Theory of Structures

Notes by-

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Analysis of Redundant Frames

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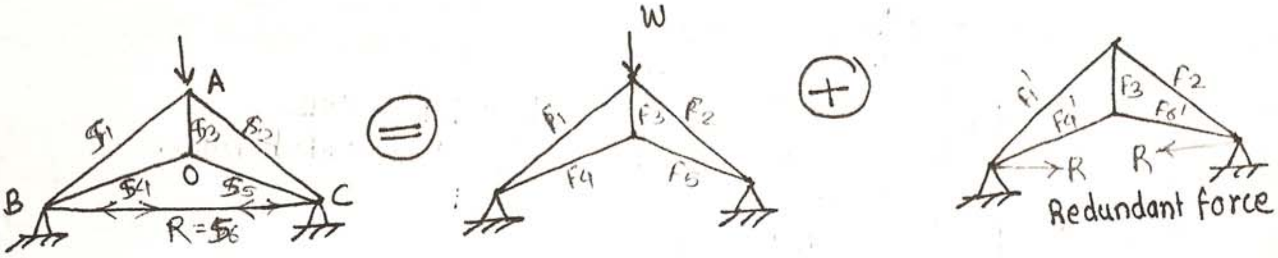
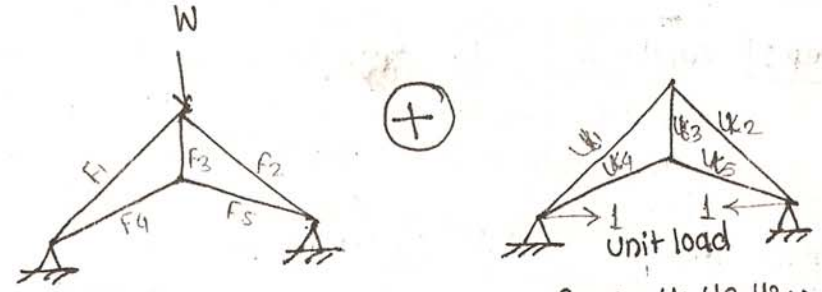
(C)

Consider a statically indeterminate frame having $D_{si} = 1$, carring external load W; tending to induce a internal forces $S_1, S_2, S_3, \dots S_6$ (Ignoring SW). Let F_6 be the Redundant force (R)



i.e. $S_1 = F_1 + F_1'$
$S_2 = F_2 + F_2'$
$S_3 = F_3 + F_3'$
$\vdots$



Force $u_1, u_2, u_3 \dots$

i.e. $S_1 = F_1 + R u_1$
$S_2 = F_2 + R u_2$
$S_3 = F_3 + R u_3$
$\vdots$
so on...

Total strain energy stored by frame $= W_i = \sum \frac{S^2 \cdot l}{2AE} = \sum [F + uR]^2 \cdot \frac{l}{2AE}$

$\therefore$ R is given by the condition that, ~~$= \sum [F_1 + R u_1$~~
stored strain energy is min.

$$\therefore \frac{\partial W_i}{\partial R} = 0$$

$$\therefore \frac{\partial W_i}{\partial R} = \sum \left[\frac{2(F + uR) l}{2AE} \frac{\partial}{\partial R}(F + uR) \right]$$

$$= \sum \left(\frac{F + uR}{AE} l \cdot u \right) = 0$$

$$\therefore \sum \frac{F \cdot u \, l}{AE} + \sum \frac{u^2 R \cdot l}{AE} = 0 \Rightarrow \boxed{R = \frac{-\sum \frac{F \cdot u \cdot l}{AE}}{\sum \frac{u^2 l}{AE}}} \; \boxed{= \frac{-\sum \frac{F \cdot l}{AE}}{\sum \frac{u l}{AE}}}$$

Same relation can be obtained by .... (J.P. George)

