Theory of Structures

Notes by-

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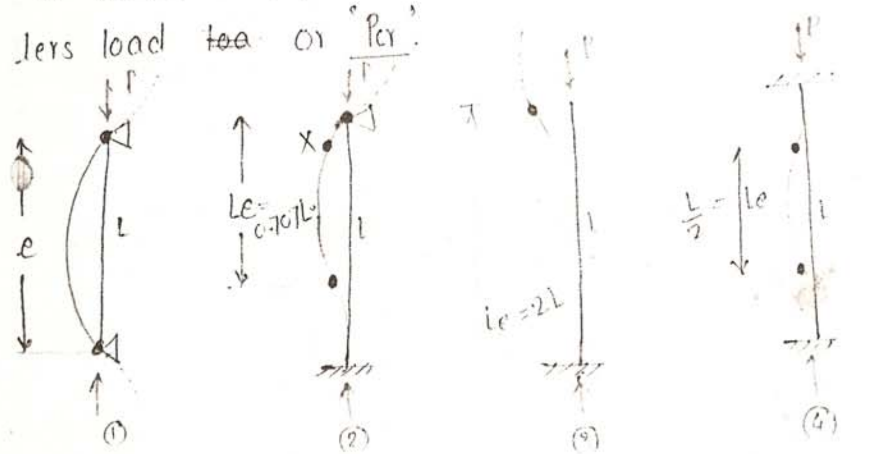
Axially loaded column 23/11/04 / Pa (1)

Stability of a column is considerably affected by the buckling behaviour the col^m which in turn depends on magnitude of load, length of col^m, matl. of col^m, gemetry (c/s) & end conditions.

Eulers buckling theory forms the basic foundation for the numerious theories that came in the last 250 years.

from the governing deflc diff eq^n $EI\frac{d^2y}{dx^2} = M$, the elastic curve a buckeled column can be traced. The buckling load is known as Eulers load ~~for~~ or 'Pcr'.



P_{cr} = Buckling load $= \frac{\pi^2 EI}{(Le)^2}$

Le = effective length (depends on end conditions)

$P_{cr} = \frac{\pi^2 EI}{l^2}$ ...... Pinned at ends (1)

Effective length (Le): It is the distance bet^n two successive points of inflection on the elastic curve.

Limitations of Eulers Theory

$P_{cr} = \frac{\pi^2 E (A k^2)}{Le^2} = \frac{\pi^2 E A^2}{(le/k)^2}$ $\frac{\pi^2 E A^2}{\lambda^2} \rightarrow \frac{P_{cr}}{A} = \frac{\pi^2 E}{\lambda^2}$

where, $\lambda = \frac{le}{k}$ = slenderness ratio.

Determine the slenderness Ratio beyond which Eulers formula is applied for a steel column. $\sigma_y = 250$ MPa, E = 200 GPa

$\frac{P_{cr}}{A} = \frac{\pi^2 E}{\lambda^2} = \sigma_y$

$\therefore \lambda^2 = \frac{\pi^2 E}{\sigma_y} = \frac{\pi^2 \times 200 \times 10^3}{250}$

$\therefore \lambda = 88.86$

$\lambda > 89$

MPSC
10M

The c/s shape of an axially loaded colᵐ could be –

1) Hollow circular (@ 90% inner dia times outer dia)

2) Solid circular ~~square~~. 3) Square

4) Rectangle (Ratio = 2)

If the area of c/s is same for any shape compare the ratio of the buckling loads. Assume case ④ to be unity.

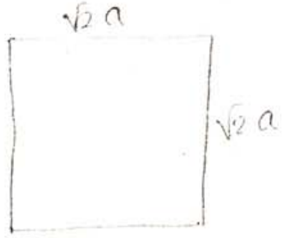
Solⁿ :- Case 4]

$I = \frac{2a \cdot a^3}{12} = \frac{a^4}{6}$

$A = 2a^2$

case 3] Square :-

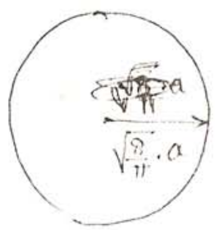
$A = 2a^2$ Rect



$I = \frac{\sqrt{2} \cdot a (\sqrt{2} \cdot a)^3}{12} = \frac{a^4}{3}$

case 2] Solid circular :-

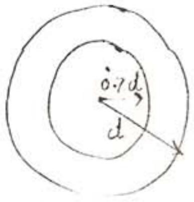
$A = 2a^2 = \frac{\pi}{4} \cdot d^2 \Rightarrow d = \sqrt{\frac{8}{\pi}} \cdot a$



$I = \frac{\pi}{64} \cdot d^4 = \frac{\pi}{64} \left[\sqrt{\frac{8}{\pi}} \cdot a \right]^4 = \frac{\pi}{64} \left(\frac{64}{\pi^2} \right) \cdot a^4 = \frac{a^4}{\pi}$

Case 4] Hollow circular :-

$$A = 2a^2 = \frac{\pi}{4}(d^2 - 0.49d^2) = \frac{\pi d^2}{4}(0.51)$$



$$\Rightarrow \frac{8a^2}{\pi \times 0.51} = d^2$$

$$\therefore \boxed{d = 2.234\,a}$$

$$\therefore I = \frac{\pi}{64}\left[(2.234a)^4 - (0.7 \times 2.234a)^4\right]$$

$$= 0.929\,($$

$$= \frac{\pi}{64}a^4\left[18.927\right]$$

$$= 0.929\,a^4$$

$$= \frac{a^4}{1.0763}$$

$\therefore$ Ratio will be, $I_{rect} : I_{sq} : I_{cir} : I_{Hcir}$

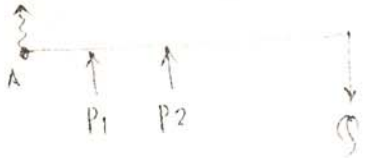
$$= \frac{a^4}{6} : \frac{a^4}{3} : \frac{a^4}{\pi} : \frac{a^4}{1.0763}$$

$$\therefore \left[1 : 2 : \frac{6}{\pi} : \frac{6}{1.0763}\right]$$

Pro :-

At what load Q will the column fail in buckling.

Sol :- FBD of the beam is,



$\Sigma M_A = 0$

$P_1 \times a + P_2 \times 2a = 4Q \times 2a$

$$\boxed{\therefore P_1 + 2P_2 = 4Q}$$

But $P_1 = P_2 = \cancel{2}\,\dfrac{\pi^2 EI}{l^2}$ ... as both col^{m} are fail simultaneously

$P_1(EI) = \cancel{EI}\ P_2(EI)$

$$\therefore \frac{\pi^2 EI}{l^2} + \frac{2\pi^2 EI}{l^2} = 4Q$$

$$\therefore \boxed{Q = \frac{3}{4}\,\frac{\pi^2 EI}{l^2}}$$

Pro:-