Strength of Materials

Notes by-

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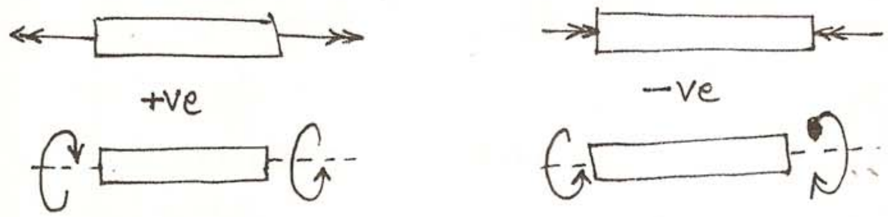
TORSION

sign convention: Right hand Thumb Rule



Imp.

Flexural formula;

$$\boxed{\frac{E}{R} = \frac{M}{I} = \frac{\sigma}{y}} \xrightarrow{\text{Analogous}} \boxed{\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}}$$

Torsion Formula:

M = Bending mmt kNm → T = Torsional Moment / Torque kNm

R = Radius of curvature → L = Length of shaft m

y = Dist. of extreme fibre from NA → R = Radius of shaft m

θ = Angle of twist

E = Young's Modulus → G = shear Modulus

σ = Bending stress → τ = shear stress

I = Moment of inertia → J = Polar moment of inertia.

① Axial stiffness: Axial force reqd. to produe unit change in length $S = \frac{AE}{L}$ (N/mm) → ① Torsional stiffness: Torque reqd. to produce unit angle of Twist $= \frac{GJ}{L}$ (Nm/rad)

Flexural stiffness $\frac{EI}{L}$

② Axial Flexibility: change in length produced by unit axial force applied. → ② Torsional Flexibility: Angle of twist produced by unit torque applied

$f = \frac{L}{AE}$ (mm/N)

$= \frac{L}{GJ}$ (rad/Nm)

$= \frac{1}{\text{Torsional stiffness}}$

Flexural flexibility $\frac{L}{EI}$

$f = \frac{1}{S}$

③ Axial Rigidity: Product of Young's Modus & Area of c/s. → ③ Torsional Rigidity: Product of shear modulus & polar M.I. of c/s.

$= A \cdot E$ (N)

$= GJ$ (Nm^2)

Flexural rigidity EI

④ Axial section Modulus:- → ④ Torsional section Modulus: Ratio of polar MI ~~of~~ of c/s to its radius.

Flexural section Modulus $Z = \frac{I}{y}$ (m^3)

$= \frac{J}{R}$ (m^3)

Flexural ↑ axial Torsional ↑

All things are analogous

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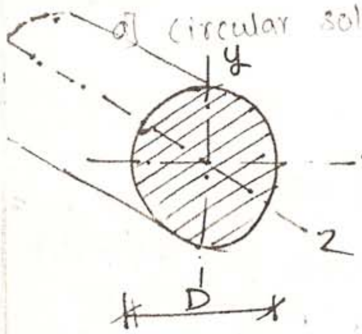
	Axial	Flexural	Torsional
stiffness: (S)	$\frac{EA}{L}$ (N/m)	$\frac{EI}{L}$ (Nm/m)	$\frac{GJ}{L}$ (Nm/Rad)
Flexibility (f)	$\frac{L}{EA}$ (m/N)	$\frac{L}{EI}$ (m/Nm)	$\frac{L}{GJ}$ (Rad/Nm)
Rigidity	EA (N)	EI (Nm^2)	GJ (Nm^2)
Section Modulus (Z)		$\frac{I}{y}$ (m^3)	$\frac{J}{R}$ (m^3)

Torsional section modulus :

a) Circular solid shaft



$$I_{xx} = I_{yy} = \frac{\pi}{64} D^4$$

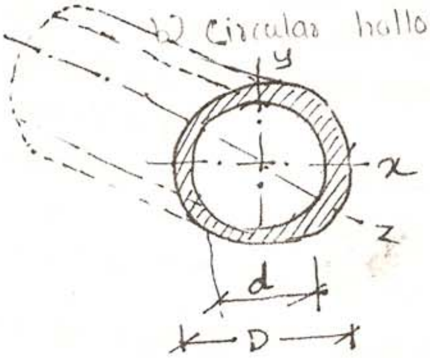
$$J = I_{zz} = I_{xx} + I_{yy} = 2\frac{\pi}{64} D^4$$

$$R = D/2$$

$$\therefore Z_T = \frac{J}{R} = \frac{2\pi}{64} D^4 \times \frac{2}{D}$$

$$\boxed{\therefore Z_T = \frac{J}{R} = \frac{\pi}{16} D^3}$$

b) Circular hollow shaft



$$I_{xx} = I_{yy} = \frac{\pi}{64}(D^4 - d^4)$$

$$J = I_{zz} = I_{xx} + I_{yy} = 2\frac{\pi}{64}(D^4 - d^4).$$

$$R = D/2$$

$$Z = \frac{J}{R} = \frac{2\pi}{64}(D^4 - d^4) \cdot \frac{2}{D}$$

$$\boxed{\therefore Z_T = \frac{J}{R} = \frac{\pi}{16D}(D^4 - d^4)}$$

* Power Transmitted by shaft :-

The power is the product of average torque & corresponding angle turned per unit duration of time.