Based of Deflection Criteria i.e. $\delta l = \frac{P.l}{AE}$

as $\delta l = 0$ i.e. $\sum \frac{Pl}{AE} = 0$

as $\Delta = \sum \frac{F \cdot u \cdot l}{AE}$; But $F = F' + Ru$

as Δ = Deflection $= \frac{dy}{dx} = \frac{\partial W_i}{\partial R} = 0$

$$\therefore \; 0 = \sum \frac{(F' + R \cdot u) \cdot u \cdot l}{AE}$$

$$\therefore \; = \sum \frac{F' \cdot u l}{AE} + \sum \frac{R \cdot u^2 l}{AE}$$

$$\boxed{R = - \frac{\sum \frac{F' \cdot l}{AE}}{\sum \frac{u \cdot l}{AE}}}$$

F' = Basic Truss
u = Unit loaded Truss.

steps:-

step I] <u>Analysis of Basic Truss:</u>

Consider any member as Redundant & Analyse the given load system; to calculate internal axial forces. i.e. (F_1', F_2', F_3', ...)

Step II] <u>Analysis of Unit loaded Truss:-</u>

Remove given load system; & apply a pair of "UNIT FORCES" in place of redundant member. & Analyse the given load system; to calculate internal axial forces ... (u_1, u_2, u_3, ...)

step III] ~~R. Final Result~~:- <u>Tabulation of Results</u>

Prepare a table to calculate $\sum \frac{F'l}{AE}$ & $\sum \frac{u \cdot l}{AE}$ as,

Member	F'	u	l	A	E	$\frac{F' \cdot l}{AE}$	$\frac{u \cdot l}{A \cdot E}$
						$\sum \frac{F'l}{AE}$	$\sum \frac{u \cdot l}{AE}$

step IV] <u>Redundant force:</u>

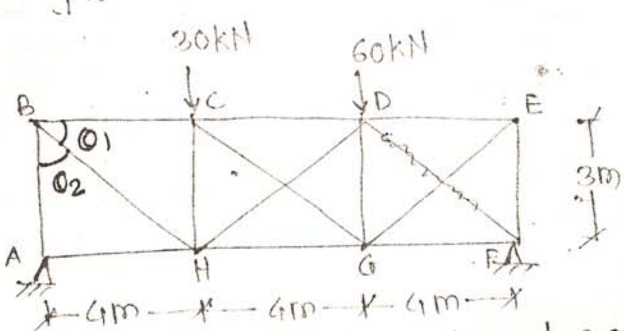
$$R = - \frac{\sum \frac{F'l}{AE}}{\sum \frac{u \cdot l}{AE}}$$

Step V] <u>Final Analysis:-</u>

$$\boxed{F = F' + u \cdot R}$$

Frame 12

Step II] Pro:

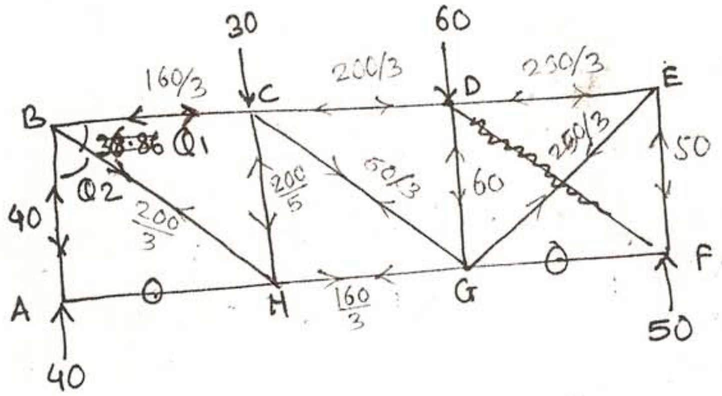


Analyse the ~~frame~~ Truss (Find the forces in the each member of truss

sectional Areas : Horizontal members = $4000\ mm^2$
Vertical members = $3000\ mm^2$
Diagonal members = $5000\ mm^2$

Step I] Analysis of Basic Truss:-

consider member DH as "Redundant"



$\sin\theta_1 = \frac{3}{5}$

$\sin\theta_2 = 4/5$

$\cos\theta_1 = 4/5$

$\cos\theta_2 = 3/5$

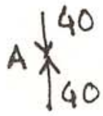
$\therefore \Sigma M_A = 0 \Rightarrow 12 R_B = 30\times4 + 60\times8$

$\therefore R_A = 40\ kN$

$R_B = 50\ kN$ → Very shortcut Powerful mtd.

