

# Theory of Structures

Notes by-

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# Slope Deflection Method

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SD  
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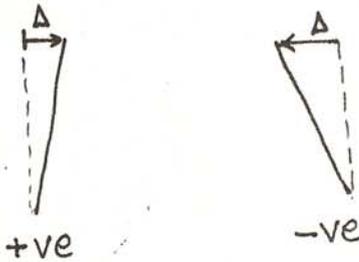
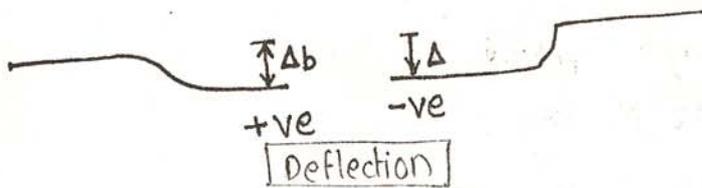
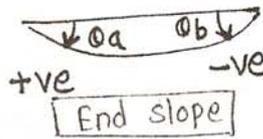
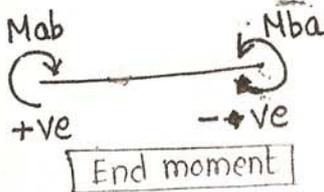
SD eq<sup>n</sup>:-

$$M_{ab} = \overline{M}_{ab} + \frac{4EI}{l} \theta_A + \frac{2EI}{l} \theta_B - \frac{6EI\Delta}{l^2}$$

$$M_{ba} = \overline{M}_{ba} + \frac{2EI}{l} \theta_A + \frac{4EI}{l} \theta_B - \frac{6EI\Delta}{l^2}$$

Sign conventions:-

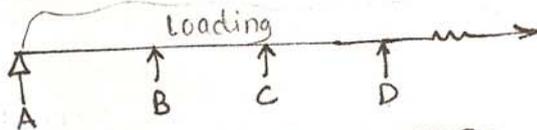
- 1] clockwise end moment  $\Rightarrow$  +ve
- 2] clockwise slope at end  $\Rightarrow$  +ve
- 3] Downward deflect<sup>n</sup> of right end of span w.r.t. left end  $\Rightarrow$  +ve
- 4] Deflection of upper end towards right w.r.t. lower end  $\Rightarrow$  +ve



[Sway]

FOR  
DETAIL SEE  
PROBLEM NO. 5  
(IDEAL SOLUTION)

\* Modification of SD eq<sup>n</sup> for simply supported end of continuous beam



$$M_{ab} = 0 = \overline{M}_{ab} + \frac{4EI}{l} \theta_A + \frac{2EI}{l} \theta_B - \frac{6EI\Delta}{l^2}$$

$$M_{ba} = \overline{M}_{ba} + \frac{2EI}{l} \theta_A + \frac{4EI}{l} \theta_B - \frac{6EI\Delta}{l^2}$$

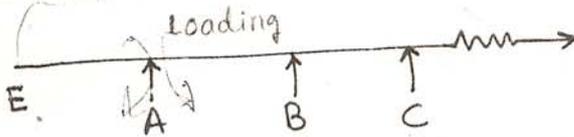
$M_{ab} = 0$  But  $\theta_A \neq 0$   
 $\therefore$  To eliminate ' $\theta$ ':  
 Make a single eq<sup>n</sup>  
 containing  $M_{ba}, \theta_B, \Delta$

i.e.  $M_{ba} = \overline{M}_{ba} - \frac{1}{2} \overline{M}_{ab} + \frac{3EI\theta_B}{l} - \frac{3EI\Delta}{l^2}$

$\leftarrow$  V. Imp.

\* Modification of SD eq<sup>n</sup> for over hang :-

Step II

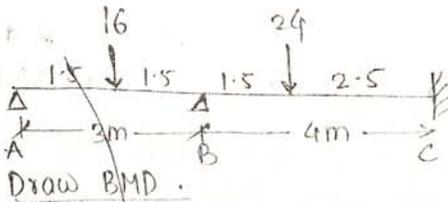


$$\bar{M}_{ab} = \bar{M}_{ae} + \bar{M}_{ab} \dots \text{(Algebraic sum)}$$

$$\therefore \bar{M}_{ba} = \bar{M}_{ba} - \frac{M_a}{2} + \frac{3EI\theta_B}{l} - \frac{3EI\Delta}{l^2} \leftarrow \text{V. Imp.}$$

Ex Pro: 1

(W)



BM  
CKN

Step I] SD eq<sup>n</sup> :- span AB

$$M_{ab} = 0 = \bar{M}_{ab} + \frac{4EI\theta_A}{l} + \frac{2EI\theta_B}{l} - \frac{6EI\Delta}{l^2}$$

$$M_{ba} = \bar{M}_{ba} + \frac{2EI\theta_A}{l} + \frac{2EI\theta_B}{l} - \frac{6EI\Delta}{l^2}$$

But modified eq<sup>n</sup> for simply supported end is,

$$M_{ba} = \bar{M}_{ba} - \frac{\bar{M}_{ab}}{2} + \frac{3EI\theta_B}{l} - \frac{3EI\Delta}{l^2}$$

$$= \frac{+16 \times 1.5 \times 1.5^2}{32} - \frac{1}{2} \left( \frac{-16 \times 1.5 \times 1.5^2}{32} \right) + \frac{3EI\theta_B}{4}$$

$$\therefore M_{ba} = 9 + EI\theta_B \quad \text{--- (a)}$$

SD eq<sup>n</sup> for span BC

$$M_{bc} = -\frac{24 \times 1.5 \times 2.5^2}{42} + \frac{4EI\theta_B}{4} + \frac{2EI\theta_C}{4}$$

$$\therefore M_{bc} = -14.0625 + EI\theta_B \quad \text{--- (b)}$$

$$M_{cb} = +\frac{24 \times 2.5 \times 1.5^2}{42} + \frac{2EI\theta_B}{4}$$

$$\therefore M_{cb} = 8.4375 + 0.5EI\theta_B \quad \text{--- (c)}$$

Also,  $M_{bc} + M_{cb} = 0$

$$\therefore 22.5 + 1.5EI\theta_B = 0$$

$$\therefore \theta_B = \frac{-15}{EI}$$

$$M_{ba} + M_{bc} = 0$$

$$\therefore 9 + EI\theta_B - 14.0625 + EI\theta_B = 0$$

$$EI\theta_B = -2.53125 + 2.53125$$

SFD

Step IV Final Moments:-

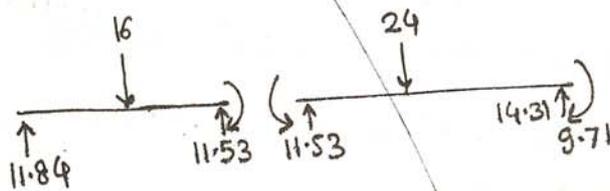
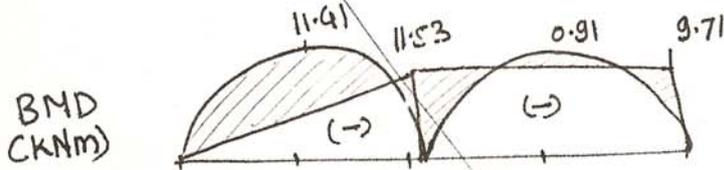
(2)

$$M_{ab} = 0$$

$$M_{ba} = 9 + 2.53125 = 11.53 \text{ kNm}$$

$$M_{bc} = -14.06 + 2.53 = -11.53 \text{ kNm}$$

$$M_{cb} = 8.43 + 0.5(2.53) = 9.71 \text{ kNm}$$



$$\therefore \Sigma M_B = 11.53 \text{ (LHS)}$$

$$\therefore 3R_A - 16 \times 1.5 = 11.53$$

$$\therefore R_A = 11.84 \text{ kN}$$

$$\therefore BM_{\text{max}} = 11.84 \times 1.5$$

$$\text{at midspan} = 17.76$$

$$\Sigma M_C = -11.53 \text{ (RHS)}$$

$$\therefore 9.71 - R_C \times 4 + 24 \times 1.5 = -11.53$$

$$\therefore R_C = 14.31$$

$$\therefore BM_{\text{max}} = 14.31 \times 4 \pm 9.71 \pm 24 \times 1.5$$

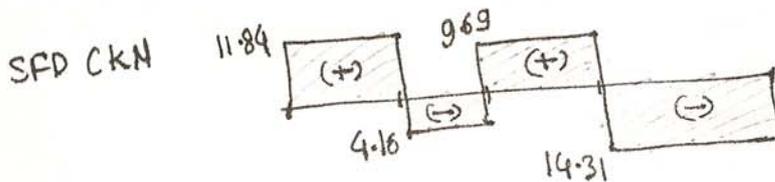
$$\text{at midspan} = +11.53$$

$$\therefore R_B = 16 + 24 - 11.84 - 14.31$$

$$= 13.85 \text{ kN}$$

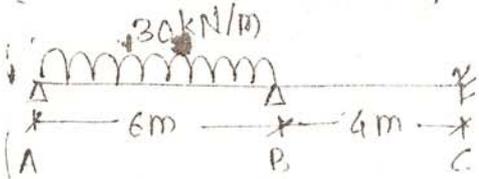
$$BM \text{ at midspan of AB} = -\left(\frac{0 + 11.53}{2}\right) + 17.76 = 14.4 + 11.41$$

$$BM \text{ at midspan of BC} = -\left(\frac{11.53 + 9.71}{2}\right) + 11.53 = 0.91$$



