta\theta$)

* Proof Resilience:- The max. strain energy which can be stored by a body without undergoing permanent deformation. i.e. it is the strain energy at elastic limit.

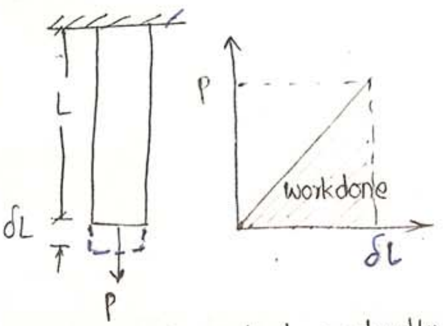
Let f_y = stress at elastic limit

$$\boxed{\therefore \text{Proof Resilience} = \frac{f_y^2}{2E} \times \text{Volume}}$$

* Modulus of Resilience:- (Proof resilience per unit vol)

$$\boxed{\therefore \text{Modulus of Resilience} = \frac{f_y^2}{2E}} \quad (\text{MPa})$$

* stress due to Gradually applied load



The load 'P' is applied gradually from zero to 'P'.

at $P = 0$; $\delta L = 0$
at $P = P$; $\delta L = \delta L$

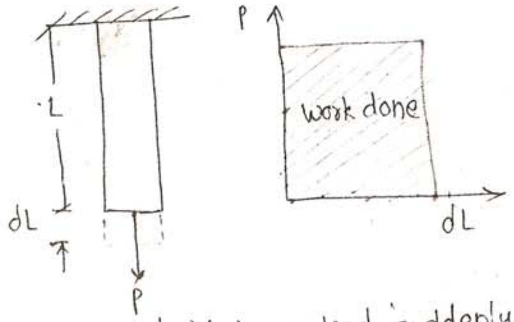
$\therefore$ Equating ; work energy to strain energy

$\therefore$ Work energy = strain energy

$$\frac{0+P}{2} \cdot \delta l = \frac{1}{2} \cdot P \cdot \delta L = \frac{1}{2} \cdot \sigma \cdot A \cdot \delta L$$

$$\boxed{\sigma = \frac{P}{A}}$$

* Stress due to suddenly applied load.



The load 'P' is applied suddenly.
$\therefore$ P is constant throughout the process of extension.

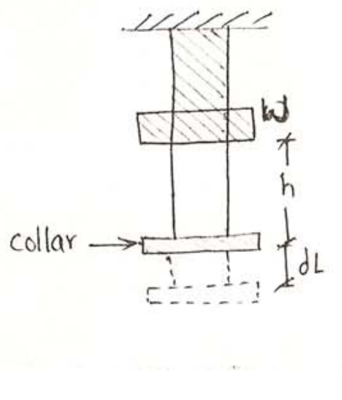
$\therefore$ work done = strain energy

$$\frac{(P+P)}{2} \cdot \delta L = P \cdot \delta L = \frac{1}{2} \cdot \sigma \cdot A \cdot \delta L$$

$$\boxed{\sigma = \frac{2P}{A}}$$

Thus ; stress produced by suddenly applied load ; is twice that of gradually applied load.

* Stress due to Impact load :-



Let the load W falls freely from dist h, before it strikes the rigid collar attached at the bottom of the bar.

$\therefore$ work done = $W(h + \delta L)$
Strain energy = $\frac{1}{2} \cdot \sigma \cdot A \cdot \delta L$

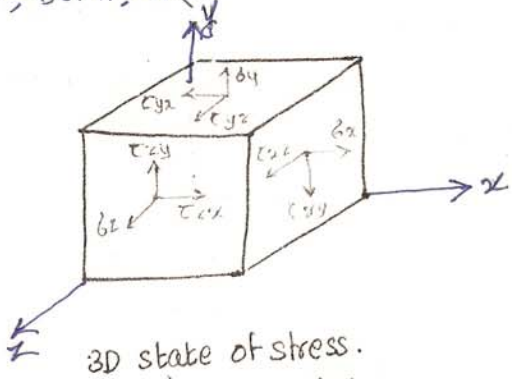
$$\boxed{\therefore \sigma = \frac{W}{A}\left[1 \pm \sqrt{\frac{2AhE}{WL} + 1}\,\right]}$$

Principal stresses & strains

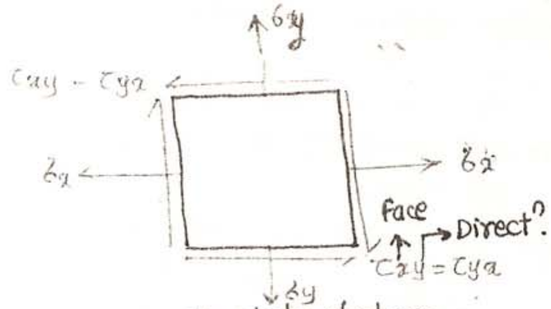
Shear stress : Tangential to plane
Normal stress : $\perp^{ar}$ (Normal) to the plane.

When shear stress & Normal stress acts together; they result into oblique stresses (NOT RESULTANT STRESSES).

उतरता, कलता, तिरपा.