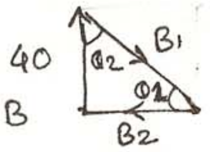
Jt. A] : By Graphical analysis [Polygon Law]

(A)



~~40 / 40 sin θ = 40 / sin(36.86) B 40 cos θ~~

(B)

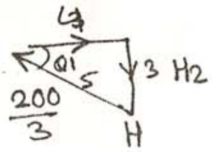


$\therefore \sin\theta_1 = \frac{40}{B_1} \Rightarrow B_1 = \frac{40}{\sin\theta_1} = \frac{40\times5}{3} = \frac{200}{3}$

$\sin\theta_2 = \frac{B_2}{B_1}$; $\cos\theta_1 = \frac{B_2}{B_1} \Rightarrow B_2 = \frac{200}{3}\times\frac{4}{5}$

$= \frac{160}{3}$

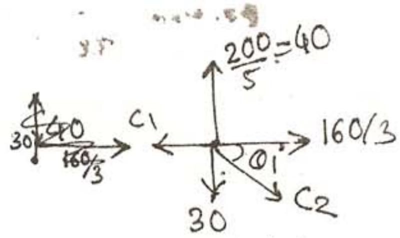
(H)



$\therefore \sin\theta_1 = \frac{H_2}{200/3}$ $\therefore H_2 = \frac{3}{5}\times\frac{200}{3} = \frac{200}{5}$

$\cos\theta_1 = \frac{H_1}{200/3}$ $\therefore H_1 = \frac{4}{5}\times\frac{200}{3} = \frac{160}{3}$

(C)



(Graphical mtd. is "Suitable" for 3 forces)
↳ Not impossible. !

$\Sigma F_y = 0 \uparrow +$

$\therefore \quad 40 - 30 = C_2 \sin\theta_1$

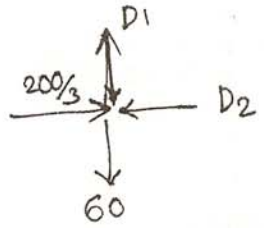
$\therefore C_2 = \frac{10 \times 5}{3} = \frac{50}{3}$

$\Sigma F_x = 0 \xrightarrow{+}$

$\therefore C_1 = \frac{160}{3} + C_2 \cos\theta$

$= \frac{160}{3} + \frac{50}{3} \times \frac{4}{5} = \frac{200}{3}$

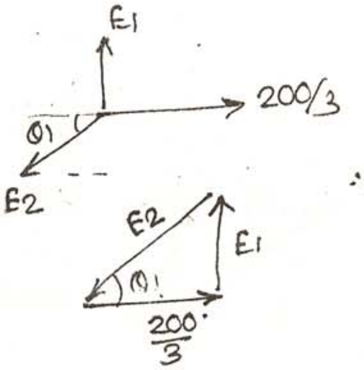
(D)



$\therefore D_1 = 60$

$D_2 = \frac{200}{3}$

(E)



$\Sigma F_x = 0$ (OR)

$\therefore$

$\therefore \sin\theta_1 = \frac{E_1}{E_2}$

$\cos\theta_1 = \frac{200/3}{E_2} \Rightarrow E_2 = \frac{200}{3} \times \frac{5}{4}$

