Theory of Structures

Notes by-

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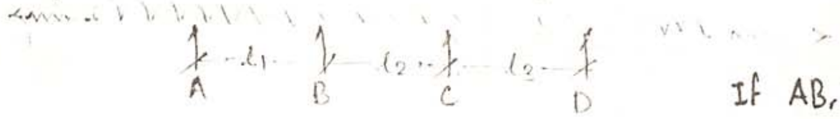
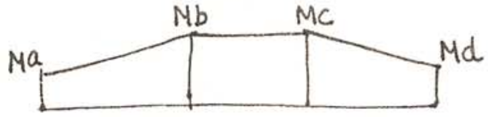
[continuous Beam]

* [Clapeyron's Theorem of three moments :]



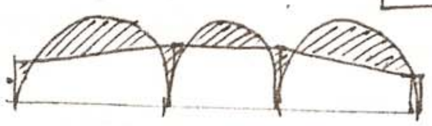
If AB, BC are two consecutive span of a continuous beams subjected to an external loading, the relation bet^n support moments A, B, C are given by,

Free BMD.



Fixed BMD

$$\boxed{M_a l_1 + 2M_b(l_1+l_2) + M_c l_2 = \frac{6a_1\bar{x}_1}{l_1} + \frac{6a_2\bar{x}_2}{l_2}}$$



Final BMD

same eq^n can be written for the span BC & CD as,

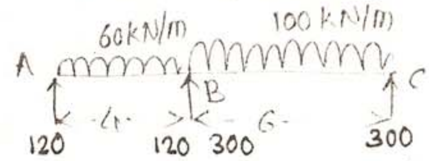
$$\boxed{M_b l_2 + 2M_c(l_2+l_3) + M_d l_3 = \frac{6a_2\bar{x}_2}{l_2} + \frac{6a_3\bar{x}_3}{l_3}}$$

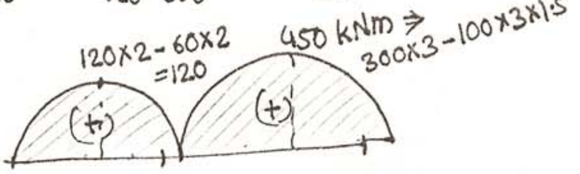
Where, a_1, a_2, a_3 = Area of free BMD for span AB, BC, CD resp.

$\bar{x}_1, \bar{x}_2, \bar{x}_3$ = Centroid dist. of free BMD on AB from A, on BC from C, on CD from D.

l_1, l_2, l_3 = Span of AB, BC, CD resp.

Pro:-

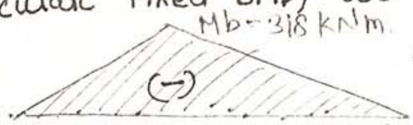




$\bar{a}$ = area of parabola = $\frac{2}{3}$ × Base × altitude

Free BMD

To calculate Fixed BMD, use clapeyron's TM eq^n;



$$\therefore 0 + 2M_b(10) + 0 = \frac{6\times\frac{2}{3}\times4\times120\times\frac{4}{2}}{4} + \frac{6\times\frac{2}{3}\times6\times450\times\frac{6}{2}}{6}$$

$\therefore M_b = 318$ kNm

fixed BMD.

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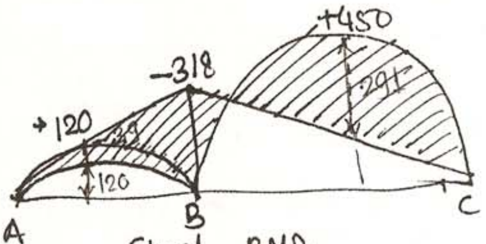


Final BMD.

$\therefore$ Max. mmt in span AB =

$= -\left(\frac{0+318}{2}\right) + 120$

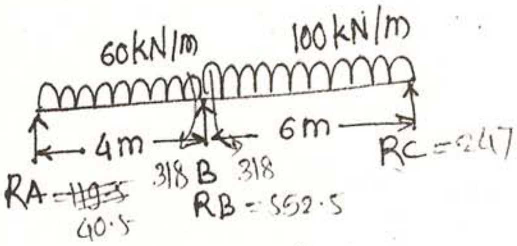
$\therefore \boxed{M_{AB} = -39 \text{ kNm}}$