Pro: 2

(W)



Pro 3

step I] Fixed End Moments:-

$$\overline{M}_{ab} = -\frac{30 \times 6^2}{12} = -90 \text{ kNm}$$

$$\overline{M}_{ba} = +90 \text{ kNm}$$

$$\overline{M}_{bc} = 0$$

$$\overline{M}_{cb} = 0$$

ste

step II] SD eq<sup>n</sup> for span AB

$$M_{ba} = \overline{M}_{ba} - \frac{\overline{M}_{ab}}{2} + \frac{3EI\theta_B}{2} - \frac{6 \cdot 3EI\Delta}{12} \dots \text{Modified eq}^n$$

$$\therefore M_{ba} = 90 + 45 + 0.5EI\theta_B = 135 + 0.5EI\theta_B$$

SD eq<sup>n</sup> for span BC

$$M_{bc} = \frac{4EI\theta_B}{4} + \frac{2EI\theta_C}{L}$$

$$M_{cb} = \frac{2EI\theta_B}{4}$$

step

step III] Equilibrium eq<sup>n</sup>:-

$$M_{ba} + M_{bc} = 0$$

$$\therefore 135 + 0.5EI\theta_B + EI\theta_B = 0$$

$$\therefore EI\theta_B = -90$$

step III

step IV] Final end Moments:-

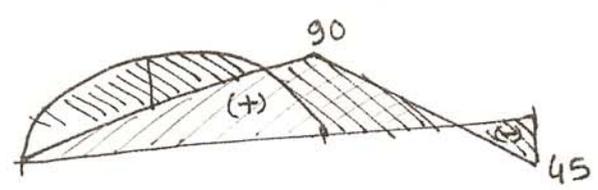
$$M_{ab} = 0$$

$$M_{ba} = 135 + 0.5 \times (-90) = 90 \text{ kNm}$$

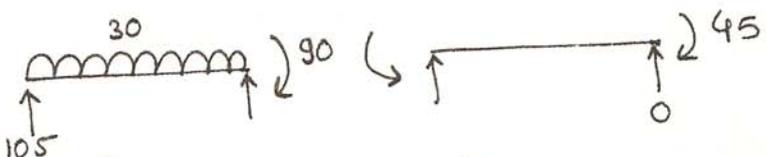
$$M_{bc} = -90 \text{ kNm}$$

$$M_{cb} = -45 \text{ kNm}$$

step IV

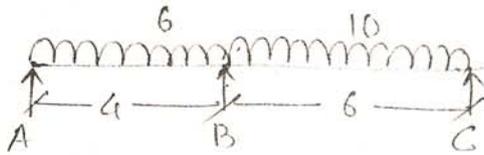


step V



$$6 R_A - \frac{30 \times 6^2}{2} = 90 \Rightarrow R_A = 105 \text{ kNm}$$

$$\therefore \text{BM at midspan} = 180 \text{ kNm}$$



(W)

Step I

$$\begin{aligned} \overline{M}_{ab} &= -\frac{6 \times 16}{12} = -8 \text{ kNm} \\ \overline{M}_{ba} &= 8 \text{ kNm} \\ \overline{M}_{bc} &= -\frac{10 \times 6^2}{12} = -30 \text{ kNm} \\ \overline{M}_{cb} &= 30 \text{ kNm} \end{aligned} \quad \left. \vphantom{\begin{aligned} \overline{M}_{ab} \\ \overline{M}_{ba} \\ \overline{M}_{bc} \\ \overline{M}_{cb} \end{aligned}} \right\} \text{FEM}$$

Step II) Eq<sup>m</sup> eq<sup>n</sup> Span AB :-

$$\begin{aligned} M_{ba} &= \overline{M}_{ba} - 0.5 \overline{M}_{ab} + \frac{3EI\Delta B}{L} \\ &= 8 + 0.5(8) + \frac{3}{4} EI\Delta B \\ &= 12 + \frac{3}{4} EI\Delta B \end{aligned}$$

Eq<sup>n</sup> eq<sup>n</sup> for span BC :-

$$\begin{aligned} M_{bc} &= \overline{M}_{bc} - 0.5 \overline{M}_{cb} + \frac{3EI\Delta B}{L} \\ &= -30 - 15 + \frac{3}{6} EI\Delta B \\ &= -45 + 0.5 EI\Delta B \end{aligned}$$

Step III) Eq<sup>m</sup> eq<sup>n</sup> :-

$$M_{ba} + M_{bc} = 0 \Rightarrow 12 - 45 = -\frac{3}{4} EI\Delta B - 0.5 EI\Delta B$$

$$\therefore \boxed{EI\Delta B = 26.4}$$

Step IV) Final end Moments :-

$$\begin{aligned} M_{ab} &= 0 \\ M_{ba} &= 31.8 \\ M_{bc} &= -31.8 \\ M_{cb} &= 0 \end{aligned}$$

Step V) Reaction & SFD

$$\begin{aligned} 4R_A - 6 \times 4 \times 2 &= 31.8 \Rightarrow R_A = 19.95 \\ 6R_C - 10 \times 6 \times 3 &= -31.8 \Rightarrow R_C = 24.70 \\ R_B &= 6 \times 4 + 10 \times 6 - 19.95 - 24.70 = 39.35 \end{aligned} \quad \left. \vphantom{\begin{aligned} 4R_A \\ 6R_C \\ R_B \end{aligned}} \right\} \text{KN}$$

$$\begin{aligned} \therefore \text{BM @ midspan of AB} &= 19.95 \times 2 - 6 \times 2 \times 1 = 27.90 \\ \text{BM @ midspan of BC} &= 24.70 \times 3 - 10 \times 3 \times 1.5 = 29.10 \end{aligned} \quad \left. \vphantom{\begin{aligned} \text{BM @ midspan of AB} \\ \text{BM @ midspan of BC} \end{aligned}} \right\} \text{KNm}$$

∴ Resultant BI

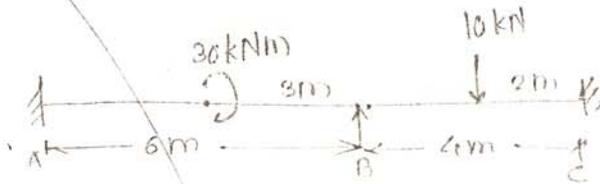
step V) BM at silent points:-

$$M_{ab} = 0$$

$$M_{ab \text{ midspan}} = -\left(\frac{31.8 + 0}{2}\right) + 27.90 =$$

sk

Pro: 4)



step I) Fixed end moments:-

$$\overline{M}_{ab} = +\frac{30}{4} = +7.5 \text{ kNm} \quad \left. \begin{array}{l} \overline{M}_{ba} = 30/4 = 7.5 \text{ kNm} \\ \overline{M}_{bc} = -\frac{10 \times 2 \times 2^2}{16} = -5 \text{ kNm} \\ \overline{M}_{cb} = 5 \text{ kNm} \end{array} \right\} \text{Both +ve}$$

$$\overline{M}_{bc} = -\frac{10 \times 2 \times 2^2}{16} = -5 \text{ kNm}$$

$$\overline{M}_{cb} = 5 \text{ kNm}$$

step II) SD eq<sup>n</sup>:

$$M_{ab} = +7.5 + \frac{2EI\theta_B}{L} = -7.5 + \frac{1}{3}EI\theta_B$$

$$M_{ba} = 7.5 + \frac{2}{3}EI\theta_B$$

$$M_{bc} = -5 + EI\theta_B$$

$$M_{cb} = 5 + 0.5EI\theta_B$$

step III) Eq<sup>m</sup> eq<sup>n</sup>:-

$$M_{ba} + M_{bc} = 0$$

$$\therefore \boxed{EI\theta_B = -1.5}$$

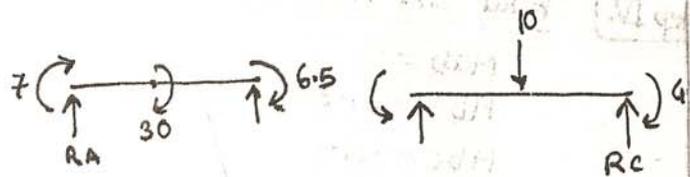
step IV) Final end moments:-

$$M_{ab} = 7 \text{ kNm}$$

$$M_{ba} = 6.5 \text{ kNm}$$

$$M_{bc} = -6.5 \text{ kNm}$$

$$M_{cb} = 4.25 \text{ kNm}$$



step V) Reactions:-

$$\sum M @ B = 6.5 \quad (\text{LHS})$$

$$\therefore +6R_A + 7 + 30 = 6.5$$

$$\therefore R_A = -7.25 \text{ kN}$$

∴ R<sub>A</sub>

$$\therefore R_B = 10 + 7.25 - 4.44$$

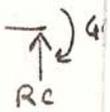
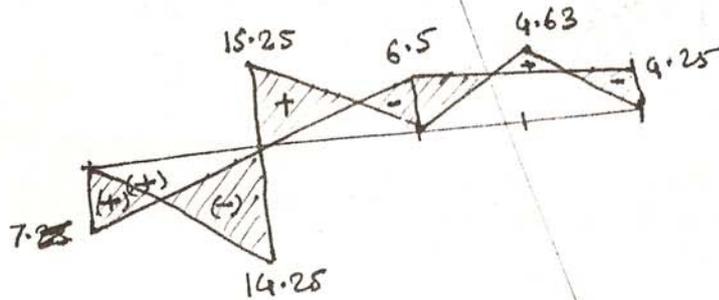
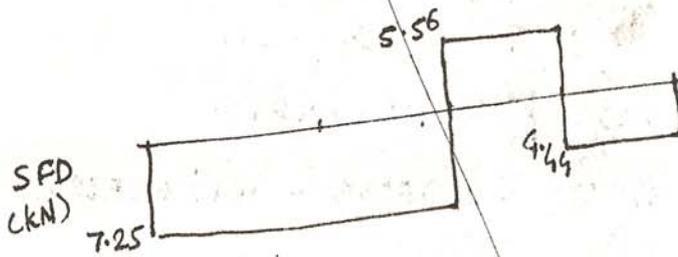
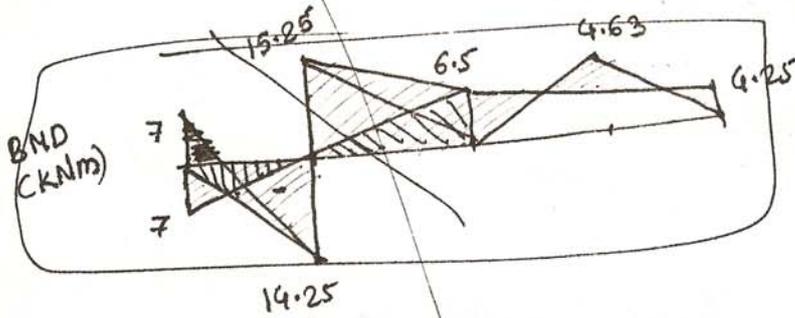
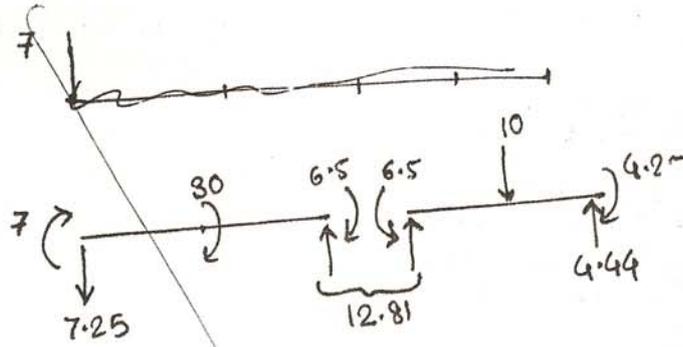
$$= 12.81 \text{ kN}$$

$$\sum M_B @ B = 6.5 \quad (\text{RHS})$$

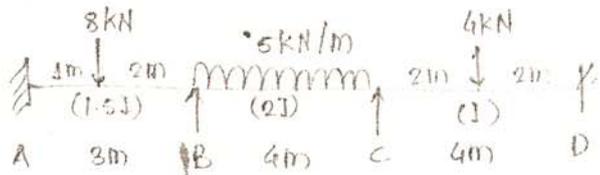
$$-4R_C + 4.25 + 10 \times 2 - 6.5 = 0$$

$$\therefore R_C = 4.44 \text{ kN}$$

$$6R_A - \frac{30 \times 6}{2} = 0 \rightarrow$$



Prob 5



(IDEAL SOLUTION)

step I] Fixed end moments:-

$$\overline{M}_{ab} = -\frac{8 \times 1.5 \times 2^2}{3^2} = -3.56 \text{ kNm}$$

$$\overline{M}_{ba} = \frac{8 \times 2 \times 1^2}{3^2} = +1.78 \text{ kNm}$$

$$\overline{M}_{bc} = -\frac{5 \times 4^2}{12} = -6.67 \text{ kNm}$$

$$\overline{M}_{cb} = +6.67 \text{ kNm}$$

$$\overline{M}_{cd} = -\frac{4 \times 2 \times 2^2}{16} = -2 \text{ kNm}$$

$$\overline{M}_{dc} = +2 \text{ kNm}$$

step II] SD eq<sup>n</sup>:-

$$M_{ab} = -3.56 + \frac{2(1.5)EI\theta_B}{3} = -3.56 + EI\theta_B$$

$$M_{ba} = +1.78 + \frac{4(1.5)EI\theta_B}{3} = +1.78 + 2EI\theta_B$$

$$M_{bc} = -6.67 + \frac{4(2)EI\theta_B}{4} + \frac{2(2)EI\theta_C}{4} = -6.67 + 2EI\theta_B + EI\theta_C$$

$$M_{cb} = +6.67 + EI\theta_B + 2EI\theta_C$$

$$M_{cd} = -2 + EI\theta_C$$

$$M_{dc} = +2 + 0.5EI\theta_C$$

step III] Eq<sup>m</sup> eq<sup>n</sup>:-

$$M_{ba} + M_{bc} = 0 \Rightarrow 1.78 - 6.67 + 4EI\theta_B + EI\theta_C = 0$$

$$M_{cb} + M_{cd} = 0 \Rightarrow 6.67 - 2 + EI\theta_B + 3EI\theta_C = 0$$

$$\therefore EI\theta_B = 1.76$$

$$EI\theta_C = -2.14$$

step IV] Final End moments:-

$$M_{ab} = -1.8 \text{ kNm}$$

$$M_{ba} = +5.3 \text{ kNm}$$

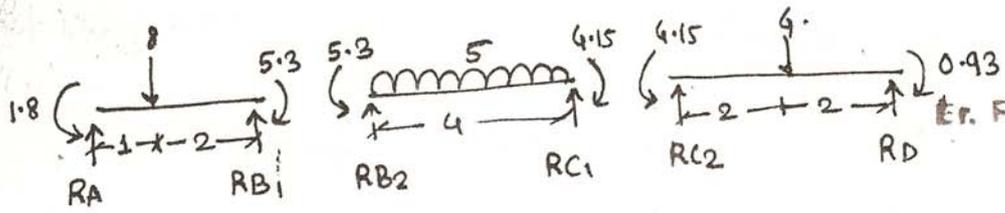
$$M_{bc} = -5.3 \text{ kNm}$$

$$M_{cd} = -4.15 \text{ kNm}$$

$$M_{dc} = 0.93 \text{ kNm}$$

$$M_{cb} = +4.15 \text{ kNm}$$

N)