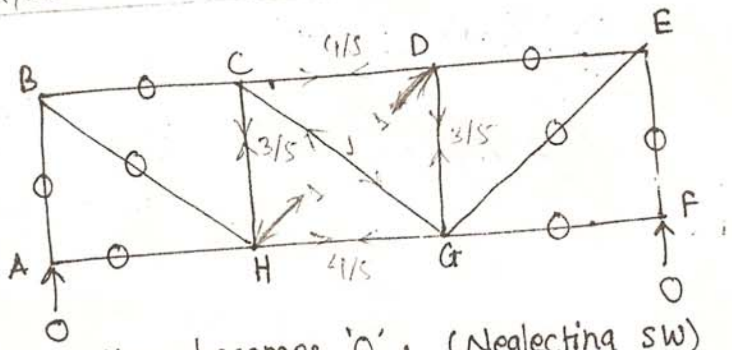
$= \frac{250}{3}$

$\therefore E_1 = E_2 \sin\theta_1 = \frac{250}{3} \times \frac{3}{5} = \frac{250}{5} = 50$

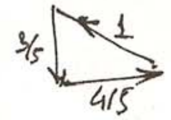
(F)

$\therefore$ O.K.

step II | Analysis of 'unit loaded Truss :-

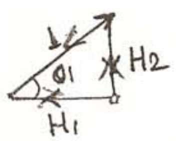


(C)



Note:- As Reactions becomes '0', (Neglecting SW)

(H)



$\cos\theta_1 = \frac{H_1}{1} \quad \therefore H_1 = 4/5$

$\sin\theta_1 = \frac{H_2}{1} \quad H_2 = 3/5$

(D)

frame

step III] Tabulation of Results :-

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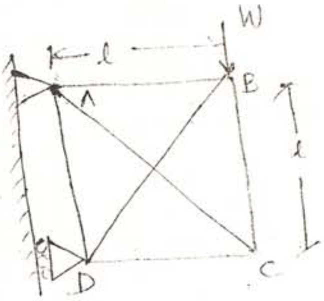
Frame
13

Member	F'	u	$l \times 10^3$ (mm)	$A \times 10^3$	$\frac{F' \cdot u l}{AE}$	$\frac{u^2 \cdot l}{AE}$
AB	40	0	3	3	0	0
BC	160/3	0	4	4	0	0
CD	200/3	−4/5	4	4	−160/3	16/25
DE	200/3	0	4	4	0	0
EF	50	0	3	3	0	0
FG	0	0	4	4	0	0
GH	−160/3	−4/5	4	4	128/3	16/25
HA	0	0	4	4	0	0
BH	−200/3	0	5	5	0	0
CH	200/5	−3/5	3	3	−24	9/25
CG	−50/3	1	4	4	−50/3	1
HD	0 (R)	1	4	4	0	1
DG	60	3/5	3	3	−36	9/25
GF	−250/3	0	5	5	0	0

Pro: 

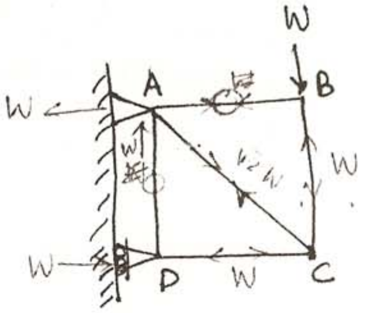
all members have same c/s area & are of same mat.

Analyse the frame

frame 15

step I] Analysis of basic truss :-

Assume BD as Redundant.



