Strength of Materials

Notes by-

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DIRECT & BENDING STRESSES

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Direct stress (due to axial load) = $\sigma a_{t}^{c} = \pm \frac{P}{A}$

Bending stress (Due to BM or eccentric load) $\sigma b_{t}^{c} = \pm \frac{M}{I} \cdot y$.

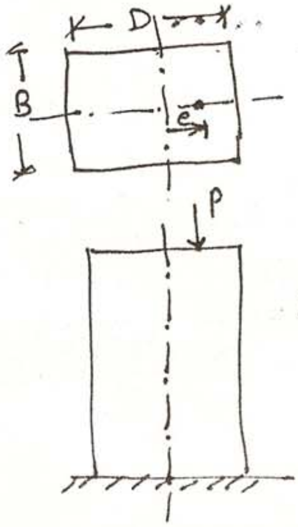
$\therefore$ Resultant stress = $\sigma_{min}^{max} = \pm \sigma a_{t}^{c} \pm \sigma b_{t}^{c}$

$$\boxed{\sigma_{min}^{max} = \pm \frac{P}{A} \pm \frac{M}{I} \cdot y}$$

$+ \Rightarrow$ Compressive
$- \Rightarrow$ Tensile.

Core / Kernel of a section :- It is defined as, "Partial area of c/s within which the load is placed, so as not to produce 'Resultant tensile stress".
i.e. It is the area of c/s, such that if load is placed within that area, no tesile stresses will be produced.

Core / kernel of Rectangular section :-



c/s of col^m

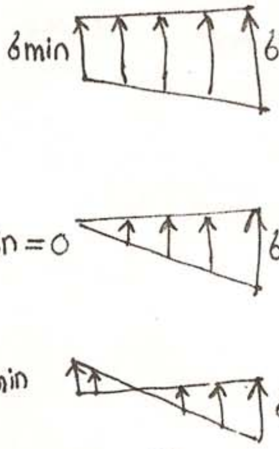
Eccentric load on col^m

$\therefore$ Direct stress = $\sigma a = \frac{P}{A} = \frac{P}{BD}$
(Axial stress)

Bending stress = $\sigma b = \pm \frac{M}{I} \cdot y$
$= \pm \frac{P \cdot e}{\frac{B \cdot D^3}{12}} \left(\frac{D}{2}\right)$
$= \pm \frac{6 P \cdot e}{B \cdot D^2}$

Depending on σa & σb magnitude possible conditions with diagram are as shown in fig.

i] $\sigma a > \sigma b$ .... comp. stress
ii) $\sigma a = \sigma b$ .... comp. stress (Limiting)
iii) $\sigma a < \sigma b$ .... Tensile + compressive.



Resultant stress diagram ($\sigma a > \sigma b$)

($\sigma a = \sigma b$)

($\sigma a < \sigma b$)

$\therefore$ Limiting condition is
$\sigma a = \sigma b$

$\therefore \frac{P}{B \cdot D} = \pm \frac{6 P \cdot e}{B \cdot D^2}$

$\therefore 1 = \pm \frac{6e}{D}$

$\therefore \boxed{e = \pm \frac{D}{6}}$

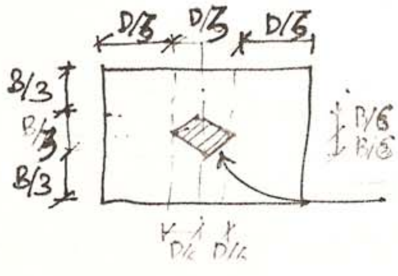
$\therefore$ for No tension condition, eccentricity $\not>$ D/6.

Similarly for y axis,

$\boxed{e = \pm B/6}$

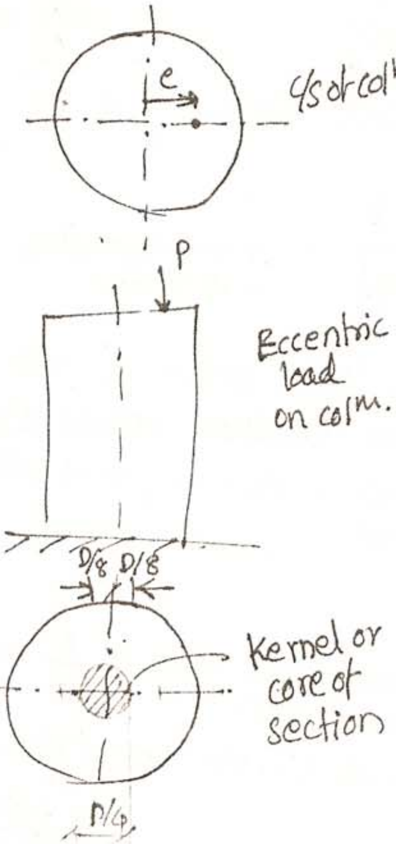
MIDDLE IIIrd RULE

Middle IIIrd Rule: Resultant should pass through middle IIIrd of sect^n for No tension condition



Core / karnel of Rectangular section.