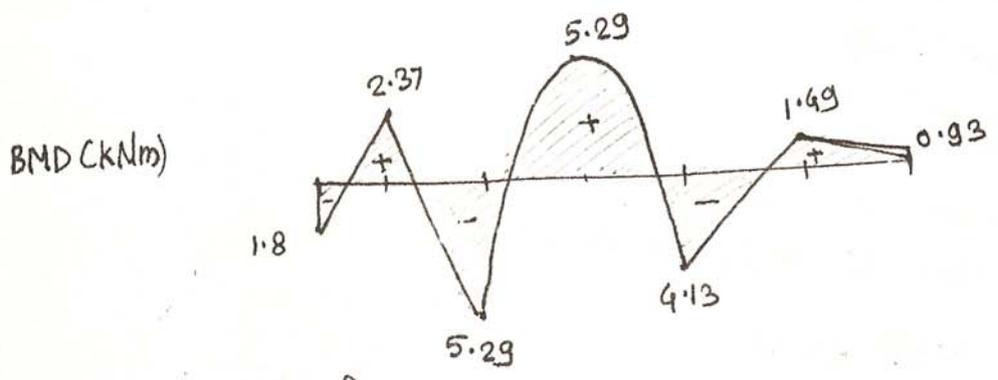
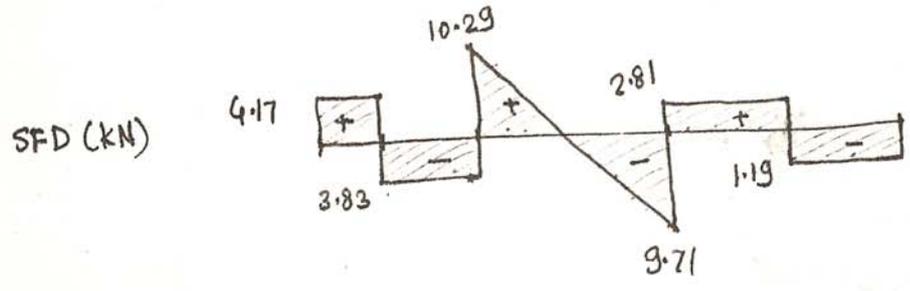
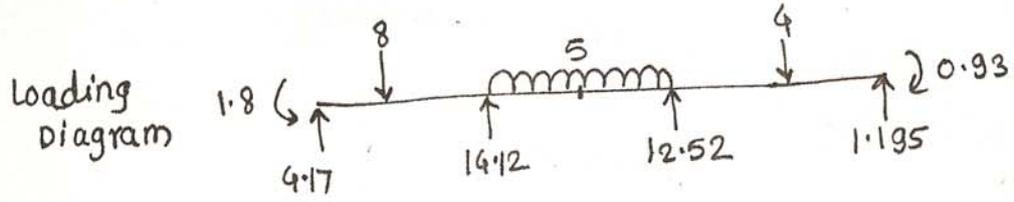


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$\sum M_B = 0 \Rightarrow 3R_A - 1.8 - 8 \times 2 + 5.3 = 0 \quad \sum M_B = 0$   
 $\therefore R_A = 4.17 \text{ kN}$   
 $R_{B1} = 3.83 \text{ kN}$

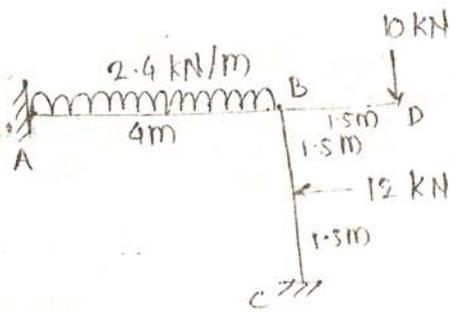
$\sum M_C = 0$   
 $\therefore -5.3 + 5 \times \frac{4^2}{2} - 4R_{C1} + 4.15 = 0$   
 $\therefore R_{C1} = 9.71 \text{ kN}$   
 $R_{B2} = 10.29 \text{ kN}$

$\sum M_D = 0$   
 $-4.15 + 4 \times 2 + 0.93 = 4R_D$   
 $\therefore R_D = 1.195 \text{ kN}$   
 $R_{C2} = 2.805 \text{ kN}$



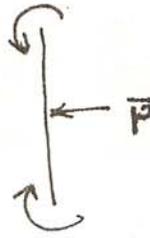
Vo. Gurdal

Prob 6



MPSC-96  
32M

step



step I Fixed end moments:-

$$\overline{M}_{ab} = -\frac{2.4 \times 4^2}{12} = -3.2 \text{ kNm}$$

$$\overline{M}_{ba} = +3.2 \text{ kNm.}$$

$$\overline{M}_{bd} = -10 \times 1.5 = -15 \text{ kNm.} \rightarrow \text{overhang.}$$

$$\overline{M}_{bc} = -\frac{12 \times 1.5^3}{32} = -4.5 \text{ kNm}$$

$$\overline{M}_{cb} = +4.5 \text{ kNm.}$$

step II SD eq<sup>n</sup>:-

$$M_{ab} = -3.2 + \frac{2EI\theta_B}{4}$$

$$M_{ba} = +3.2 + EI\theta_B$$

$$M_{bc} = -4.5 + \frac{4}{3}EI\theta_B$$

$$M_{cb} = +4.5 + \frac{2}{3}EI\theta_B$$

step III Eq<sup>n</sup> eq<sup>n</sup>:-

$$M_{ba} + M_{bc} + M_{bd} = 0$$

$$\therefore 3.2 - 4.5 - 15 + EI\theta_B + \frac{4}{3}EI\theta_B = 0$$

$$\therefore EI\theta_B = 6.99$$

step IV Final end moments:-

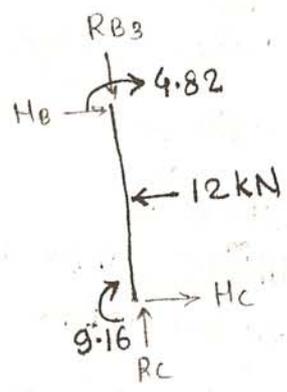
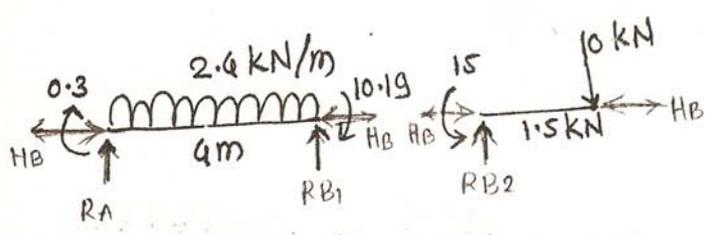
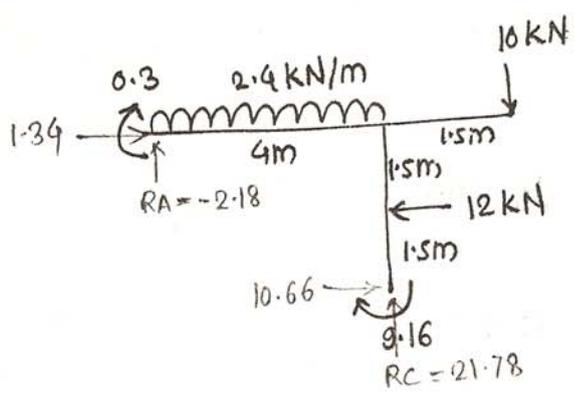
$$M_{ab} = 0.3 \text{ kNm}$$