$\Sigma M_A = 0.$

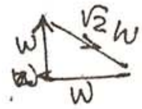
$\therefore Wl = H_D \cdot l$

$\therefore H_D = W.$

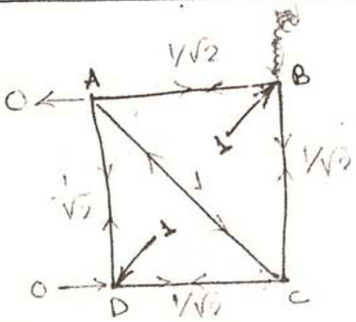
$\therefore H_A = -W.$

$\Sigma f_y = 0$

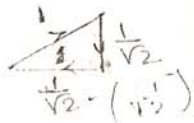
$\therefore R_A = W.$

Ⓐ 

step II] Analysis of unit loaded Truss :-



Ⓐ

Ⓑ 

step III] Tabulation of Results :-

Member	F'	u	l	F'·u·l	u²·l
AB	0	$-1/\sqrt{2}$	l	0	$l/2$
BC	W	$-1/\sqrt{2}$	l	$-Wl/\sqrt{2}$	$l/2$
CD	W	$-1/\sqrt{2}$	l	$-Wl/\sqrt{2}$	$l/2$
DA	0	$-1/\sqrt{2}$	l	0	$l/2$
AC	$-\sqrt{2}W$	1	$l\sqrt{2}$	$-Wl\sqrt{2}$	$l\sqrt{2}$
BD	0	1	$l\sqrt{2}$	0	$l\sqrt{2}$
			$\Sigma =$	$-3.4142\,Wl$	$4.8284\,l$

$$\therefore R = \frac{-3.4142\,Wl}{4.8284\,l}$$

$$\therefore \boxed{R = -0.7071\,W} \Rightarrow \boxed{R = \frac{-W}{\sqrt{2}}}$$

$$R = -\left[\frac{\Sigma F' \cdot u \cdot l}{\Sigma u^2 \cdot l}\right]$$

$$= -\left[-\frac{W}{\sqrt{2}}\right]$$

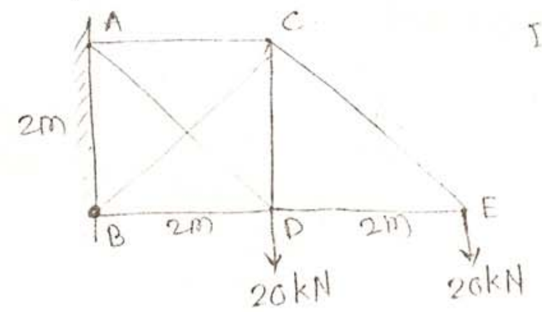
$$= W/\sqrt{2}$$

$\therefore$

Step IV] Final Results :-

Member	F'	U	R	$U \cdot R$	$F = F' + UR$
AB	0	$-1/\sqrt{2}$	$+W/\sqrt{2}$	$-W/2$	$-W/2$ (T)
BC	W	$-1/\sqrt{2}$		$-W/2$	$W/2$ (C)
CD	W	$-1/\sqrt{2}$		$-W/2$	$W/2$ (C)
DA	0	$-1/\sqrt{2}$		$-W/2$	$-W/2$ (T)
AC	$-\sqrt{2}W$	1		$+W/\sqrt{2}$	$-W/\sqrt{2}$ (T)
BD	0	1		$+W/\sqrt{2}$	$W/\sqrt{2}$ (C)

Pro:-



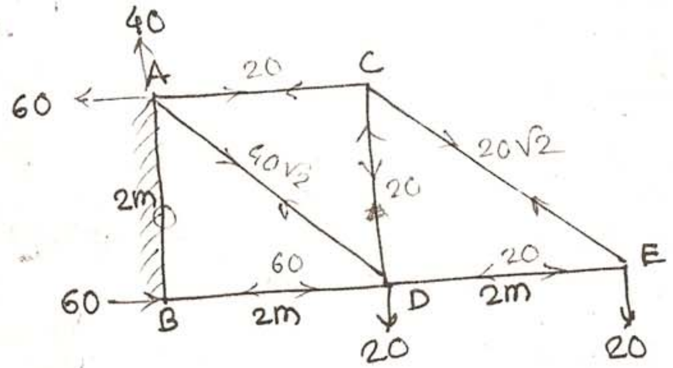
If Area of each member $= 1000\,mm^2$

$E = 2 \times 10^5\ N/mm^2$

Analyse the frame.

step I) Analysis of basic frame :-

Assume BC as Redundant.