Max. mmt. at midspan BC

$= -\left(\frac{0+318}{2}\right) + 450$

$\therefore \boxed{M_{BC} = 291 \text{ kNm}}$

$\boxed{\text{BM at support } B = -318 \text{ kNm}}$



$\therefore \Sigma M @ B = 0$ (LHS)

$\therefore 4RA - \frac{60 \times 4^2}{2} + 318 = 0$

$\therefore RA = 40.5 \text{kN}.$

$RB = 60 \times 4 - 40.5$

$= 199.5$

$\Sigma MB = 0$ (RHS)

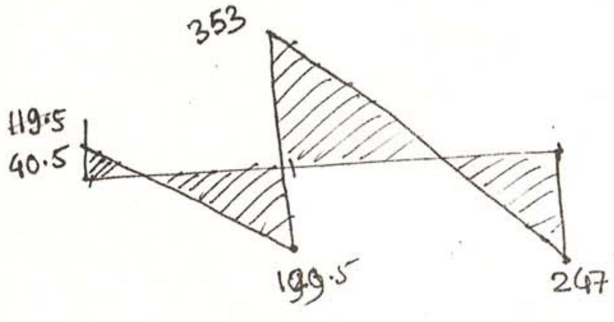
$6RC - \frac{100 \times 6^2}{2} + 318 = 0$

$\therefore RC = 247 \text{ kN}$

& $RB = 100 \times 6 - 247$

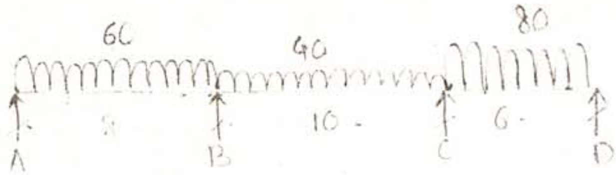
$= 353 \cdot \text{kN}$

$\therefore RB = 552.5$



SFD.

Pro:

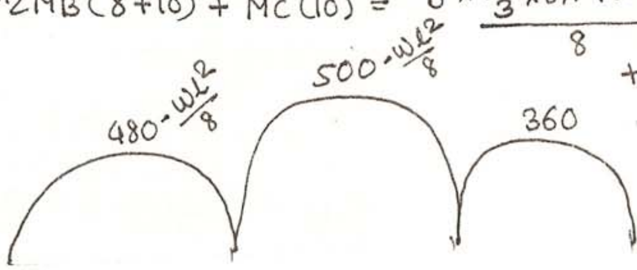


* Analyse the Beam
* Draw BMD.

Using clapeyron's theorem of 3 moments for span ABC & BCD.

Span ABC

$\therefore 0 + 2M_B(8+10) + M_C(10) = \frac{6 \times \frac{2}{3} \times 8 \times 480 \times 4}{8} + \frac{6 \times \frac{2}{3} \times 10 \times 500 \times 5}{10} \Rightarrow 36M_B + 10M_C = 17680 \cdots (i)$



Free BMD

Span BCD

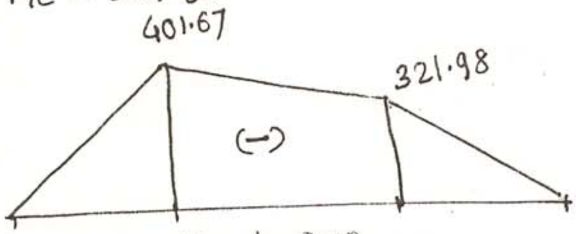
$10M_B + 2M_C(10+6) + 0 = \frac{6 \times \frac{2}{3} \times 10 \times 500 \times 5}{10} + \frac{6 \times \frac{2}{3} \times 360 \times 6 \times 3}{6}$

$\therefore 10M_B + 32M_C = 14320 \cdots (ii)$

$\therefore$ Solving (i) & (ii)

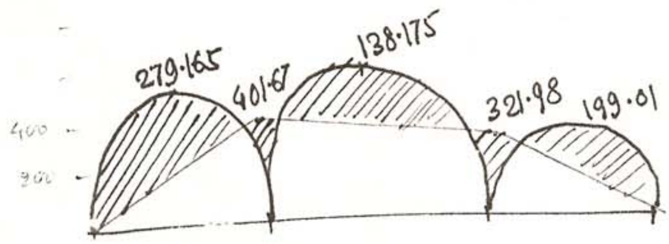
$M_B = 401.67$

$M_C = 321.98$



Fixed BMD

BMD



Final BMD

$\therefore$ BM at midspan of AB $= -\dfrac{401.67}{2} + 480 = 279.165$ kNm

BM at B $= 401.67$ kNm

BM at midspan of BC $= -\left(\dfrac{401.67+321.98}{2}\right) + 500 = 138.175$ kNm

BM at C $= 321.98$ kNm

BM at midspan of CD $= -\dfrac{321.98+0}{2} + 360 = 199.01$ kNm

Reactions.