$$M_{ba} = 10.19 \text{ kNm}$$

$$M_{bc} = 4.82 \text{ kNm}$$

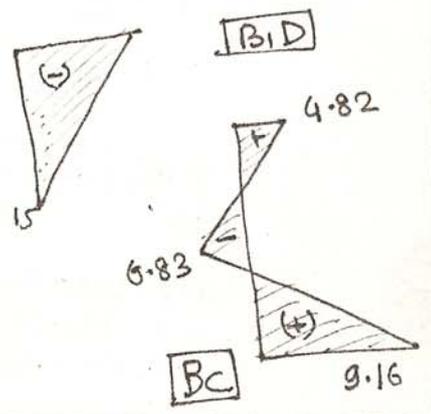
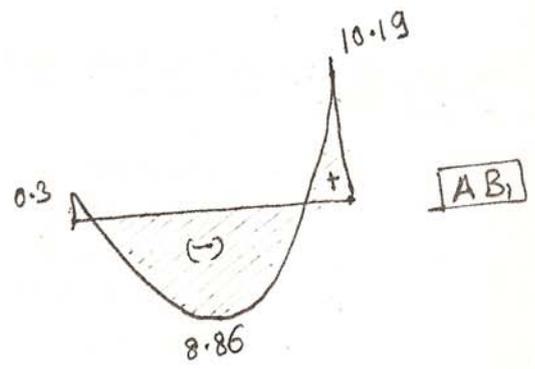
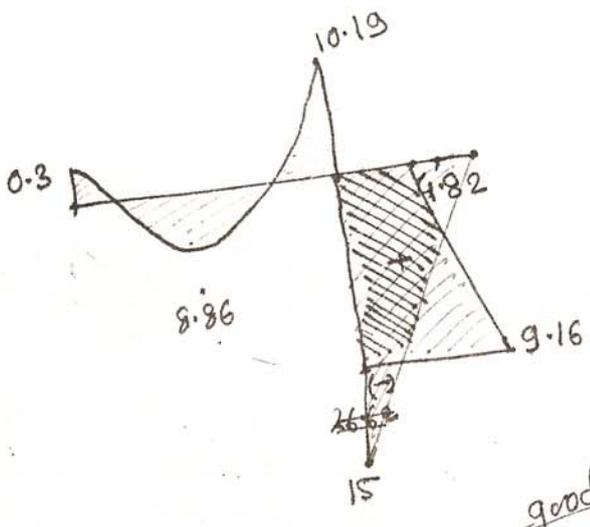
$$M_{cb} = 9.16 \text{ kNm.}$$

step V Reactions :-



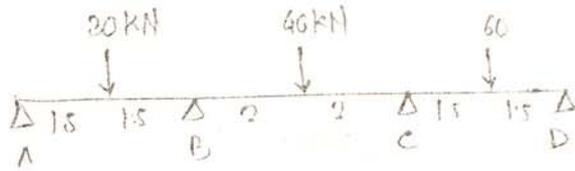
$\sum R_{B1} = 0$   
 $\therefore 0.3 + 4R_A - 2.4 \times \frac{4}{2} + 10.19 = 0$   
 $\therefore R_A = -2.18 \text{ kN. } (\downarrow)$   
 $R_{B1} = +2.4 \times 4 - 2.18 = +11.78 (\uparrow)$   
 $R_{B2} = 10 \text{ kN. } (\uparrow)$   
 $R_{B3} = R_{B1} + R_{B2} = 21.78 \text{ kN } (\uparrow)$   
 $\therefore R_C = 21.78 \text{ kN. } (\uparrow)$

$\sum M_C = 0$   
 $\therefore 3H_B + 4.82 + 9.16 - 12 \times 1.5 = 0$   
 $\therefore H_B = 1.34 \text{ kN.}$   
 $\therefore H_C = 10.66 \text{ kN.}$



good.

Pro: 7)



step I

step I) FEM

$$\overline{M}_{ab} = -\frac{30 \times 15^2}{9} = -11.25 \text{ kNm}$$

$$\overline{M}_{ba} = +11.25 \text{ kNm}$$

$$\overline{M}_{bc} = -\frac{40 \times 2^3}{42} = -20 \text{ kNm}$$

$$\overline{M}_{cb} = +20 \text{ kNm}$$

$$\overline{M}_{cd} = -\frac{60 \times 15^2}{9} = -22.5 \text{ kNm}$$

$$\overline{M}_{dc} = +22.5 \text{ kNm}$$

Loadi  
diar  
(Anal)

CSPD

step II) SD eq<sup>n</sup>:-

Modified for s.s. end

$$\left. \begin{aligned} M_{ba} &= \overline{M}_{ba} - 0.5\overline{M}_{ab} + \frac{3EI\theta_B}{L} = +16.88 + EI\theta_B \\ M_{cd} &= \overline{M}_{cd} - 0.5\overline{M}_{dc} + \frac{3EI\theta_C}{L} = -33.75 + EI\theta_C \end{aligned} \right\}$$

$$M_{bc} = -20 + EI\theta_B + 0.5EI\theta_C$$

$$M_{cb} = +20 + 0.5EI\theta_B + EI\theta_C$$

BMD

step III) Eq<sup>m</sup> eq<sup>n</sup>:-

$$M_{ba} + M_{bc} = 0 \Rightarrow 16.88 - 20 + EI\theta_B + EI\theta_B + 0.5EI\theta_C = 0$$

$$M_{cb} + M_{cd} = 0 \Rightarrow 20 - 33.75 + 0.5EI\theta_B + EI\theta_C + EI\theta_C = 0$$

$$\therefore \text{solving, } EI\theta_B = -0.17$$

$$EI\theta_C = +6.92$$

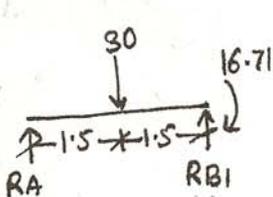
SI Pr

step IV) Final end moments:-

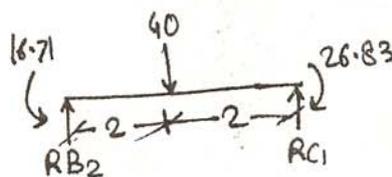
$$\left. \begin{aligned} M_{ab} &= 0 \\ M_{ba} &= +16.71 \\ M_{bc} &= -16.71 \\ M_{cb} &= +26.83 \\ M_{cd} &= -26.83 \\ M_{dc} &= 0 \end{aligned} \right\} \text{ kNm}$$

step

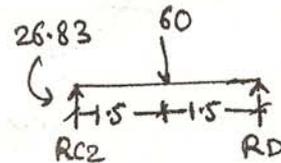
step V) Reactions:-



$$\begin{aligned} \therefore 3R_A - 30 \times 15 + 16.71 &= 0 \\ \therefore R_A &= 9.43 \text{ kN} \\ R_B &= 20.57 \text{ kN} \end{aligned}$$



$$\begin{aligned} -16.71 + 26.83 + 4R_B - 40 \times 2 &= 0 \\ \therefore R_B &= 17.47 \text{ kN} \\ R_C &= 22.53 \text{ kN} \end{aligned}$$

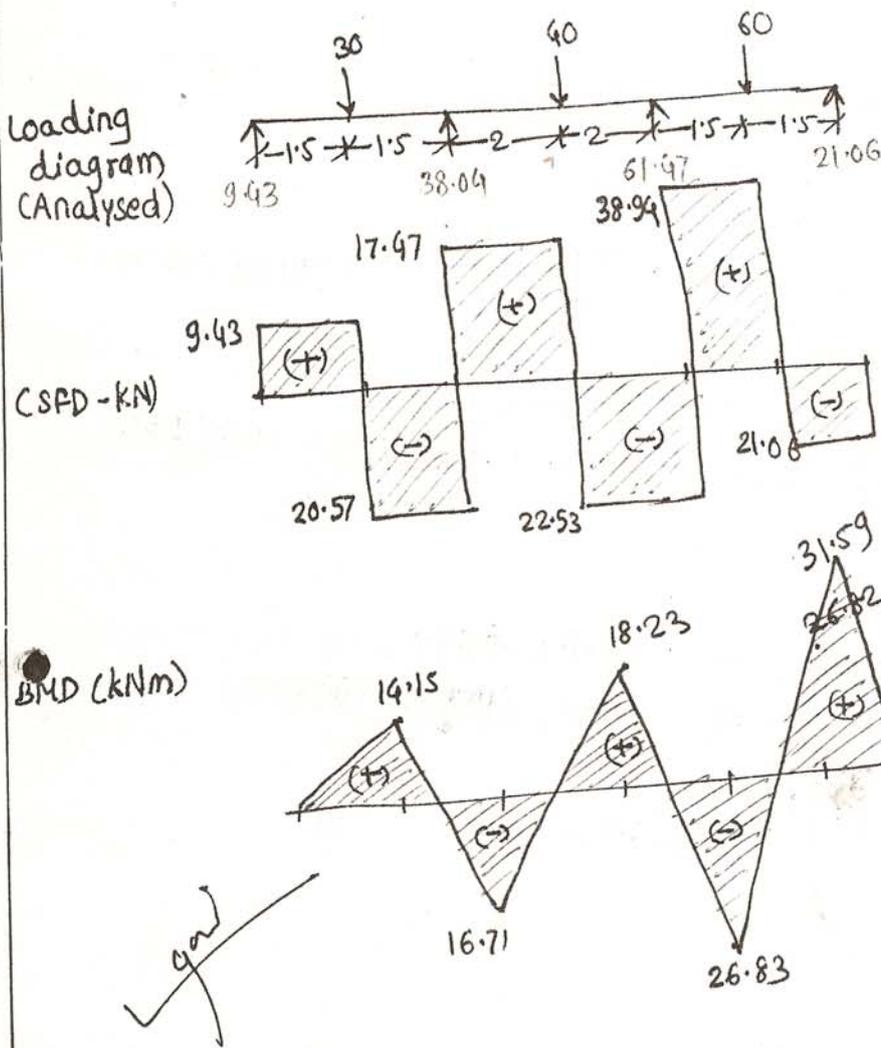


$$\begin{aligned} -26.83 + 3R_C - 60 \times 15 &= 0 \\ \therefore R_C &= 38.94 \text{ kN} \\ R_D &= 21.06 \text{ kN} \end{aligned}$$