$\therefore P =$ Force x Distance ... Axial
$\therefore P_T =$ Torque x Rotation .... Torsional
} Per unit Time.

$$= T_a \times \frac{2\pi N}{60}$$

$$\boxed{\therefore P = \text{Power} = \frac{2\pi N T_a}{60}}$$

Pro 1] Find Power transmitted by a shaft having 60 mm dia, rotating at 150 rpm. Max. per. stress = 80 MPa.

$$\frac{E}{R} = \frac{M}{I} = \frac{\sigma}{y} \qquad \bigg| \qquad \frac{G\theta}{L} = \frac{T}{J} = \frac{\tau}{R}$$

$$\therefore \frac{G\theta}{L} = \underbrace{\frac{T}{J} = \frac{\tau}{R}} \qquad \& \quad \text{Power} = T\times\frac{2\pi N}{60}$$

$$\therefore T = \frac{\tau J}{R} = \frac{80\times\frac{2\pi}{64}(60)^4}{30} = 3.393\times10^6 \text{ Nmm}$$

$$\therefore \text{Power} = 3.393\times10^6\times2\times\pi\times\frac{150}{60}$$

$$= \frac{53.295\times10^6}{10^3}\ \text{W}$$

$$= 53.295\ \cancel{\text{kW}}\times10^3\ \text{W}$$

$$= 53.295\ \text{kW}$$

Pro: A hollow circular shaft of 150 mm external dia, thk. of metal 20 mm is rotating at 200 rpm. The angle of twist on 3m length was found to be 0.7°. Calculate power transmitted & max. shear stress induced in matl. G = 80 GPa.

$$\frac{E}{R} = \frac{M}{I} = \frac{\sigma}{y} \qquad \underbrace{\frac{G\theta}{L} = \frac{T}{J}} = \frac{\tau}{R}$$

$$\sigma = \frac{My}{I},$$

$G = 80\ \text{GPa} = 80\times10^3\ \text{N/mm}^2$

$\theta = 0.7^\circ = 0.0122 \quad \left(\frac{0.7\times\pi}{180}\right)$

$L = 3000\ \text{mm}$

$J = \frac{2\pi}{64}(150^4 - 110^4)$

$= 35.327\times10^6\ \text{mm}^4.$

$$\therefore T = \frac{J\cdot G\cdot\theta}{L} = \frac{35.327\times10^6\times80\times10^3\times0.0122}{3000} = 11.49\times10^6\ \text{Nmm} = 11.49\ \text{kNm}$$

$$P = T\times\frac{2\pi N}{60} = 11.49\times10^3\times\frac{2\times\pi\times200}{60}$$

$$\therefore P = 240.71\times10^3\ \text{W}$$

$$\therefore \boxed{P = 240.71\ \text{kW}}$$

$$\frac{T}{J} = \frac{\tau}{R} \Rightarrow \tau = \frac{T\cdot R}{J} = \frac{11.49\times10^6\times\cancel{35.327\times10^6}}{35.327\times10^6}\times\frac{150}{2}$$

$$\therefore \boxed{\tau = 24.39\ \text{MPa}}$$

Pro 3) A hollow circular shaft has external dia. of 100 mm & internal dia. 80mm. Find safe power that can be transmitted if allowable shear stress is 100 MPa & max. angle of twist is 3° for 2m length. Take speed of shaft = 2.5 revolution per second & max. torque to exceed by mean torque by 20% G = 80 GPa

$$\frac{E}{R} = \frac{M}{I} = \frac{6}{y} \qquad \underbrace{\frac{G\theta}{L} = \frac{T}{J} = \frac{\tau}{R}}$$

$$\therefore T_1 = \frac{\tau J}{R} = 100 \times \frac{2\frac{\pi}{64}(100^4 - 80^4)}{100/2} = \underline{\cancel{370.96}\ 11.592 \text{ kNm.}}$$

$$T_2 = \frac{G\theta J}{L} = \frac{80\times10^3 \times \frac{3\times\pi}{180} \times \frac{\pi}{32}(100^4 - 80^4)}{2000}$$

$$\therefore \underline{T_2 = 12.1396 \text{ kNm}}$$

$\therefore$ Safe max. Torque = T_1 = 11.592 kNm.

$\therefore$ $P = T_1 \times 2\pi N = 11.592 \times 2 \times \pi \times 2.5 = \underline{182.09 \text{ kW}}$.

But, factor of safty = 1.2 (20%)

$\therefore$ $\boxed{P_{safe} = 151.75 \text{ kW}}$

good

Pro 4] A steel bar 38 mm dia & 450 mm long when tested under axial tensile load 100 kN found to stretch by 0.2 mm. The same bar when subjected to a torque of 1.27 kNm is found to twist by 1.922°. Determine the value of 4 elastic constants.