* Core of solid circular section



$$\sigma a = \frac{P}{A} = \frac{P}{\frac{\pi}{4}D^2} = \frac{4P}{\pi D^2}$$

$$\sigma b = \pm \frac{M}{I} \cdot y$$

$$= \pm \frac{P \cdot e \times D/2}{\frac{\pi}{64} D^4} = \frac{32 P \cdot e}{\pi D^3}$$

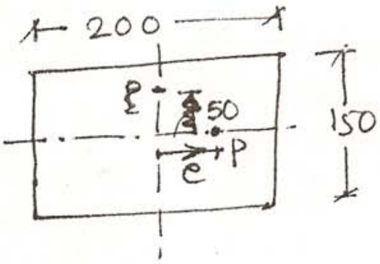
$\therefore \sigma a = \sigma b$ ... No tension condition (Limition condition)

$$\therefore \frac{4P}{\pi D^2} = \pm \frac{32 \cdot P \cdot e}{\pi D^3}$$

$$\therefore 1 = \pm \frac{8 e}{D}$$

$$\boxed{\therefore e = \pm \frac{D}{4}}$$

Pro: 1] A rectangular col^m 200x150 mm carring a comp. load 50 kN at 50 mm in a plane bisecting 150 mm side. Fid σmax, σmin & $e max$ for no tension cond^n.



$$\sigma a = \frac{P}{A} = \frac{50 \times 10^3}{200 \times 150} = 1.67 \text{ MPa}$$

$$\sigma b = \pm \frac{M}{I} \cdot y = \frac{50 \times 10^3 \times 50 \times \frac{200}{2}}{\frac{200^3 \times 150}{12}}$$

$$= 2.5 \text{ MPa}.$$

$$\therefore \sigma_{min}^{max} = 1.67 \pm 2.5 \text{ MPa}$$

$\therefore \sigma max = 4.167$ MPa ... Comp.

$\sigma min = -0.833$ MPa ... Tensile

For No tension cond^n

$$\sigma a = \sigma b$$

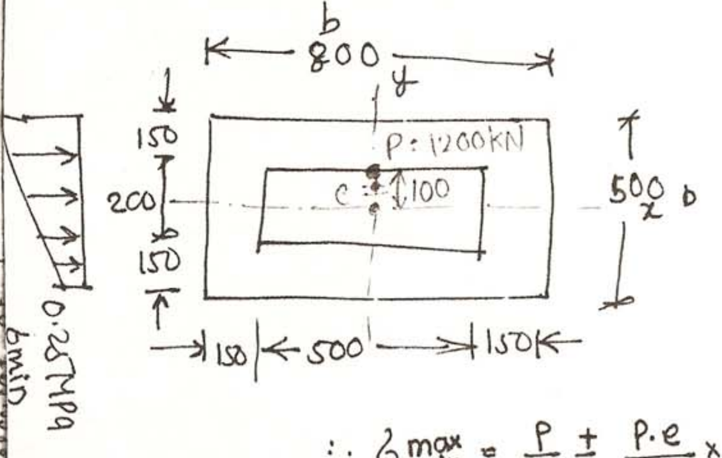
$$\therefore \frac{50 \times 10^3}{200 \times 150} = \frac{50 \times e \times 10^3}{\frac{150 \times 200^3}{12}} \times \frac{200}{2} \Rightarrow \boxed{e = 33.34 \text{ mm}}$$ good

Pro: 2] A hollow rectangular pier of 500x800 mm externally; with uniform thk. of 150 mm carries a comp. load 1200 kN at 100 mm on a plane bisecting 800 mm side. Find ① σmax, σmin

② If $\sigma a max = 5$ MPa ; find Pallowable

③ If $\sigma a max = 5$ MPa ; find e_{max} for 1200 kN load.

