Theory of Structures

Notes by-

Pravin S Kolhe,

BE(Civil), Gold Medal, MTech (IIT-K)

Assistant Executive Engineer,

Water Resources Department,

www.pravinkolhe.com

structural Analysis.

Theory of Bending
[Beams / frames / column* ⇒
(*subjected to eccentric loads)]

Hookes law
[trusses : axial loads]

Arches, suspension cables.

Analysis effectively means to trace deformation response. (DR)
The mmt & SF is only force Response. (FR) → force. conc. force, mmt.

quence :-

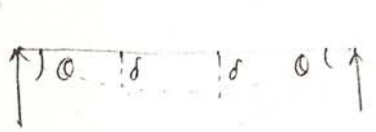
DR $\begin{cases} y - \text{deformation} \\ \theta = \dfrac{dy}{dx} = \text{slope} \end{cases}$

FR $\begin{cases} M = \dfrac{d^2y}{dx^2} \cdot EI = \text{Moment} \\ V = \dfrac{d^3y}{dx^3} \cdot EI = \text{Shear} \end{cases}$

The calculatⁿ of DR & FR combined is Analysis.

Methods of Analysis :-
Any mtd. of analysis in the world is either Stiffness (Displacement) mtds. or Flexibility (Force) Methods.

stiffness / Displacement mtd :
Holding displacement of str. as unknown in the analysis



δ & θ are unknown displacements.
by some technique, you find unknown
mtd. is disp. mtd.

Deformatⁿ is Reality

Force Mtd / Flexibility mtds.
→ forces are unknown.

R & M are unknown.

BM & SF are concepts

This is superior & widely used & modern pacakages are based on stiffness mtd.

Frame is assembly of beams & col^m connected by rigid Joints.

(Welded Joint)

Angle is maintained in case of rigid Joint.

⇝ Angular displacement takes place.

Pinned Joint

As we are not considering axial deformation, $\Delta = \Delta$

At every beam there are 4 known conditions

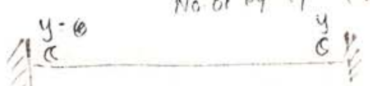
& we have,

$$EI \frac{d^4 y}{dx^4} = \omega.$$

[known quantities at free ends.]

static in = 0

static in = 3 - 2 = 1 (No. of eq^m eq^n) Not allowed. ($\Sigma F_x = 0$)



static in = 4 - 2 = 2

static indetermancy :- (IR)

static indetermancy = 2 - 2 = 0

= No. of unknown − No. of eq^m eq^n

Unknown displacements:-

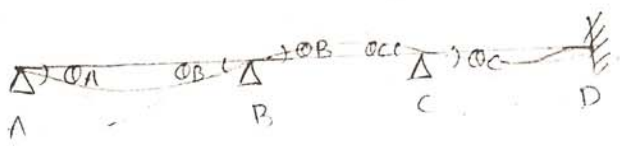
Rotat^n are unknown at both end i.e. θA & θB ...
(2)

Deflect^n is unknown.
(1)

slope is unknown
(1)

Nothing is unknown at boundaries.
(0)

The choice of any particular method for analysis depends upon number unknown displacements & the No. of unknown forces that the structure as.



Unknown disp = 3.
(unknowns: θA, θB, θC i.e. 3.)
~~No. o'~~
Forces : 3.
Any mtd

stiffness/Flexibility both can work.

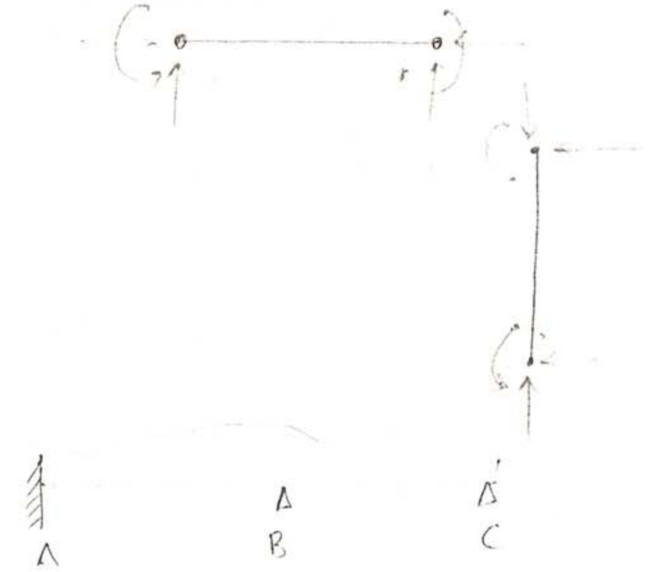
Disp: 2
Forces: 4
= (6-2)

: Displacement mtd is preferred.

