Strength of Materials

Notes by-

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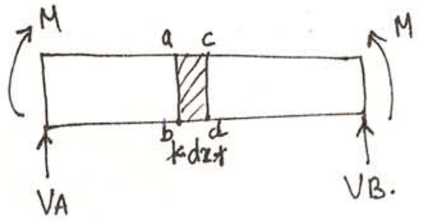
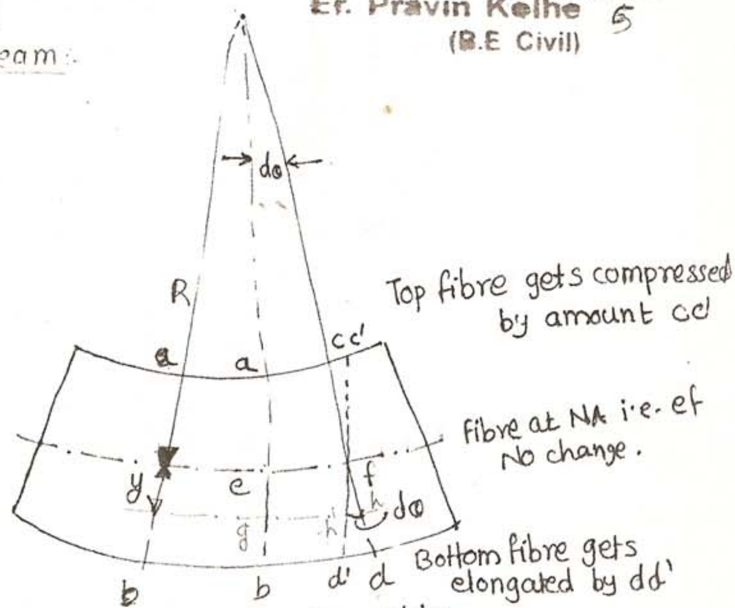
Deflection of Beam

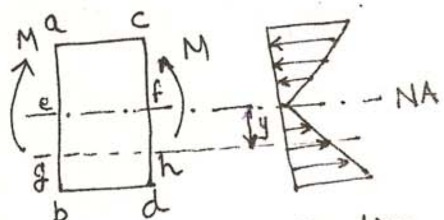
* Differential Eq^n of elastic curve of a beam :-
* Derivation of flexural formula :-



Pure Bending / Simple bending
Only BM acts throughout beam.
No. SF.



Bending Deformation.



FBD of strip 'abcd'. Bending stress diagram.

The deformation of fibre 'gh' located at 'y' from NA.
The deformation of fibre 'gh' located at 'y' from NA; subtened at an angle. Its elongation hh' is the arc of circle of raidus y ; subtened at an angle dθ & is given by,

$$\delta L = hh' = y \cdot d\theta$$

$$\therefore \text{Longitudinal strain} = \varepsilon = \frac{\delta L}{L} = \frac{y \cdot d\theta}{\cancel{gh}\ ef} = \frac{y \cdot d\theta}{R \cdot d\theta} = \frac{y}{R}$$

Where ; R = Radius of curvature of NA.

As ; matl. obeys Hookes law ;

$$\varepsilon = \frac{\delta L}{L} \Rightarrow \sigma = \varepsilon \cdot E = \frac{\delta L}{L} \cdot E = \frac{y}{R} \cdot E$$

$$\therefore \boxed{\frac{\sigma}{E} = \frac{y}{R}}$$

& $\frac{M}{I} = \frac{E}{R}$

$$\therefore \boxed{\frac{\sigma}{y} = \frac{E}{R} = \frac{M}{I}}$$ ---- This is known as flexural formula.

∴ Where ; σ = Bending stress
y = Dist. of fibre from NA
E = Mod. of elasticity or Youngs Modulus
R = Radius of curvature
M = Moment
I = MI

$$\therefore \frac{E}{R} = \frac{M}{I} \quad \text{— (a)}$$

$$ds = R \cdot d\theta$$

$$\tan\theta = \frac{dy}{dx}$$

$$\therefore \frac{1}{R} = \frac{d\theta}{ds} = \frac{d\theta}{dx} \quad \text{—②}$$

$$\therefore \theta = \frac{dy}{dx}$$

$$\therefore \frac{d\theta}{dx} = \frac{d^2y}{dx^2} \quad \text{—①}$$

$$\therefore \frac{1}{R} = \frac{d^2y}{dx^2} \quad \text{... equating ① \& ②}$$

$$\frac{1}{R} = \frac{M}{IE} \quad \text{... from (a)}$$

$$\therefore \frac{M}{IE} = \frac{d^2y}{dx^2}$$

$$\boxed{EI\frac{d^2y}{dx^2} = M} \quad \text{... DE of elastic curve}$$

$$\therefore EI \cdot \frac{dy}{dx} = \int M\,dx + C_1 \quad \text{.... Eq}^n \text{ for slope.}$$