$\Sigma M @ B = \cancel{0}\ 401.67$ (LHS)

$\therefore 8V_a - 60\times\dfrac{8^2}{2} + 401.67 = 0$

$\therefore V_a = 189.79$ kN.

$\Sigma M @ C = \cancel{0}\ 321.98$ (RHS)

$6V_D - 80\times\dfrac{6^2}{2} + 321.98$

$\therefore V_D = 186.34$ kN.

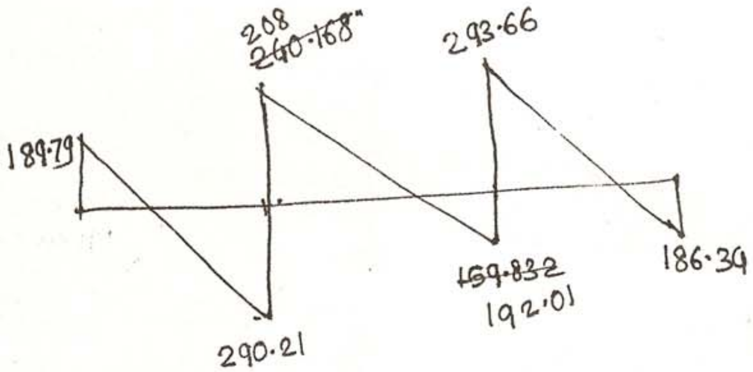
$\Sigma M @ C = \cancel{0}\ 321.98$ (LHS)

$189.79\times 18 - 60\times 8\times 14 - 40\times 10\times 5 + R_B\times 10 = \cancel{0}\ 321.98$

$\therefore R_B = \cancel{530.378\text{ kN}}\ .498.2$

& $R_C = 60\times 8 + 40\times 10 + 80\times 6 - 189.79 - 186.34 - \cancel{530.378}\ 498.2$

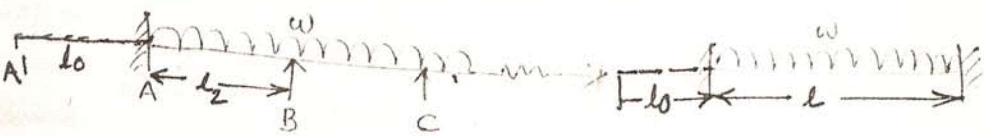
$\therefore R_C = \cancel{453.492\text{ kN}}\ 485.7$ kN.



O.K.

(SFD)

* Fixed end continuous beam :-



To find FEM ; from clapeyron's theorem of TM, consider an imaginary span of $l_1 = 0$.

$\frac{\omega l^2}{4} - \frac{\omega l^2}{8}$

$\therefore 0 + 2M_a(l) + M_b(l) = \frac{\omega l^3}{4}$ as $M_a = M_b$

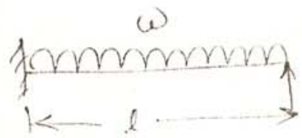
$\therefore 3M_a l = \frac{\omega l^3}{4}$

$\therefore \boxed{M_a = \frac{\omega l^2}{12}}$

$\frac{6a\bar{x}}{l} = \frac{6 \times \frac{2}{3} \times l \times \frac{\omega l^2}{8} \times \frac{l}{2}}{l}$

$= \frac{4}{8} \quad \frac{\omega l^3}{4}$.