Disp.= 4
Forces = 2

Force mtd. is preferred.

In a frame, the beam & column elements will be subjected to 3 end forces [in FBD, in general.] (Torsion Neglected)



Internal Hinge

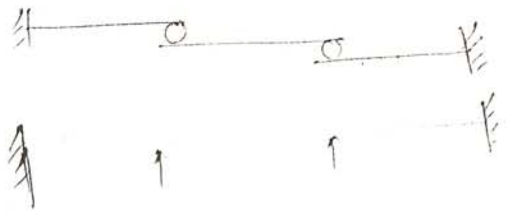
FR → M = 0
DR ⇒ slope discontinuity at that point.

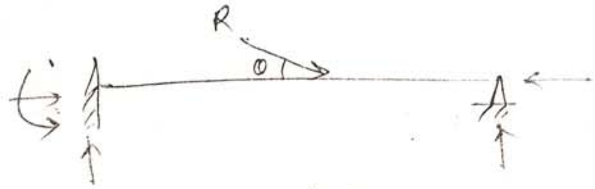
There is only SF

FBD.

... cancelled or balanced moments at hinge.

⇑ = ↑ ... addition of forces from left & Right part of struct

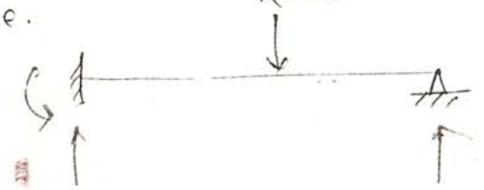




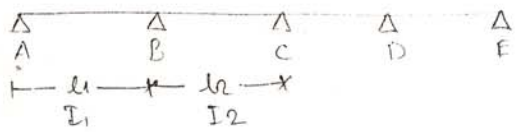
In such case also, horizontal forces are not considered as they did not affect vertical force. i.e. in SFD & BMD.

They affects only in case of Axial force diagram.

i.e.



Neglect Rcosθ.



Unknown React^n = 5 - 2 = 3
Unknow Disp = 5
∴ Use FB

1) TME is best mtd. amoung Force Mtd.
2) SD is best mtd. amoung Disp. mtd.

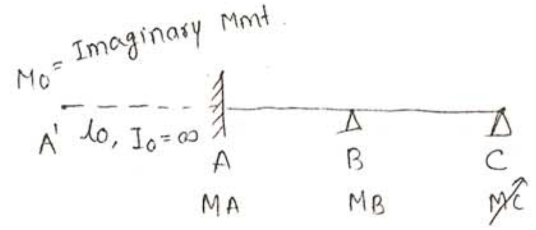
Three Moment Eq^n :

$$M_A\left(\frac{l_1}{I_1}\right) + 2M_B\left(\frac{l_1}{I_1}+\frac{l_2}{I_2}\right) + M_C\left(\frac{l_2}{I_2}\right) = -\frac{6A_1a_1}{l_1 I_1} - \frac{6A_2a_2}{l_2 I_2} + \underbrace{\frac{6Eh_A}{l_1} + \frac{6Eh_C}{l_2}}_{\text{When the supports are yielding}}$$

Where A_1 & A_2 :- Simply supported BMD area.
a_1, a_2 :- Dist. of C.G. of area from extremes.

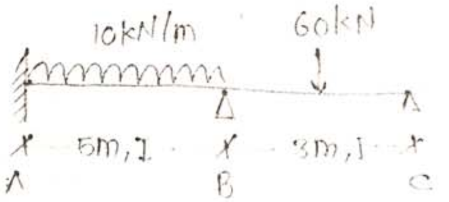
Special case:-

eg:-

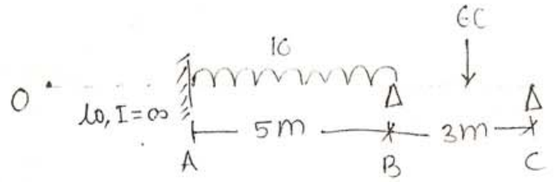


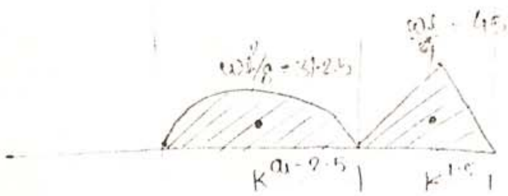
∴ We will get 2 eq^{n}s. considering span A'AB & ABC

Pro:-



Sol^n:- Introduce an imaginary span so that we have 2 eq^{ns}.