NOTE :- ... side ... will be 'b' & ... 'd' for calculation



$\sigma_a = \frac{P}{A}$

$A = 800\times500 - 200\times500$

$= 300\times10^3 \text{ mm}^2$

$I = \frac{800\times500^3}{12} - \frac{500\times200^3}{12} = \cancel{8.33\times10}$

$= 8\times10^9 \text{ mm}^4$

$y = \frac{D}{2} = 250 \text{ mm}.$

$\therefore \sigma_{\substack{max\\min}} = \frac{P}{A} \pm \frac{P.e}{I}\times y.$

$= \frac{1200\times10^3}{300\times10^3} \pm \frac{1200\times10^3\times100}{8\times10^9}\times250$

$\left.\begin{array}{l}\sigma_{max} = 7.750 \text{ MPa}\\ \sigma_{min} = 0.250 \text{ MPa} \quad \text{v. v. good.}\end{array}\right\}$ Ans. .. (1)

Now: (2) $\sigma_{a\,max} = 5$ MPa ; $P = ?$

$\therefore \sigma_{a\,max} = 5 = \frac{P}{300\times10^3}$

$\therefore \boxed{P_{allowable} = 1500 \text{ kN}}$ .. If no eccentricity.

$\sigma_{max} = 5 = \frac{P\times10^3}{300\times10^3} \pm \frac{P\times10^3\times100}{8\times10^9}\times250$

$\therefore 5 = \frac{(8\times10^9\ P\times10^3) + (300\times10^3\times P\times10^3\times100)250}{300\times10^3\times8\times10^9}$

$\therefore \frac{5}{\cancel{250}} = \frac{(8\times10^{12}P + 3\times10^{10}P)P}{\cancel{24\times10^{15}}}$

$\therefore P = 774.19$ kN — check!

$1.2\times10^{16} = (8\times10^{12} + 75\times10^{11})P$

$\therefore \boxed{P = 774.194 \text{ kN}}$ .. Ans.

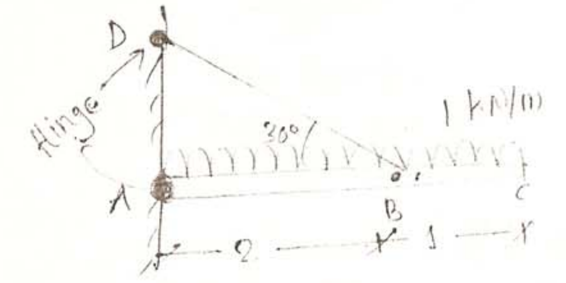
3) For ecc.

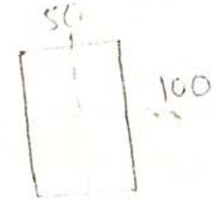
$\therefore 5 = \frac{\overset{1200}{\cancel{P}}\times10^3}{300\times10^3} + \frac{1200\times10^3\ e}{8\times10^9}\times250$

$\therefore \boxed{e = 26.667 \text{ mm}}$ ... emax. for $\sigma_{max} = 5$ MPa & $P = 1200$ kN

NOTE :- 5211 side of load, that side will be 'b' & other will be 'd' for calculation of I

Pro: A horizontal wooden cantilever 3m long & 50 mm wide, 100 mm deep. The beam is supported as shown. Find max. normal comp. stress & state the position of c/s where it occurs.



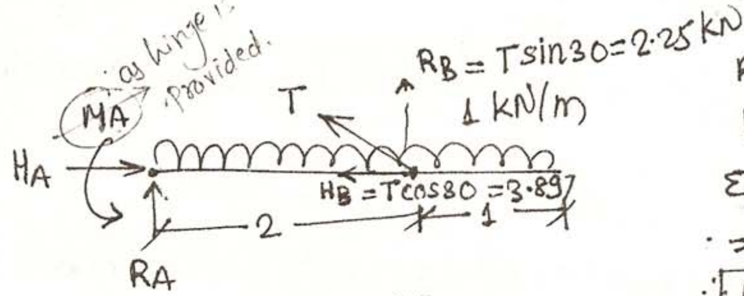


We have to find $\sigma_{max\ comp} = \sigma_a + \sigma_b$

$$= \frac{P}{A} + \frac{M}{I} \times y.$$

∴ We have to find 'P' & 'M'.

After analysing the str. the max. reaction & mmt. is taken to evaluate max. compressive stress.