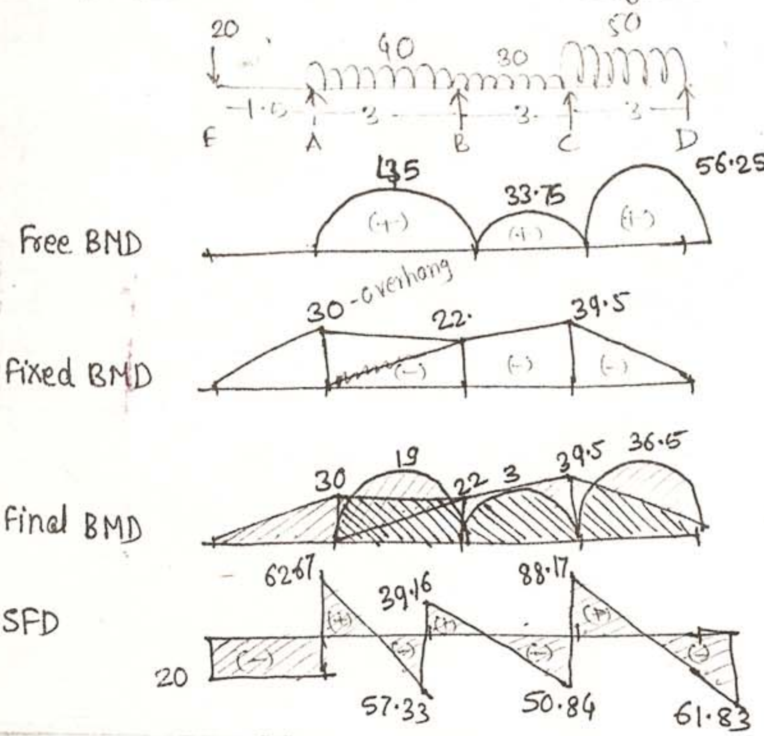
* Propped cantilever :-

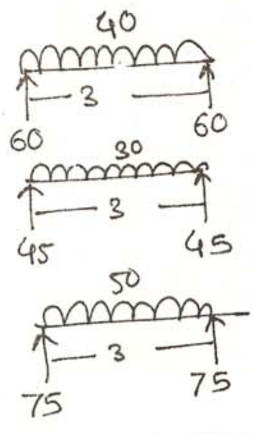


$\therefore M \cdot 0 + 2M_a(l+0) + 0 = \frac{\omega l^3}{4}$

$\boxed{M_a = \frac{\omega l^2}{8}}$

* continuous beam with overhang :-





, Mmax = 45

Mmax = 33.75

Mmax = 56.25

step I $\boxed{\text{Free BM}}$

Step II) Fixed BM.

span ~~EA~~ ABC : Apply Clapeyron's theorem of 3 moments.

$$\therefore M_a \times 3 + 2M_b \times 6 + M_c \times 3 = \frac{6 \times \frac{2}{3} \times 3 \times 45 \times \frac{3}{2}}{3} + \frac{6 \times \frac{2}{3} \times 3 \times 33.75 \times \frac{3}{2}}{3}$$

But $M_a = 20 \times 1.5 = 30$ kNm.

$$\therefore 30 \times 3 + 12M_b + 3M_c = 472.5$$

$$\therefore \boxed{12M_b + 3M_c = 382.5} \text{ — (a)}$$

Span BCD

$$3M_b + 2M_c(6) = \frac{6 \times \frac{2}{3} \times 3 \times 33.75 \times 1.5}{3} + \frac{6 \times \frac{2}{3} \times 3 \times 56.25 \times \frac{3}{2}}{3}$$

$$\therefore \boxed{3M_b + 12M_c = 540} \text{ — (b)}$$

$\therefore M_b = 22$ kNm

$M_c = 39.5$ kNm.

Step III) $\therefore$ Final BM :

BM at A = 30

BM at midspan of AB $= -\left(\frac{30+22}{2}\right) + 45 = \cancel{34}\ +19$ kNm

BM at B = −22 kNm

BM at midspan of BC $= -\left(\frac{22+39.5}{2}\right) + 33.75 = +3$ kNm

BM at C = −39.5 kNm

BM at midspan of CD $= -\left(\frac{0+39.5}{2}\right) + 56.25 = +36.5$ kNm.

BM at D = 0

Step IV) Reactions :

$\Sigma M @ B = -22$ ... (LHS)

$\therefore -20 \times 4.5 + \cancel{20} - 40 \times \frac{3^2}{2} + R_A \times 3 = -22 \quad \Rightarrow \quad \boxed{R_A = 82.67 \text{ kN}}$

$\Sigma M @ C = -39.5$ ... (LHS)

$\therefore -20 \times 7.5 + 82.67 \times 6 - 40 \times 3 (4.5) + 3R_B - 30 \times 3 \times 1.5 = -39.5$

$\Rightarrow \boxed{R_B = 96.49 \text{ kN}}$

$\Sigma M @ C = -39.5$ ... (RHS)

$\therefore +3R_D - 50 \times 3 \times 1.5 = -39.5 \quad \Rightarrow \boxed{R_D = 61.83 \text{ kN}}$