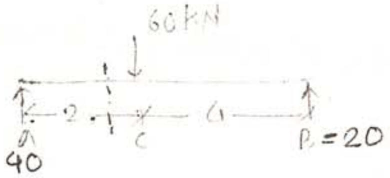
$$EI \cdot y = \iint M \cdot dx + C_1x + C_2 \quad \text{... Eq}^n \text{ for deflection.}$$

× Methods of displacement analysis :-

① Macaulay's Mtd
② Moment area mtd
③ conjugate beam mtd.

① Macaulay's Method :-

find θ_A, θ_B, Δ_C, Δ_{max}



$$6R_B = 120$$

$$EI \cdot \frac{d^2y}{dx^2} = |40x| - |(60)(x-2)|$$

$$\therefore EI\frac{dy}{dx} = \left|40\frac{x^2}{2}\right| - \left|60\frac{(x-2)^2}{2}\right| + C_1 \qquad \text{at } x=0 \,;\, y=0$$

$$EI\,y = \left|\frac{40}{2}\frac{x^3}{3}\right| - \left|\frac{60}{2}\frac{(x-2)^3}{3}\right| + C_1x + C_2 \,;\ \text{at } x=6,\, y=0$$

$$\therefore EI \cdot \frac{dy}{dx} = 20x^2 - 30(x-2)^2 + C_1$$

$$EI\,y = 6.67x^3 - 10(x-2)^3 + C_1x + C_2$$

$\therefore$ at $x=0$; $y=0$

$$\therefore 0 = 0 - 0 + 0 + C_2$$

$$\therefore \boxed{C_2 = 0}$$

at $x=6$, $y=0$

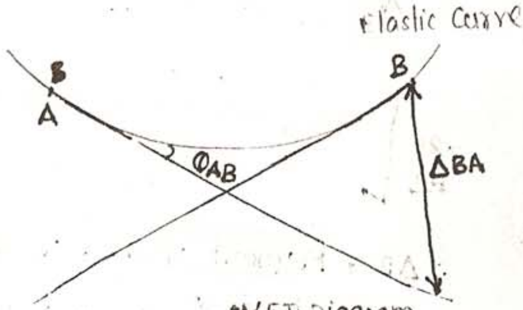
$$\therefore 0 = 6.67 \times 6^3 - 10(6-2)^3 + 6C_1 + 0$$

$$\therefore \boxed{C_1 = -133.33}$$

③ Moment Area Method · Mohr's Theorems:

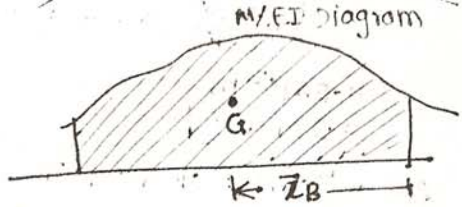
suitability: slope & deflections at a given point are read. instead of complete eqⁿ of deflection (elastic) curve.

$$\boxed{\theta_{AB} = \int_{\theta_A}^{\theta_B} \cdot d\theta = \frac{1}{EI}\int_{x_A}^{x_B} M \cdot dx}$$



First Moment-Area Theorem:-

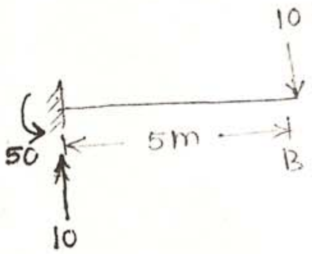
$$\boxed{\Delta BA = \frac{1}{EI}\int_{x_A}^{x_B} \cdot M \cdot x\,dx}$$



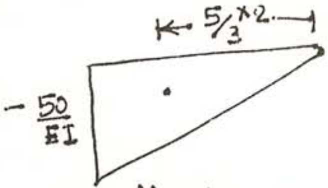
Second Moment-Area Theorem:-

$$\boxed{\Delta BA = (\text{Area})_{AB} \cdot \bar{x}_B}$$

Pro:



$EI = 13 \times 10^3$



$\frac{M}{EI}$ diagram.