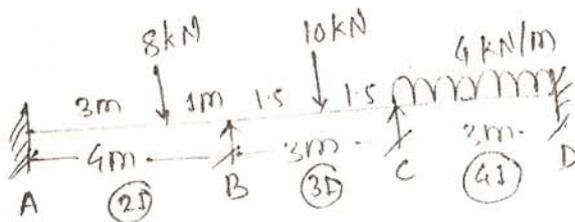
step VII] SFD & BMD :-

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(B.E Civil)

SD  
7



ST P80: 8



step I] Fixed end moments :-

$$\begin{aligned}
 \overline{M}_{ab} &= -1.5 \text{ kNm} \\
 \overline{M}_{ba} &= +4.5 \text{ kNm} \\
 \overline{M}_{bc} &= -3.75 \text{ kNm} \\
 \overline{M}_{cb} &= +3.75 \text{ kNm} \\
 \overline{M}_{cd} &= -3 \text{ kNm} \\
 \overline{M}_{dc} &= +3 \text{ kNm}
 \end{aligned}$$

2-60X1  
8.94 kN  
1.06 kN

Step II) SD eq<sup>n</sup>:-

~~Mba = 8 kNm~~

$$\therefore M_{ab} = -1.5 + 2(2) \frac{EI\theta_B}{4} = -1.5 + EI\theta_B$$

$$M_{ba} = 4.5 + 4(2) \frac{EI\theta_B}{4} = +4.5 + 2EI\theta_B$$

$$M_{bc} = -3.75 + 4(3) \frac{EI\theta_B}{3} + 2(3) \frac{EI\theta_C}{3} = -3.75 + 4EI\theta_B + 2EI\theta_C$$

$$M_{cb} = +3.75 + 2EI\theta_B + 4EI\theta_C$$

~~M<sub>cd</sub> = -3 + 4(4) \frac{EI\theta\_C}{3} = -3 + \frac{16}{3} EI\theta\_C~~

Modified eq<sup>n</sup>: ~~M<sub>cd</sub> = -3.75 - (\frac{+3}{2}) + \frac{3(4)EI\theta\_C}{3}~~  

$$M_{dc} = +3 + \frac{8}{3} EI\theta_C = -4.5 + 4EI\theta_C$$

Step III) Eq<sup>m</sup> eq<sup>n</sup>:-

$$M_{ba} + M_{bc} = 0 \Rightarrow 4.5 - 3.75 + 6EI\theta_B + 2EI\theta_C = 0$$

$$M_{cb} + M_{cd} = 0 \Rightarrow 3.75 - 4.5 + 2EI\theta_B + 8(\frac{16+16}{3}) EI\theta_C = 0$$

$$\therefore EI\theta_B = -0.17$$

$$EI\theta_C = -0.08 \text{ or } 0.126$$

Step IV) Final end moments:-

$$M_{ab} = -1.67$$

$$M_{ba} = +4.16$$

$$M_{bc} = -4.16$$

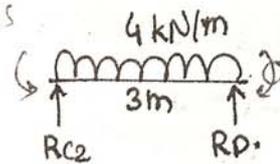
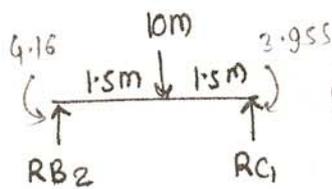
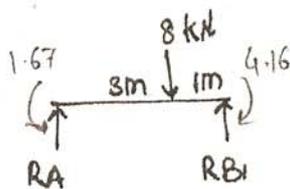
$$M_{cb} = +3.955$$

$$M_{cd} = -3.955$$

$$M_{da} = 0$$

} kNm

Step V) Reactions:-



$$4.16 + 4R_A - 1.67 - 8 \times 1 = 0$$

$$\therefore R_A = 1.38 \text{ kN}$$

$$R_{B1} = 6.62 \text{ kN}$$

$$-4.16 + 3R_{B2} - 10 \times 1.5 + 3.955 = 0$$

$$\therefore R_{B2} = 5.07 \text{ kN}$$

$$R_{C1} = 4.93 \text{ kN}$$

$$-3.955 + 3R_D + 4 \times \frac{3^2}{2} = 0$$

$$\therefore R_D = 4.68 \text{ kN}$$

$$R_{C2} = 7.32 \text{ kN}$$

Step

Loadin  
CAN

SFD  
CKN

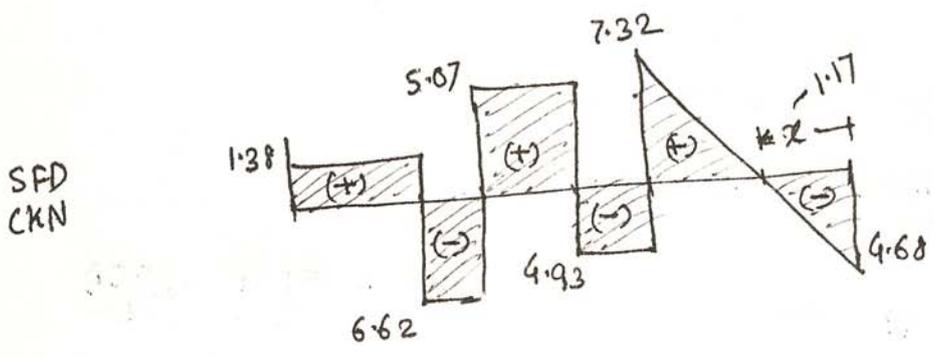
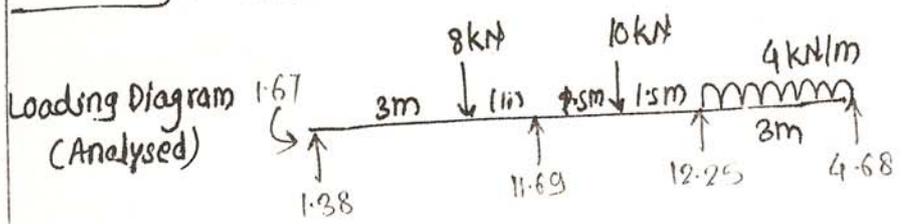
BMD  
CKN

Pr

S

SI

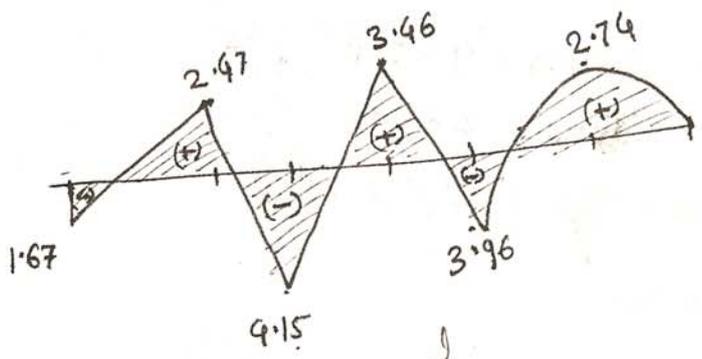
Step VI) SFD & BMD



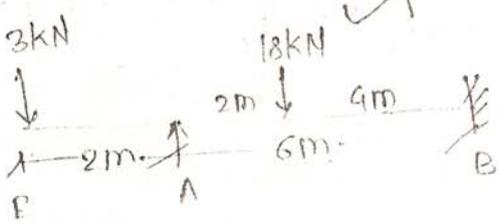
$$\frac{7.32}{3-x} = \frac{4.68}{x}$$

$$\therefore x = 1.17m$$

BMD (kNm)