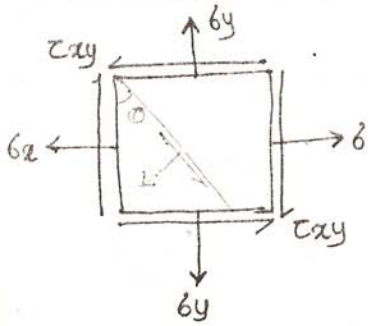


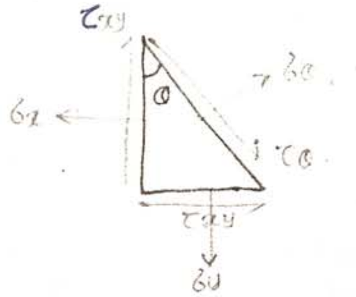
3D state of stress.
9 components of stress



2D state of stress

* Transformation of plane stress :-





$$\sigma_\theta = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cdot\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$\tau_\theta = \left(\frac{\sigma_x-\sigma_y}{2}\right)\cdot\sin 2\theta + \tau_{xy}\cos 2\theta$$

} stress Transformat^n eq^n.

$$\tan\phi = \frac{\tau_\theta}{\sigma_\theta} \qquad R = \sqrt{\tau_\theta^2 + \sigma_\theta^2}$$

The planes on which max. or min. normal stress occurs ; & ther is no shear stress are known as "Principal Planes". & stress acting on these planes – max. & min. normal stress are called as "principal stresses."

$\therefore$ To locate plane of max. and min normal stresses ;

$$\boxed{\tan 2\theta_n = -\frac{2\tau_{xy}}{\sigma_x - \sigma_y}}$$

where θ_n = Orientat^n of plane of max. normal stresses.

Also; $$\sigma_{1,3} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

where σ_1 = Major principal stress
σ_3 = Minor principal stress.

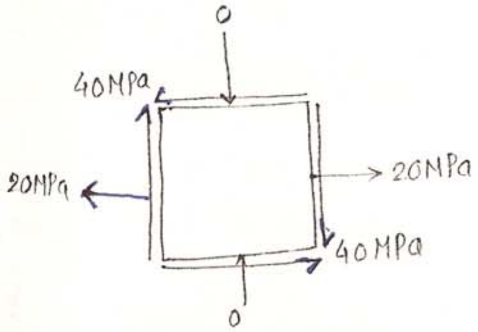
* Max. shear stress :-

$$\boxed{\tan 2\theta_s = \frac{\sigma_x - \sigma_y}{2\tau_{xy}}}$$

θ_s = plane of max. shear stress

$$\tau_{\substack{max\\min}} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Pro:- At a pt. in a stressed mat ; the state of stress is as shown in fig.
Determine :-
1. Principal stresses
2. Principal plane
3. Max. shear stress
4. Plane of max. shear



sol^n :- a] Principal stresses :-

$$\sigma_{\substack{1\\3}} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

[Here ; $\sigma_x = 20$ MPa
$\sigma_y = 0$
$\tau_{xy} = 40$ MPa]

$$\therefore \sigma_{\substack{1\\3}} = \frac{20+0}{2} \pm \sqrt{\left(\frac{20-0}{2}\right)^2 + 40^2}$$

$$\boxed{\begin{aligned} \therefore \sigma_1 &= 51.23 \text{ MPa (Tensile)} \\ \sigma_3 &= -31.23 \text{ MPa (Compressive)} \end{aligned}}$$

... Major principal stress
... Minor principal stress

b] plane of principal stress (Principal Plane)

$$\tan 2\theta_n = \frac{-2\tau_{xy}}{(\sigma_x - \sigma_y)} = -\frac{2 \times \overset{4}{40}}{(\underset{10}{\cancel{20}} - 0)} = \tan^{-1} 4 = \frac{75.96}{2} = 37.98^\circ$$

$$\boxed{\therefore \theta_n = -37.98^\circ ; -127.98^\circ}$$ ...... Major principal plane

$= 90 + 37.98^\circ = 127.98^\circ$.

c] Max. shear stress :

$$\tau_{\substack{max\\min}} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \pm \sqrt{\left(\frac{20-0}{2}\right)^2 + 40^2}$$

$$\boxed{\tau_{\substack{max\\min}} = \pm 41.23 \text{ MPa.}}$$ ... Max. & min. shear stress.

d] Plane of max. shear stress.

$$\therefore \tan 2\theta_s = \frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$$= \frac{20-0}{2 \times 40}$$

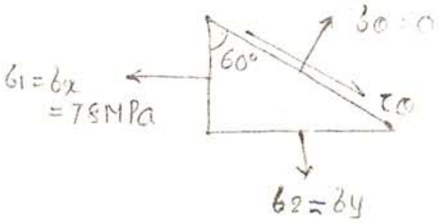
$$\therefore \boxed{\theta_s = 7.02^\circ ; 97.02^\circ}$$

Prc: At a point in a strained matl. subjected to 2D state of stress ; one of the principal stress is 78 MPa ; tensile on a plane at 60° to this principal plane ; the normal stress is zero.

Determine (1) Other principal stress

(2) shear stress on the plane of zero normal stress

~~(3) The plane on which normal & shear stresses are equal in magnitude~~



1] Principal stress:-

$$\sigma\theta = \left(\frac{\sigma x+\sigma y}{2}\right)+\left(\frac{\sigma x-\sigma y}{2}\right)\cdot \cos 2\theta - \tau xy \sin 2\theta .$$

As it is given that principle stress $= \sigma\theta = 0$

$$\tau xy = 0$$

$$\sigma x = 78 \text{ MPa}$$

$$\therefore 0 = \left(\frac{78+\sigma y}{2}\right)+\left(\frac{78-\sigma y}{2}\right)\cdot \cos(2\times 60)$$

$$\therefore\ 39 + 39 \times \cos(120) + 0.5\sigma y - 0.5\sigma y \cos 120$$

$$\boxed{\therefore \sigma y = -26.67 \text{ MPa} \quad (\text{comp.})}$$

2] Shear stress:-

$$\tau\theta = \frac{\sigma x-\sigma y}{2}\ \sin 2\theta + \tau xy \cos 2\theta$$

$$= \left[\frac{78-(-26.67)}{2}\right]\times \sin 120$$

$$\boxed{\therefore \tau\theta = 45.32 \text{ MPa (Tensile)}}$$