$\therefore \theta_{AB} = 0$

$\theta_B = \frac{1}{EI}\int_0^5 (50 \cdot \cancel{dx} \times \frac{5}{2} \times 2)\,dx$ = Area of $\frac{M}{EI}$ dia. betⁿ A & B i.e Area of triangle $= \frac{1}{2} \cdot A \cdot B$

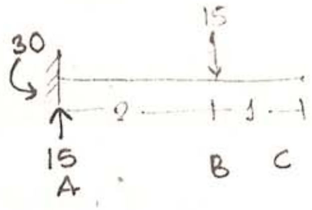
$\therefore \theta_B = -\frac{50 \times 5 \times 0.5}{EI} = \frac{-125}{13 \times 10^3} = 9.615 \times 10^{-3}$ rad

ΔB = Moment of area of M/EI diagram from B.

$= \frac{50}{EI} \times 0.5 \times 5 \times \frac{5}{3} \times 2$

$= -0.03205$ m

$= -32.051$ mm.



find ΔB $\quad \theta B$
ΔC
$E = 2\times10^5$ MPa
$I = 4\times10^7$ mm^4 $\quad$: $EI = 8\times10^3$ kNm^2

$\therefore \Delta B$ = Moment of area of M/EI dia. from B

$= \frac{1}{2}\times\left(\frac{-30}{EI}\right)\times 2\times \underline{\frac{2}{3}(2)}$ centre of gravity of triangle

$= -5\times10^{-3}$ m

$= -5$ mm $(\downarrow)$

ΔC = Moment of area of M/EI dia. from C

$= \frac{1}{2}\times\frac{-30}{EI}\times 2\times\left(1+\frac{2\times2}{3}\right)$

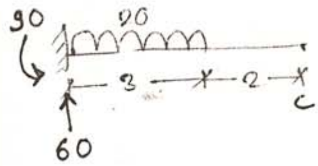
$= -8.75\times10^{-3}$ m $= 8.75$ mm $(\downarrow)$

$\theta c = \theta B$ = ~~Moment~~ Area of M/EI dia. betn AB.

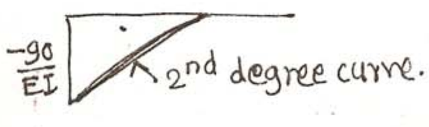
$= \frac{1}{2}\times\left(-\frac{30}{EI}\right)\times 2$

$= -3.75\times10^{-3}$ rad.

$= -3.75\times10^{-3}\times\frac{180}{\pi} = 0.214.86^{\circ}$ (↺)



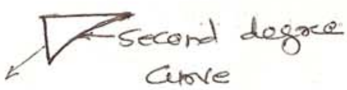
$\Delta C = ?$
$\theta c = ?$
EI = constant



θc = ~~A~~rea of M/EI dia. betn AC

$= \frac{1}{3}\times\left(-\frac{90}{EI}\right)\times 3$

$= \cancel{\frac{-135}{EI}}\ -\frac{90}{EI}$ ~~rad.~~

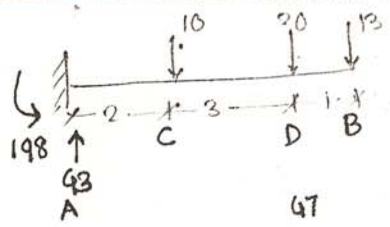


$\Delta C = \frac{1}{3}\times\left(-\frac{90}{EI}\right)\times 3\ \left[2+\frac{3}{4}(3)\right)$ $\quad \bar{x} = \frac{3}{4}\times$ Base

$= \frac{-382.5}{EI}$ (↓)

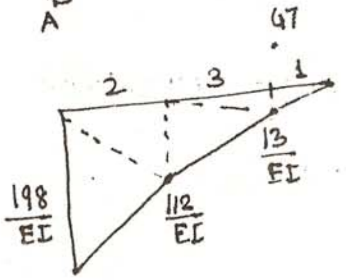
~~... 2mm thk , 2 m long used as a strut ; hinged~~

Er. Pravin Kolhe (B.E. Civil) $\frac{SOM}{7}$



Find
θB
θC
ΔB
ΔC

$EI = \frac{2\times10^5 \times 5\times10^8}{10^3\times10^6}$
$= 1\times10^5\ kN/m^2$