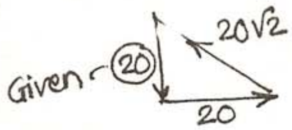
$$\Sigma M_A = 0 \Rightarrow H_B \times 2 = 20 \times 2 + 20 \times 4$$

$$\therefore H_B = 60$$

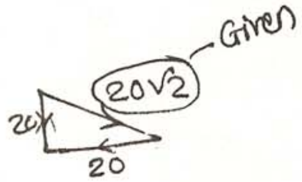
$$H_A = 60$$

$$R_A = 40$$

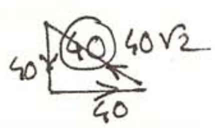
(E)



(C)



(D)



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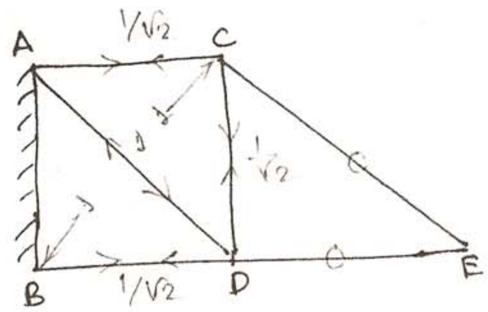
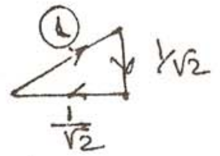
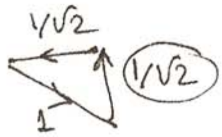
step II] Analysis of unit loaded Truss :-



(C)



(D)



step III] Tabulation of Result :- $CAE = 1000 \times 2 \times 10^5 = 200 \times 10^6$ N) = constant

Member	F'	u	l	$F'u.l$	$u^2.l$
AC	-20	$-1/\sqrt{2}$	2	$480/\sqrt{2}$	1
CE	$-20\sqrt{2}$	0	$2\sqrt{2}$	80	0
ED	20	0	2	0	0
DB	60	$-1/\sqrt{2}$	2	-120 $\cancel{-180}/\sqrt{2}$	+1
CD	20	$-1/\sqrt{2}$	2	$-40/\sqrt{2}$	+1
AD	$-40\sqrt{2}$	1	$2\sqrt{2}$	-1[illegible] $\cancel{-40/\sqrt{2}}$	$2\sqrt{2}$
CB	0	1	$2\sqrt{2}$	0	$2\sqrt{2}$
			$\Sigma =$	$\cancel{-216.57}$ -244.85	8.657

$$\therefore R = -\left[\frac{\Sigma F'ul}{\Sigma u^2 l}\right] = -\left[\frac{-244.85}{8.657}\right] = 28.284$$

step IV] Final Result :-

Member	F'	u	R	$u.R$	$F = F' + u.R$
AC	-20	$-1/\sqrt{2}$			-40 (T)
CE	$-20\sqrt{2}$	0			$-20\sqrt{2}$ (T)
ED	20	0	28.284		20 (C)
DB	60	$-1/\sqrt{2}$			40 (C)
CD	20	$-1/\sqrt{2}$			0
AD	$-40\sqrt{2}$	1			-28.28 (T)
CB	0	1			28.28 (C)

* Degree of Redundancy :- (Degree of indetermancy) Frame 19

Statically Indeterminate structures

- Externally indeterminate (Suppot Reactions) (RA, HA, MA)
- Internally indeterminate. (Internal Reactions) (Axial force)

For beam D_{si} ext $= R - \underline{\underline{2}}$ ($\Sigma Fx = 0$)

For frames, D_{si} ext $= R - 3$

For Multistoried frames :-

First Method

GATE que.

$$\boxed{D_{si}\ ext = (r + 2h + 3f) - 3}$$

Where, r = No. of Roller supports
h = No. of hinged supports
f = No. of fixed supports.

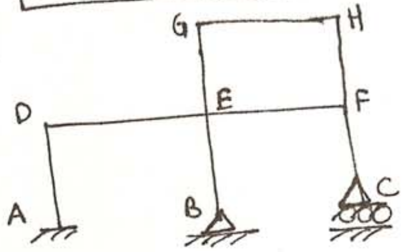
$$\boxed{D_{si}\ int = 3(m - n + 1)}$$

Where, m = Total No. of colm in upper storeys leaving the first story.
n = No. of storeys.