$R_A + R_B = 1 \times 3 = 3$

$H_A = H_B = T\cos 30.$

$\Sigma M_A = 0$

$\therefore \cancel{-M_A} + 1 \times 3 \times 1.5 - T\sin 30 \times 2 = 0$

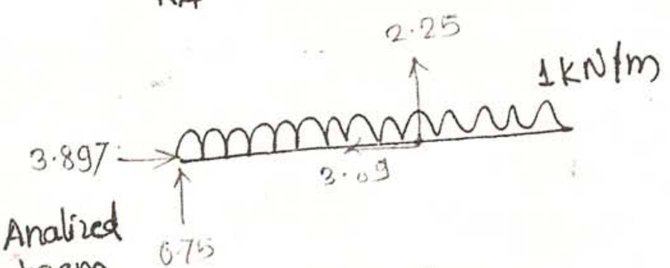
$\therefore \boxed{\cancel{M_A} + T = 4.5}$

$\Sigma M_B = 0$

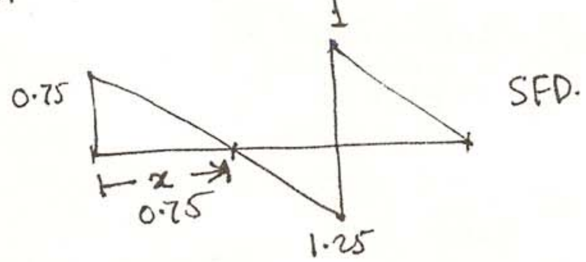
$\therefore -M_A + R_A \times 2 - 1 \times 2 \times 1 + 1 \times 1 \times 0.5$

$\boxed{\cancel{M_A} - 2R_A = -1.5}$

$\boxed{R_A = 0.75 \text{ kN}}$

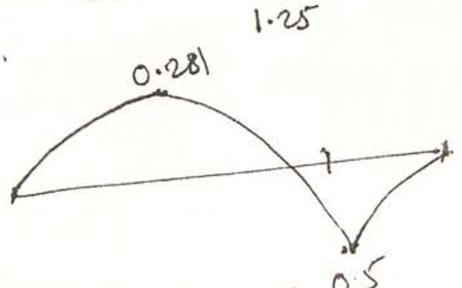


Analized beam



$$\frac{0.75}{x} = \frac{1.25}{2-x}$$

$x = 0.75$ m

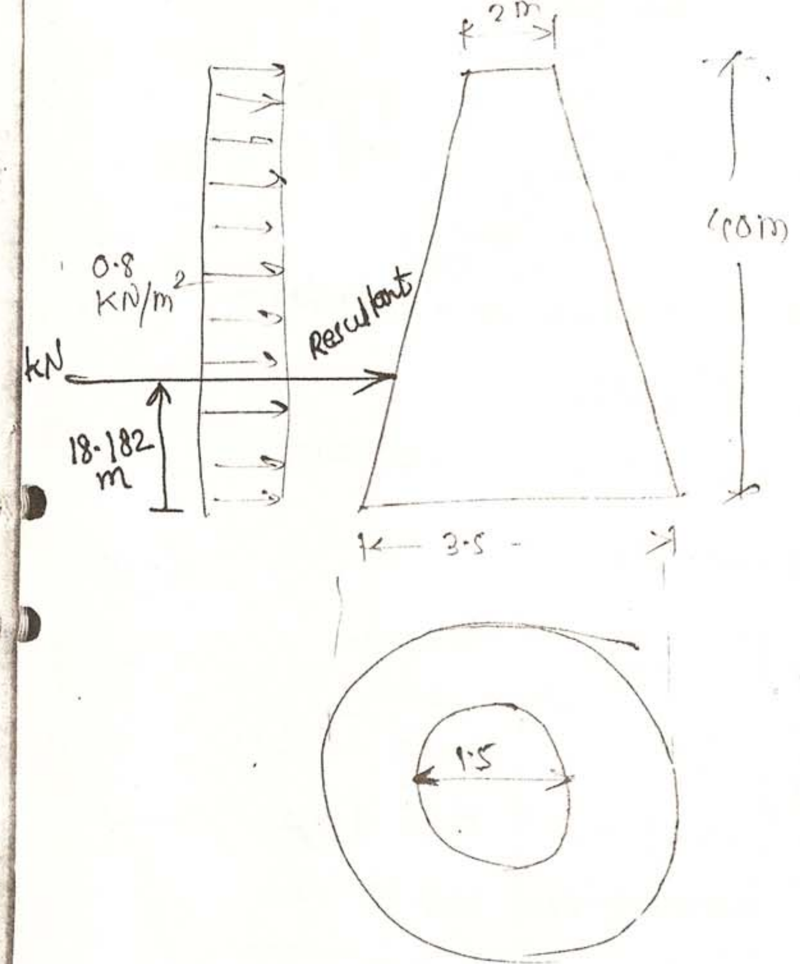


$$\boxed{\begin{aligned} \therefore P_{max} &= 3.897 \text{ kN} \\ M &= 0.5 \text{ kNm} \end{aligned}}$$