θB = Area of M/EI dia. from AB

$= \frac{198}{EI}\times0.5\times2 + \frac{112}{EI}\times2\times0.5 + \frac{112}{EI}\times0.5\times3 + \frac{13}{EI}\times0.5\times3 + \frac{13}{EI}\times1\times0.5$

$\boxed{\begin{array}{l} \therefore \theta_B = 5.04\times10^{-3}\ rad \\ \theta_B = 0.228^\circ\ (\downarrow) \end{array}}$

θC = Area of M/EI dia. from AC

$= \frac{198}{EI}\times0.5\times2 + \frac{112}{EI}\times0.5\times2$

$\boxed{\begin{array}{l} \theta_C = 3.1\times10^{-3}\ rad \\ \theta_C = 0.1776^\circ\ (\downarrow) \end{array}}$

ΔB = Moment of area of M/EI dia. from B

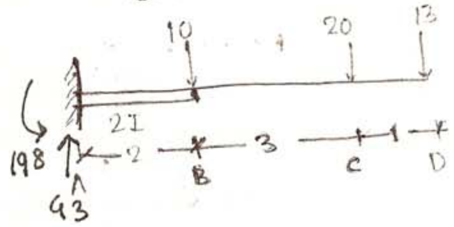
$\therefore \Delta B = \frac{198\times0.5\times2}{EI}\left(4+\frac{4}{3}\right) + \frac{112}{EI}\times2\times0.5\left(4+\frac{2}{3}\right) + \frac{112\times0.5\times3}{EI}\left(1+\frac{6}{3}\right) + \frac{13}{EI}\times0.5\times3\left(1+\frac{3}{3}\right) + \frac{13}{EI}\times1\times0.5\times\frac{2}{3}$

$\boxed{\begin{array}{l} \Delta B = 0.02126\ m \\ \Delta B = 21.26\ mm \downarrow \end{array}}$

ΔC = Moment of M/EI dia. from C

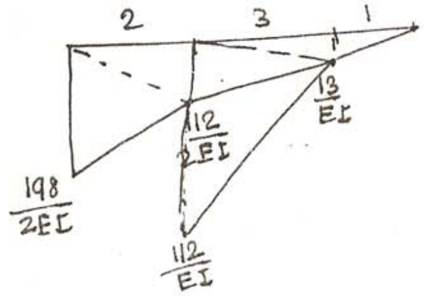
$= \frac{198\times0.5\times2}{EI}\left(\frac{4}{3}\right) + \frac{112}{EI}\times2\times0.5\left(\frac{2}{3}\right)$

$\boxed{\begin{array}{l} = 3.337\times10^{-3}\ m \\ \Delta C = 3.387\ mm\ (\downarrow) \end{array}}$



$\theta_D = \theta_B = \Delta_D = \Delta_B = ?$

$EI = 2\times10^5 \times 5\times10^8 = 10^5\ kNm^2$



$EI\,\theta_D = \frac{198\times0.5\times2}{2} + \frac{112}{2}\times0.5\times2 + \frac{112}{1}\times0.5\times3 + \frac{13}{EI}\times0.5\times3 + 13\times0.5\times1$

$\boxed{\theta_D = 3.49\times10^{-3}\ rad,\ \theta_D = 0.199^\circ\ (\circlearrowright)}$

$EI\,\theta_B = \frac{198}{2}\times0.5\times2 + \frac{112}{2}\times0.5\times2$

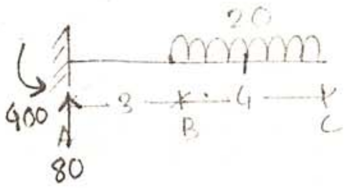
$\boxed{\theta_B = 1.52\times10^{-3} = 0.0871^\circ\ (\circlearrowright)}$

$\Delta B = \frac{198}{2EI}\times0.5\times2\left(\frac{4}{3}\right) + \frac{112}{2EI}\times0.5\times2\left(\frac{2}{3}\right)$

$\boxed{\Delta B = 1.693\times10^{-3}\ m\ (\downarrow)}$

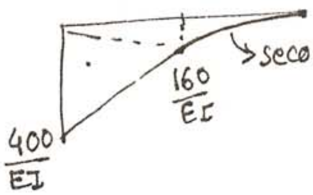
$\Delta D = \frac{198\times0.5\times2}{2EI}\left(\frac{4}{3}+4\right) + \frac{112}{2EI}\times0.5\times2\left(\frac{2}{3}+4\right) + \frac{112}{EI}\times0.5\times3\,(1+2) + \frac{13}{EI}\times0.5\times3\,(1+1) + \frac{13}{EI}\times0.5\times1\times\frac{2}{3}$