10:9



step I) FEM:

$$M_{ae} = 3 \times 2 = 6 \text{ kNm}$$

$$\overline{M}_{ab} = \frac{-18 \times 2 \times 4^2}{36} = -16 \text{ kNm}$$

$$\overline{M}_{ba} = \frac{+18 \times 4 \times 2^2}{36} = +8 \text{ kNm}$$

$$\overline{M}_a = M_{ae} + \overline{M}_{ab} = 6 - 16 = -10 \text{ kNm}$$

step II) SD eqn :-

ij Modified SD eqn :-  $M_{ab} = \overline{M}_{ab} - \frac{\overline{M}_a}{2} + \frac{3EI\theta_A}{L}$  (incomplete)

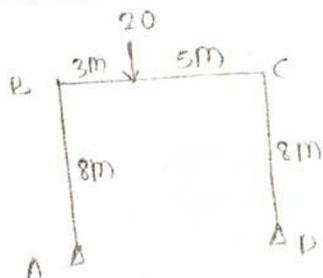
$$M_{ab} = -16 - 5 + 0.5EI\theta_A$$

$$M_{ba} = 8 + \frac{2}{6}EI\theta_B$$

step III) Eqm eqn :-  $M_{ab}$

$\frac{\partial U}{\partial \theta} = 0$

Pro: 10



Step I

Step I) FEM:-

$$\overline{M_{ab}} = \overline{N_{ba}} = \overline{M_{cd}} = \overline{N_{dc}} = 0$$

$$\overline{M_{bc}} = \frac{-20 \times 3 \times 5^2}{8^2} = -23.44 \text{ kNm}$$

$$\overline{M_{cb}} = \frac{20 \times 5 \times 3^2}{64} = +14.06 \text{ kNm}$$

Step II) SD eq<sup>n</sup>:-

Simply supported end:-  $\overline{M_{ab}} = \overline{N_{ba}} = \overline{M_{ba}} - \frac{\overline{M_{ab}}}{2} + \frac{3EI\theta_B}{L} - \frac{3E\epsilon_D}{L^2}$

$$\therefore \overline{N_{ba}} = \frac{3}{8} EI\theta_B$$

$$\overline{M_{bc}} = -23.44 + \frac{4}{8} EI\theta_B + \frac{2}{8} EI\theta_C$$

$$\overline{M_{cb}} = 14.06 + \frac{2}{8} EI\theta_B + \frac{4}{8} EI\theta_C$$

$$\overline{M_{cd}} = \overline{N_{dc}} - \frac{\overline{M_{dc}}}{2} + \frac{3EI\theta_C}{L} - \frac{3E\epsilon_D}{L^2}$$

$$= \frac{3}{8} EI\theta_C$$

(BC)

(BA)

(CD)

Step III) Eq<sup>m</sup> eq<sup>n</sup>:-

$$\overline{M_{ba}} + \overline{M_{bc}} = 0 \Rightarrow -23.44 + \left(\frac{3}{8} + \frac{4}{8}\right) EI\theta_B + \frac{2}{8} EI\theta_C = 0$$

$$\Rightarrow \frac{7}{8} EI\theta_B + \frac{2}{8} EI\theta_C = 23.44$$

$$\overline{M_{cb}} + \overline{M_{cd}} = 0 \Rightarrow 14.06 + \frac{2}{8} EI\theta_B + \left(\frac{4}{8} + \frac{3}{8}\right) EI\theta_C = 0$$

$$\Rightarrow \frac{2}{8} EI\theta_B + \frac{7}{8} EI\theta_C = -14.06$$

$$\therefore EI\theta_B = 34.17$$

$$EI\theta_C = -25.83$$

Step III) Final end moments:-

$$M_{ab} = 0$$

$$M_{ba} = 12.81 \text{ kNm}$$

$$M_{bc} = -12.81 \text{ kNm}$$

$$M_{cb} = 9.687 \text{ kNm}$$

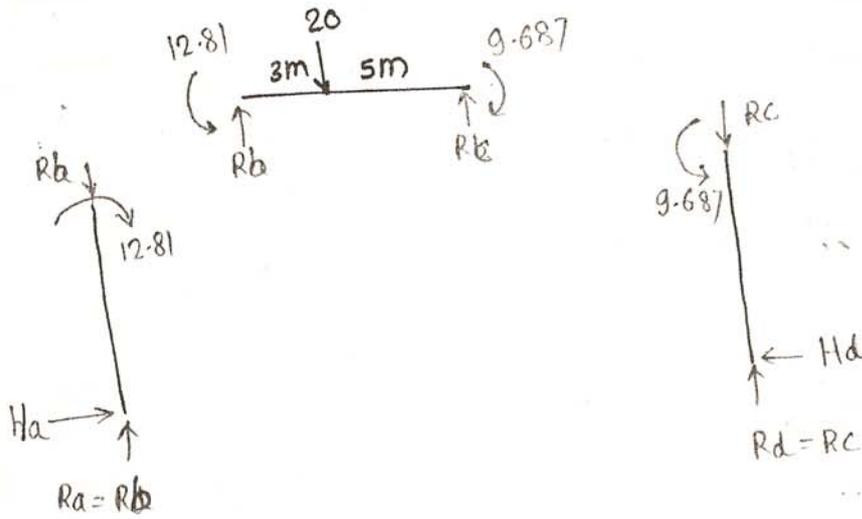
$$M_{cd} = -9.687 \text{ kNm}$$

$$M_{dc} = 0$$

Step I) Reactions:-

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(B.E. Civil)

SD  
9



$$(BC) \sum M_C = 0 \Rightarrow 8R_b - 12.81 + 9.687 - 20 \times 5 = 0$$

$$\therefore R_b = 12.89 \text{ kN.}$$

$$R_c = 7.11 \text{ kN.}$$

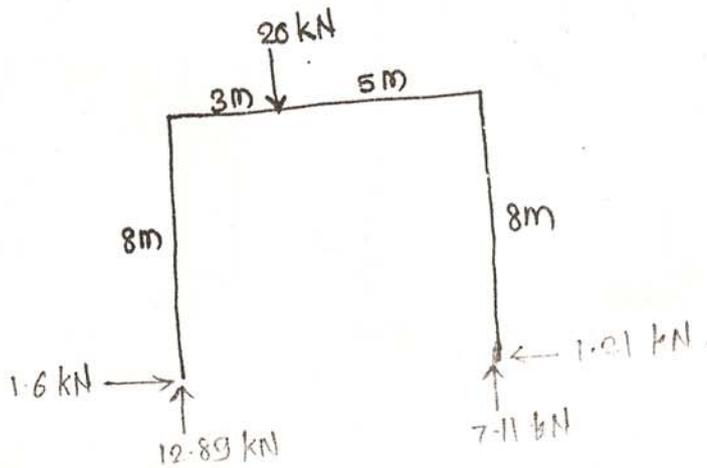
$$(BA) \sum M_b = 0 \Rightarrow -H_a \times 8 + 12.81 = 0 \Rightarrow H_a =$$

$$\underline{H_a = 1.6 \text{ kN}}$$

$$(CD) \sum M_c = 0 \Rightarrow 8H_d = 9.687$$

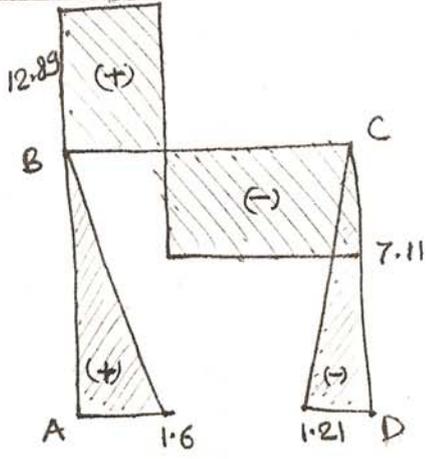
$$\therefore \underline{H_d = 1.21 \text{ kN}}$$

should balance each other.

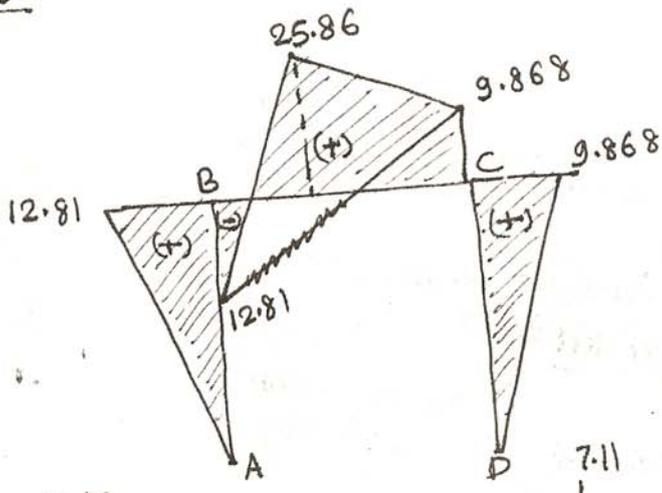


correct.