$$\sigma_{max} = \frac{3.897 \times 10^3}{50 \times 100} + \frac{0.5 \times 10^6}{\frac{50 \times 100^3}{12}} \times \frac{100}{2} \qquad \therefore \boxed{\sigma_{max} = 6.779 \text{ MPa}}$$

v.v.v. good.

Q:- A chimney as shown in fig. is subjected to uniform wind pressure of 0.8 kN/m² on projected area. calculate overturning mmt. at base. If total wt. of chimney is 5000 kN & internal dia. at base is 1.5 calculate normal stress intensities at base.



$\therefore \sigma_{max} = 1030$ kN/m²
$\sigma_{min} = 243.23$ kN/m²

We have to find σ_{max} & σ_{min} at base.

$$\therefore \sigma_{\substack{max \\ min}} = \frac{P}{A} \pm \frac{M_{overturning} \cdot y}{I}$$

$$\therefore P = 0.8 \left[\frac{2+3.5}{2}\right] \times 40$$

$$\boxed{P = 88 \text{ kN}}$$

'P' acting at $\left(\frac{a+2b}{a+b}\right) \cdot \frac{H}{3}$ from base

i.e. $\left(\frac{3.5+2\times2}{3.5+2}\right) \times \frac{40}{3} = 18.182$ m from base

$\therefore$ overturning mmt $= P \times e$
$= 88 \times 18.182$

$$\boxed{M = 1600 \text{ kNm}}$$

$\therefore A = \frac{\pi}{4}[3.5^2 - 1.5^2]$
$= 7.854$ m²

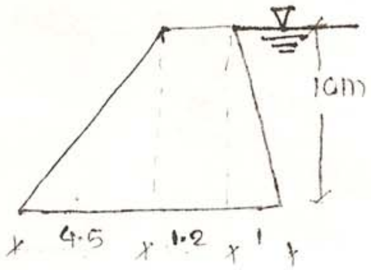
$I = \frac{\pi}{64}(3.5^4 - 1.5^4)$
$= 7.118$ m⁴.

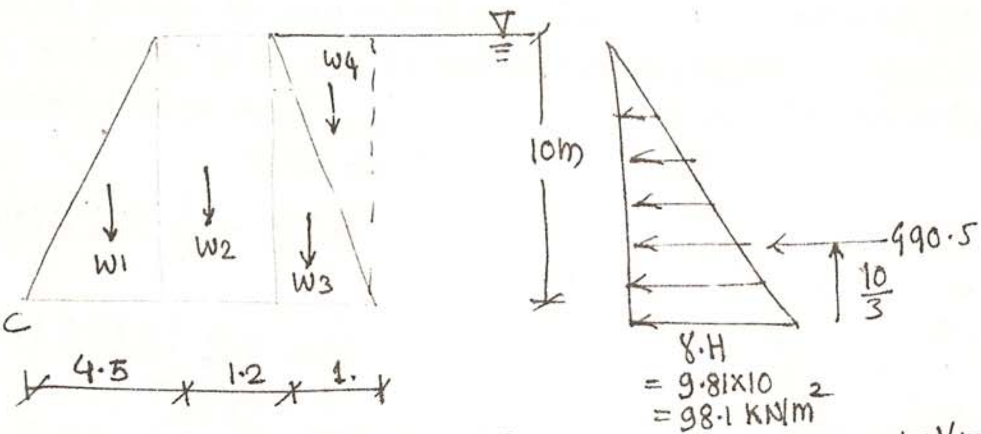
$$\therefore \sigma_{\substack{max \\ min}} = \frac{\cancel{88}\ 5000}{7.854} \pm \frac{1600}{7.118} \times \frac{3.5}{2}$$

$= \cancel{11.204}\ 636.62 \pm 393.387$ kN/m²

$\frac{\times 10^3}{(10^3)^2}$

Q:- A masonary dam of trapezoidal c/s, 10m high as shown in fig.
If density of concrete = 22 kN/m³
density of water = 9.81 kN/m³
find σ_{max}, σ_{min} at base





Horizontal force of water $= P_H = \frac{1}{2} \times 98.1 \times 10 = 490.5\ kN/m$.