$\boxed{\Delta D = 0.01337\ m = 13.367\ mm\ \downarrow}$



ΔC ? ?
θC

$EI = 2\times10^5\times8\times10^8 = 1.6\times10^5$



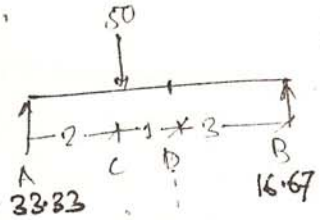
$\Delta C = \frac{400}{EI}\times0.5\times3\,[4+2] + \frac{160}{EI}\times0.5\times3\,[4+1] + \frac{1}{3}\times\frac{160}{EI}\times4\left[\frac{3}{4}(4)\right]$

$= 0.034\ m$

$\boxed{\Delta C = 34\ mm\ (\downarrow)}$ good

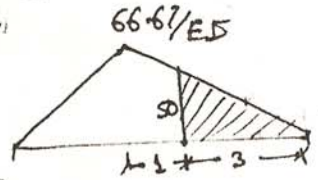
$\theta c = \frac{400}{EI}\times0.5\times3 + \frac{160}{EI}\times0.5\times3 + \frac{1}{3}\times\frac{160}{EI}\times4$

$\boxed{\theta c = 6.583\times10^{-3}\ rad,\ \theta c = 0.377^\circ\ (\circlearrowright)}$ good



θC
θmax
Δmax
ΔD
?

EI = constant



$\theta D = \frac{1}{2} \times \frac{50 \times 3}{EI} = \frac{75}{EI}$ (↓) v.good. ... one step answer.

$\Delta D = \frac{1}{2} \times \frac{50 \times 3}{EI} \times \frac{2}{3}(3) = \frac{150}{EI}$ (↓)

[complicated ; somewhat & not included in syllabus]

③ Conjugate beam method :-

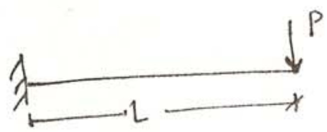
$EI \cdot y$ = Deflection

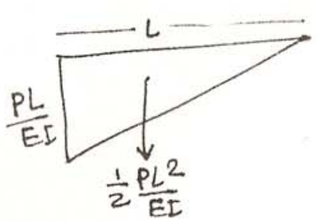
$EI \frac{dy}{dx}$ = slope

$EI \frac{d^2y}{dx^2}$ = Moment

$EI \frac{d^3y}{dx^3}$ = shear

$EI \frac{d^4y}{dx^4}$ = loading intensity

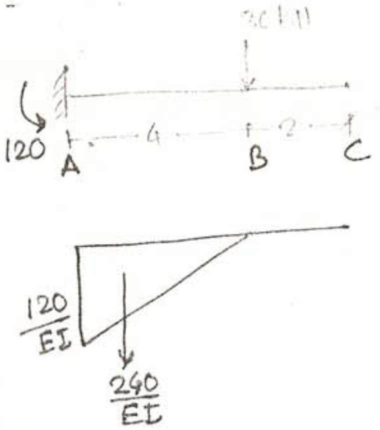




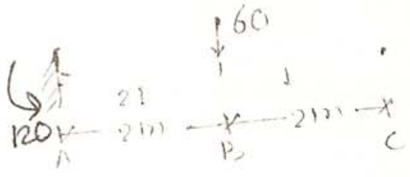
Conjugate Beam.

$M_B = \frac{PL^2}{2EI}\left(\frac{2}{3}L\right) = \frac{PL^3}{3EI}$ (↓) $= y_B$

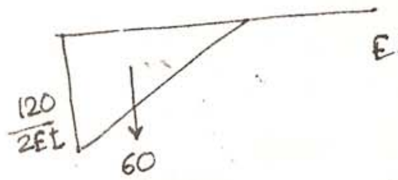
$R_B = \frac{PL^2}{2EI}$ (↑) $= \theta_B$



EI = 24717



EI = 63000



$$\frac{1120}{EI} = \Delta_C$$

$$\frac{240}{EI} = \theta_C$$

$$\frac{200}{EI} =$$

$$\frac{60}{EI} = \theta_B = 0.9546'$$

0.9000