$\therefore$ Total $D_i = D_{si}$ ext $+ D_{si}$ int

$$\boxed{D_i\ total = r + 2h + 3f + 3(m - n)}$$

eg:-



Here, r = 1
h = 1
f = 1
m = 2
n = 2

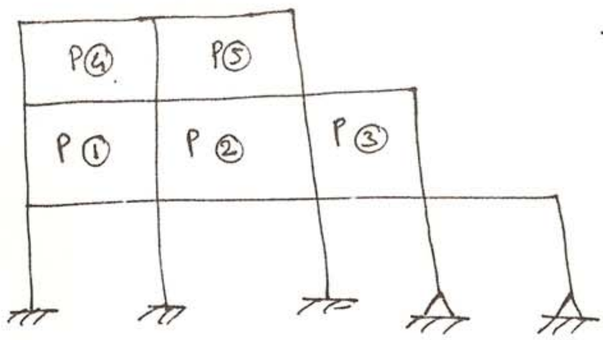
$\therefore$ D_i total $= 1 + 2\times1 + 3\times1 + 3(2-2)$
$= \underline{\underline{6}}$

Second Method :-

$$\boxed{D_{si}\ ext = (r + 2h + 3f) - 3}$$

$$\boxed{D_{si}\ int = 3 \times P}$$ where, P = No. of bays in uppor storey.

For eg :-



$\therefore$ r = 0
h = 2
f = 3
p = 5

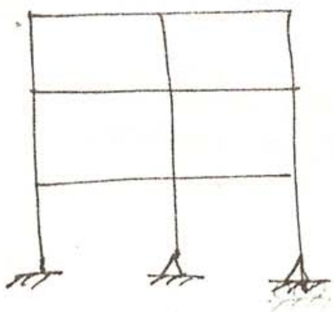
$\therefore$ ~~Dsi~~

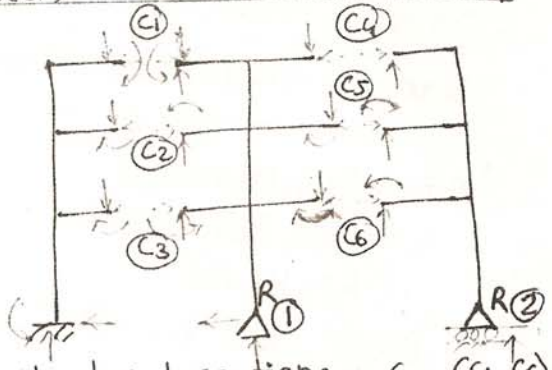
D_i total $= 0 + 2\times2 + 3\times3 - 3 + 3\times5$
$= \underline{\underline{25}}$

Third method:

$$\boxed{Di\ total = 3 \times \text{No. of cut sections} - \text{No. of release at support}}$$
(cut horizontal sectⁿ)

eg:-





∴ No. of cut sections = 6 (C1, C6)

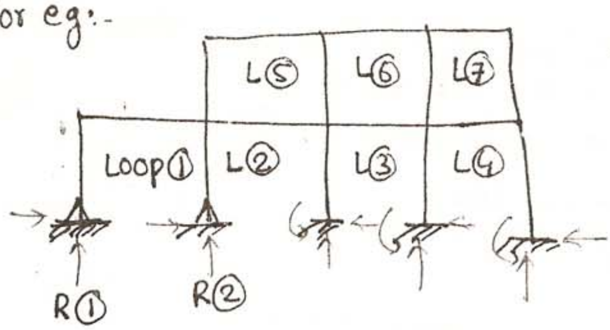
No. of Release at support = $R_1 + 2R_2$ = ③

$\therefore Di\ total = 3 \times 6 - 3 = 15.$

Fourth Method:-

$$\boxed{Di\ total = 3 \times \text{No. of loop} - \text{No. of release at supports}}$$

For eg:-



∴ No. of loop = 7

No. of release = 2

∴ $Di\ total = 3 \times 7 - 2$
$= \underline{\underline{19}}$ ✓