Strength of Materials

Notes by-

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... 2mm thk, 2m long used as a strut; hinged (at) ...

COLUMN & STRUT

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$\frac{SOM}{9}$

column: Vertical Compression member

Strut: Non-vertical compression member.

Long col^m: Fails by buckling

short col^m: Fails by crushing.

Critical load: (Buckling load):- Max. axial load to which col^m does not buckle; but very small lateral load will cause buckling of col^m.

3000

* Eulers Theory of long column:-

Assumptions: ① col^m is long. i.e. direct stresses are insignificant to bending stresses.

② col^m is prismatic, homogeneous & uniform.

③ Load is axial compressive.

④ Modulus of elasticity (E) is same in Tension as well as in comp.

⑤ Plane c/s of col^m remains plane after bending

⑥ Longitudinal fibres are free to expand or contract.

$\frac{2X}{200}$ = EI =

1B

8596°

Case I] When both ends of col^m are pinned or hinged.	$P_E = \frac{\pi^2 EI}{L^2}$	$Le = L$
Case II] One end fixed; other end free	$P_E = \frac{\pi^2 EI}{4L^2}$	$Le = 2L$
case III] Both end fixed	$P_E = \frac{4\pi^2 EI}{L^2}$	$Le = L/2$
case IV] One end fixed other end Hinged.	$P_E = \frac{2\pi^2 EI}{L^2}$	$Le = L/\sqrt{2}$

$$\boxed{\text{General formula: } P = \frac{\pi^2 EI}{Le^2}}$$

Le = Effective length of col^m
= Dist. bet^n pt. of contraflexure.

* Limitation of Eulers formula:-

Let $P_E = \frac{\pi^2 EI}{L_e^2}$

$I = A \cdot r^2$ $\quad$ A = c/s area

$r = \sqrt{I/A}$ $\quad$ r = Radius of gyration.

$$\therefore P_E = \frac{\pi^2 E (A \cdot r^2)}{L_e^2}$$

$$\therefore \frac{P_E}{A} = \frac{\pi^2 E}{\left(\frac{Le}{r}\right)^2}$$

$$\boxed{\therefore F_{cc} = \frac{\pi^2 E}{\lambda^2}}$$

F_{cc} = Elastic critical stress
i.e. stress corresponding to buckling load P_E

λ = Slenderness ratio
$= \frac{Le}{r}$

But $F_{cc} \ngtr F_y$

i.e. Elastic critical stress $\ngtr$ Yield stress of col^m matl.

$\therefore$ If λ is very less, F_{cc} will be more than F_y.

$\therefore$ For Eulers formula holds good; $\left| \lambda \geqslant \sqrt{\frac{\pi^2 E}{F_y}} \right|$ → Limitation (Only for Long col^m)

* Rankine's formula:

Used for long as well as short col^m.

$$\boxed{\frac{1}{P_R} = \frac{1}{P_C} + \frac{1}{P_E}}$$

Where P_R = Rankine's Load

P_C = Crushing load = $F_y \cdot A$

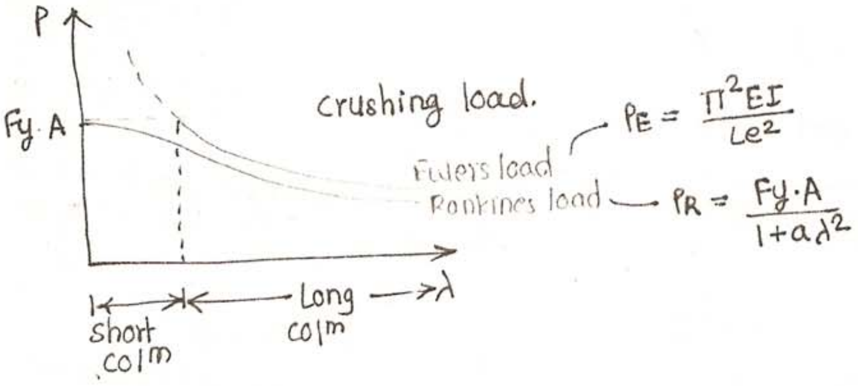
P_E = Euler's load. /critical/ buckling load.

$$\therefore P_R = \frac{P_C \cdot P_E}{P_C + P_E} = \frac{P_C}{1 + \frac{P_C}{P_E}} = \frac{F_y \cdot A}{1 + \frac{F_y \cdot A}{P_E}}$$

$$= \frac{F_y \cdot A}{1 + \left(F_y \cdot A / \pi^2 EI / L_e^2\right)} = \frac{F_y \cdot A}{1 + \left(\frac{F_y \cdot A \cdot L_e^2}{\pi^2 E \cancel{A} r^2}\right)} \qquad I = A \cdot r \quad \frac{L_e}{r^2} = \lambda;$$

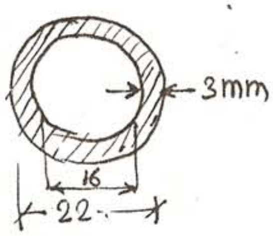
$$= \frac{F_y \cdot A}{1 + \frac{F_y \cdot \lambda^2}{\pi^2 E}}$$

$$\therefore \boxed{P_R = \frac{F_y \cdot A}{1 + a\lambda^2}} \qquad \text{Where } a = \frac{F_y}{\pi^2 E} = \text{Rankine's constant.}$$