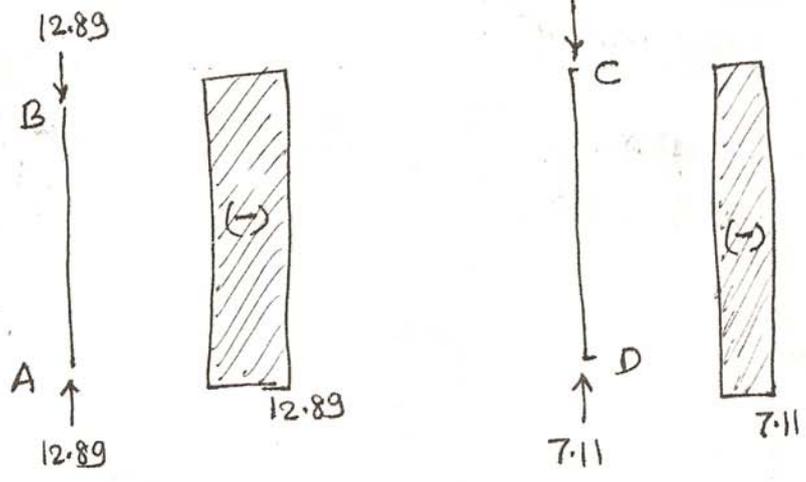
step VI) SFD & BMD



BMD:



Axial Force Diagram



Pro: ]

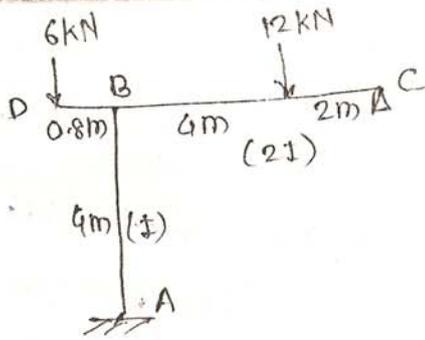
Step I

Step I

Step I

step

Pro: 1



SD  
10

Step I) FEM:-

$$M_{ab} = M_{ba} = 0$$

$$M_{bc} = -\frac{12 \times 4 \times 2^2}{36} = -5.33 \text{ kNm}$$

$$M_{cb} = +10.67 \text{ kNm}$$

$$M_{bd} = 6 \times 0.8 = 4.8 \text{ kNm}$$

Step II) SD eqn:-

$$M_{ab} = \frac{2EI\theta_B}{4}$$

$$M_{ba} = \frac{4EI\theta_B}{4}$$

$$M_{bc} = -5.33 + \frac{4(2)EI\theta_B}{6} + \frac{2(2)EI\theta_C}{6}$$

$$M_{cb} = 10.67 + \frac{4EI\theta_B}{6} + \frac{8EI\theta_C}{6} = 0 \quad \text{--- (a)}$$

Step III) Eqn eqn:-

$$M_{bd} + M_{bc} + M_{ba} = 0$$

$$\Rightarrow 4.8 + (-5.33) + \frac{8}{6}EI\theta_B + \frac{4}{6}EI\theta_C + 10.67EI\theta_B = 0$$

$$\Rightarrow \left(\frac{8}{6} + \frac{4}{6} + 1\right)EI\theta_B + 0.67EI\theta_C = 0.53 \quad \text{--- (b)}$$

$$\therefore EI\theta_B = 0.18$$

$$\therefore \text{Solving (a) \& (b)} \\ EI\theta_B = 2.95 \text{ kNm} \\ EI\theta_C = -9.48 \text{ kNm}$$

Step III) Final end moments:-

$$M_{ab} = 0.09 \text{ kNm}$$

$$\therefore M_{ab} = 1.475 \text{ kNm}$$

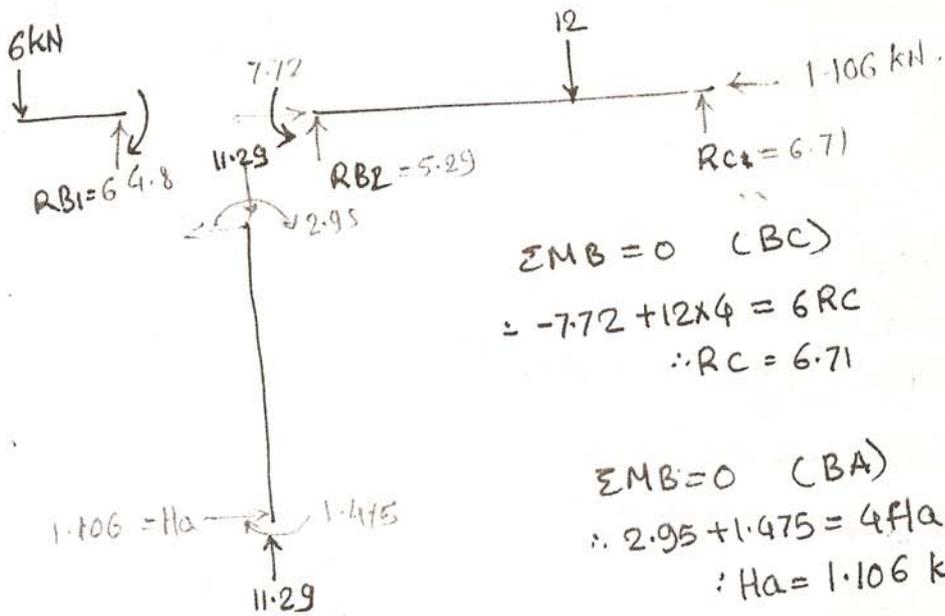
$$M_{ba} = 2.95 \text{ kNm}$$

$$M_{bc} = -7.72 \text{ kNm}$$

$$M_{cb} = 0$$

$$M_{bd} = 4.8 \text{ kNm} \quad \text{gwt}$$

step II] Reactions :-



$$\sum M_B = 0 \text{ (BC)}$$

$$= -7.72 + 12 \times 4 = 6R_C$$

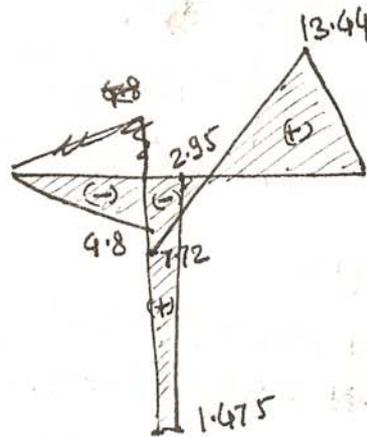
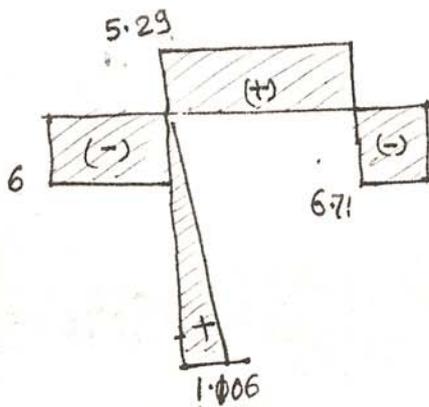
$$\therefore R_C = 6.71$$

$$\sum M_B = 0 \text{ (BA)}$$

$$\therefore 2.95 + 1.475 = 4H_A$$

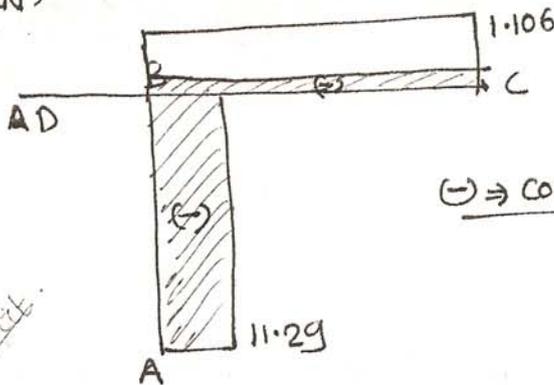
$$\therefore H_A = 1.106 \text{ kN}$$

step III] SFD, BMD, AFD :-



SFD (kN)

BMD (kNm)



(-)  $\Rightarrow$  compressive

Axial force diagram (kN)

correct

(E) M V

Carri

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(6)