"CONSIDER UNIT WIDTH OF DAM"

$\therefore\ P_H = 490.5\ kN$; $M_{PH} = 490.5 \times \frac{10}{3} = 1635\ kNm$

Calculation of $P_V = W_1 + W_2 + W_3 + W_4$.

Area (W't)	Force (kN)	Lever arm w.r.t. "Toe" (m)	Moment (kNm)
$W_1 = \frac{1}{2} \times 4.5 \times 10 \times 22$	495 ↓	$\frac{2}{3}(4.5) = 3$	1485 (↺)
$W_2 = \frac{1}{2}$ 1.2×10×22	264 ↓	$4.5 + \frac{1.2}{2} = 5.1$	1346.4 (↺)
$W_3 = \frac{1}{2} \times 1 \times 10 \times 22$	110 ↓	$4.5 + 1.2 + \frac{1}{3} = 6.033$	663.67 (↺)
$W_4 = \frac{1}{2} \times 1 \times 10 \times 9.81$	49.05 ↓	$4.5 + 1.2 + \frac{2}{3} = 6.37$	312.285 (↺)

$\boxed{\Sigma P_V = \Sigma W = 918.05\ kN\ (\downarrow)}$

$\boxed{P_H = 490.5\ kN\ (\leftarrow)}$

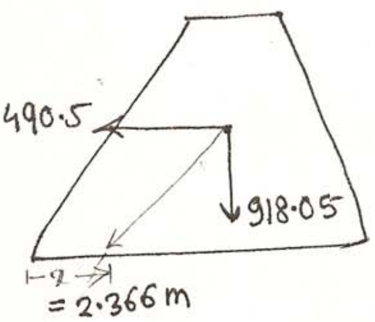
$\boxed{\Sigma M_{PV} = 3807.355\ kNm\ (↺)}$

$\boxed{\Sigma M_{PH} = 1635\ kNm\ (↻)}$

$\boxed{\therefore \Sigma M_O = 2172.355\ kNm\ (↺)}$

Let Resultant of P_H & P_V strikes at x from 'c' at base.

$$\therefore x = \frac{\Sigma M}{\Sigma W} = \frac{2172.355}{918.05} = 2.366\ m \text{ from } c$$



$\therefore$ eccentricity $= e = \frac{B}{2} - x$

$$= \frac{(4.5+1.2+1)}{2} - 2.366$$

$$= \underline{0.984\ m}$$

for No tension condition,

$$e \not> \frac{B}{6} \Rightarrow 0.984 \not> \frac{6.7}{6} = 1.12$$

$\therefore$ O.K

$$\therefore \sigma_{\substack{max\\min}} = \frac{\Sigma W}{A} \pm \frac{M}{I} \times y.$$

$$= \frac{\Sigma W}{B \times 1} \pm \frac{\Sigma W \times e}{Z}$$

$A = B \times 1$
$= 6.7 m^2$

$I = \frac{1 \times B^3}{12} = 25.06\ m^4$

$Z = \frac{I}{y} = \frac{25.06}{6.7/2} = 7.482\ m^3$

$Z = I/y$

Z = section modulus.

$$\therefore \sigma_{\substack{max\\min}} = \underbrace{\frac{918.05}{6.7}}_{137.02} \pm \underbrace{\frac{918.05 \times 0.984}{7.482}}_{120.74}$$

$$\boxed{\begin{aligned} \sigma_{max} &= 257.76\ kN/m^2 \\ \sigma_{min} &= 16.28\ kN/m^2 \end{aligned}}$$

* Stability of dam against overturning :-

Factor of safty against overturning $= \frac{M_R}{M_O} = \frac{\text{Resisting mmt}}{\text{overturning mmt.}}$

$= \frac{MPV}{MPH}$

$= \frac{3807.355}{1635} =$

$= 2.3286 > 1.5$

$\therefore$ O.K.

V.V. Gude

* Stability of dam against sliding :-

FOS against sliding $= \frac{\mu . \Sigma W}{PH}$

$= \frac{0.6 \times 918.05}{490.5}$

$= 1.123$

$\not> 1.5$

Not Safe.

If $\mu = 0.6$.

or it will given as $\phi = 30^\circ$

$K_p = \frac{1 - \sin\phi}{1 + \sin\phi} = 0.33$

$\therefore \mu = \tan\phi = 0.577$