Pro:- A MS tube 22mmϕ, 3mm thk, 2m long used as a strut; hinged at two ends. calculate crippling load by Eulers formula. $E = 200\,GPa$ 25 10

Soln:- $P_E = \dfrac{\pi^2 EI}{Le^2}$



For both ends hinged; $Le = L$

$\therefore P_E = \dfrac{\pi^2 EI}{L^2}$

$\therefore I = \frac{\pi}{64}[D^4 - d^4] = \frac{\pi}{64}[22^4 - 16^4] = 8282\ mm^4.$

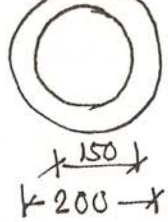
$E = 200\times10^3\ N/mm^2$

$\therefore P_E = \dfrac{\pi^2 \times 200\times10^3 \times 8282}{(2000)^2}$

$= 4087\ N$

$\boxed{P_E = 4.087\ kN.}$

→ Find crushing load by Rankines formula for a hollow CI colm of 200mm ext. dia & 25mm thk. of metal. If length of colm = 8m; both ends fixed. $Fy = 550\,MPa$, $a = \frac{1}{1600}$



$P_R = \dfrac{Fy \cdot A}{1 + a\lambda^2}$

$A = \frac{\pi}{4}(200^2 - 150^2) = 13.74\times10^3\ mm^2$

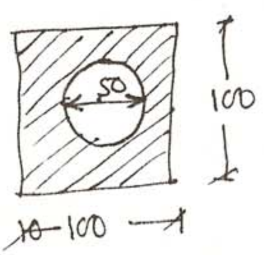
$I = \frac{\pi}{64}(200^4 - 150^4) = 53.69\times10^6\ mm^4.$

$\lambda = \dfrac{Le}{r}$; $\dfrac{Le}{\sqrt{I/A}} = \dfrac{0.5\times8\times10^3}{\sqrt{\dfrac{53.69\times10^6}{13.74\times10^3}}} = 63.98$

$\therefore P_R = \dfrac{550\times 13.74\times10^3}{1 + \frac{1}{1600}\times63.98^2} = \cancel{72}\ \underline{2123.7\ kN}$

Pro:- A square colm of 100x100mm c/s has a concentric longitudinal hole of 50mm dia. The length of colm is 5m; one end fixed & other end hinged. Determine the Eulers buckling load assuming $E = 200\,GPa$

Soln:- $P_E = \dfrac{\pi^2 IE}{Le^2}$ $\quad Le = \dfrac{L}{\sqrt{2}}$

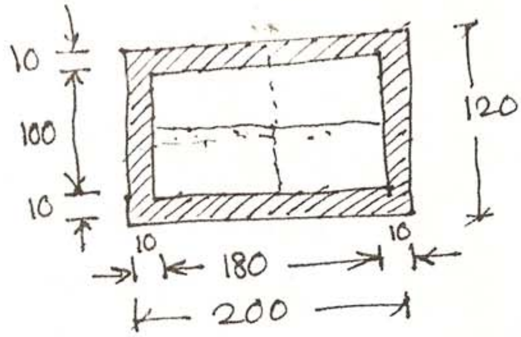


$I = \dfrac{100\times100^3}{12} - \frac{\pi}{64}(50)^4 = 8026.54\ mm^4.$

$A = 100\times100 - \frac{\pi}{4}(50)^2 = 8036.5\ mm^2$

$\therefore P_E = \dfrac{\pi^2 \times 8026.54 \times 200}{(5000/\sqrt{2})^2}$ $\quad \boxed{\therefore P_E = 1267.5\ kN}$

Pro: The c/s of col^m is hollow rectangular section having outside dimensions 200×120 mm & inside dimension 180×100 mm having uniform thk. 10 mm. It is fixed at one end & hinged at other end. If the buckling load given by 800 kN (Rankine's formula) find actual length of col^m. $f_y = 300$ MPa, E = 200 GPa, a = 1/7500.



$$P_R = \frac{f_y \cdot A}{1 + a\lambda^2}$$

$$\lambda = \frac{L_e}{r} \qquad L_e = \frac{L}{\sqrt{2}} \therefore$$

$$r = \sqrt{I/A} \qquad \boxed{\lambda = \frac{L}{r\sqrt{2}}}$$

$$I = \frac{200 \times 120^3}{12} - \frac{180 \times 100^3}{12}$$

$$= 13.8 \times 10^6 \text{ mm}^4$$

$$A = 200 \times 120 - 180 \times 100 = 6 \times 10^3 \text{ mm}^2$$

$$\therefore r = 47.96 \text{ mm}$$

$$\because P_R = \frac{f_y \cdot A}{1 + a\lambda^2} \Rightarrow 800 \times 10^3 = \frac{300 \times 6 \times 10^3}{1 + \frac{\lambda^2}{7500}}$$

$$\therefore 1 + 1.333 \times 10^{-4} \lambda^2 = \frac{300 \times 6 \times 10^3}{800 \times 10^3} = 2.25$$

$$\therefore \boxed{\lambda = 96.82}$$

$$\therefore 96.82 = \frac{L}{r\sqrt{2}} \Rightarrow L = 96.82 \times 47.96 \times \sqrt{2}$$

$$\boxed{\therefore L = 6.5672 \text{ m}}$$

v. good