$\Sigma F_y = 0$

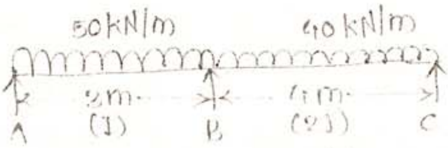
$\therefore 82.67 + 96.49 + 61.83 + R_C = 20 + 40 \times 3 + 30 \times 3 + 50 \times 3$

$\therefore \boxed{R_C = 139.01 \text{ kN}}$

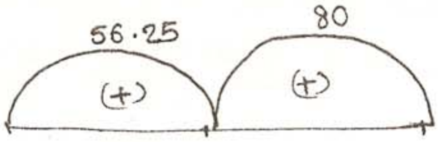
Draw SFD.

Continuous beam with different MI for diff. span

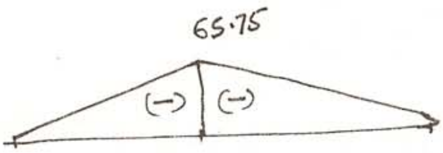
Loading Diagram (Given)



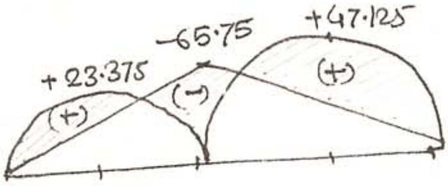
Free BMD (kNm)



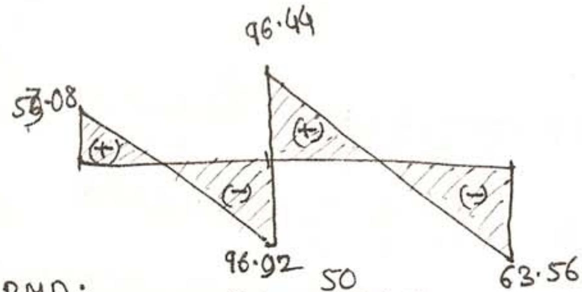
Fixed BMD (kNm)



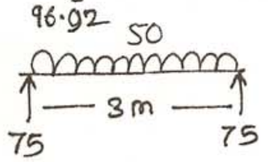
Final BMD (kNm)

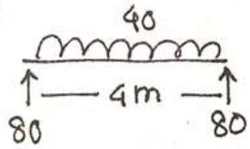


SFD (kN)



step I] Free BMD:





$$BM_{max} = 75 \times 1.5 - 50 \times \frac{1.5^2}{2} = 56.25$$

$$BM_{max} = 80 \text{ kNm}$$

step II] Fixed BMD:

Applying Clapeyron's Three moment theorem for span ABC,

$$\therefore M_a\left(\frac{l_1}{I_1}\right) + 2M_b\left(\frac{l_1}{I_1} + \frac{l_2}{I_2}\right) + M_c\left(\frac{l_2}{I_2}\right) = \frac{6a_1\bar{x}_1}{l_1 I_1} + \frac{6a_2\bar{x}_2}{l_1 I_2}$$

$$\therefore 0 + 2M_b\left(\frac{3}{I} + \frac{4}{2I}\right) + 0 = \frac{6 \times \frac{2}{3} \times 3 \times 56.25 \times 1.5}{3 \times I} + \frac{6 \times \frac{2}{3} \times 4 \times 80 \times 2}{4 \times 2I}$$

$$\therefore 10M_b = 657.5$$

$$\therefore M_b = 65.75 \text{ kNm}.$$

Step III) Final BM:

BM at $A = 0$

BM at midspan of $AB = -\left(\frac{0+65.75}{2}\right) + 56.25 = 23.375$ kNm

BM at $B = 65.75$ kNm

BM at midspan of $BC = -\left(\frac{65.75+0}{2}\right) + 80 = 47.125$ kNm

BM at $D = 0$.

$\therefore$ Draw BMD.

Step IV) Reactions:-

$\Sigma M@B = -65.75$ (LHS)

$\therefore 3R_A - 50\times\frac{3^2}{2} = -65.75$

$\therefore R_A = 53.08$ kN.

$\Sigma M@B = -65.75$ (RHS)

$\therefore 4R_C - 40\times\frac{4^2}{2} = -65.75$

$\therefore R_C = 63.563$ kN

$\Sigma fy = 0$

$\therefore 53.08 + 63.565 + R_B = 50\times3 + 40\times4$

$\therefore R_B = 193.36$ kN.

Draw SFD.