$A_1 = \frac{2}{3} \times$ Base $\times$ Ht
$= \frac{2}{3} \times 5 \times 31.25$
$= 104.2$

$A_2 = \frac{1}{2} \times 3 \times 45$
$= 67.5$

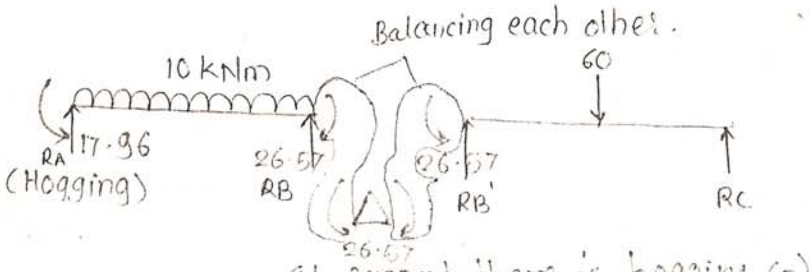
$$M_0\left(\frac{l_0}{\infty}\right) + 2M_A\left(\frac{l_0}{\infty}+\frac{5}{I}\right) + M_B\left(\frac{5}{I}\right) = -6(0) - \frac{6(104.2)\times 2.5}{I(5)}$$

$$\Rightarrow 10M_A + 5M_B = -312.5 \quad \cdots \quad (a)$$

$\therefore M_A(5) + 2M_B(5+3) + \cancel{M_C}(3) = -\frac{6(104.2)\times 2.5}{5} - \frac{6\times 67.5\times 1.5}{3}$

$\therefore 5M_A + 16M_B = -515 \quad \ldots (b)$

$\therefore \boxed{\begin{array}{l} M_A = -17.96 \text{ kNm} \\ M_B = -26.57 \text{ kNm} \end{array}}$



Reactions: $R_A + R_B' = 10\times 5$

$5R_B' + 26.57 - 17.96 + 10\times 5^2/2 = 0$

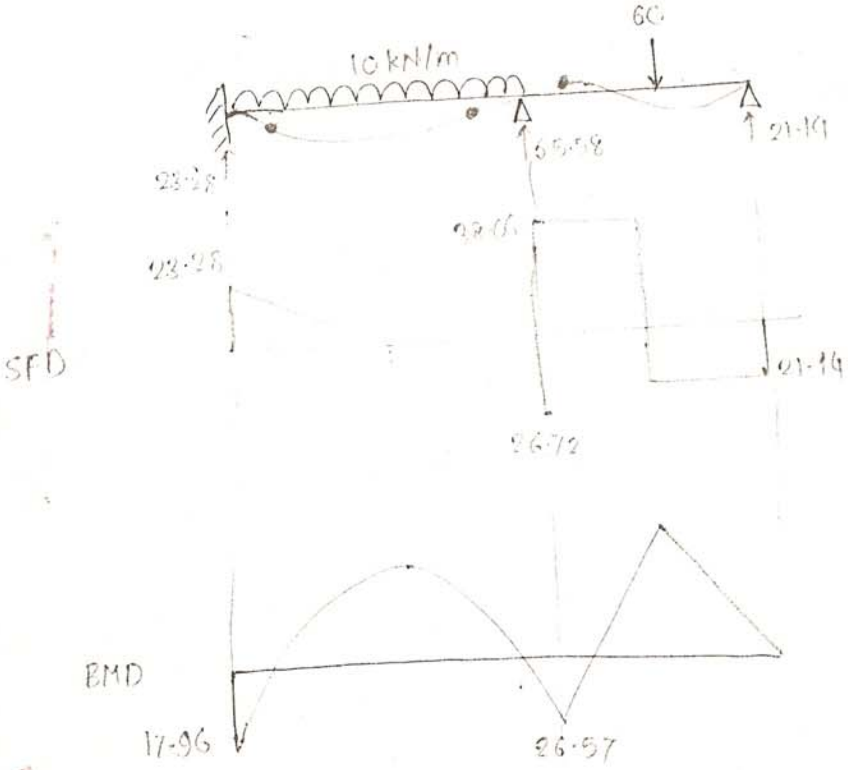
$\boxed{\begin{array}{l} R_B' = 26.72 \text{ kN} \\ R_A = 23.28 \text{ kN} \end{array}}$

$R_B'' + R_C = 60$

$\therefore -3R_C + 60\times 1.5 - 26.57 = 0$

$\therefore \boxed{\begin{array}{l} R_C = 21.14 \text{ kN} \\ R_B'' = 38.86 \text{ kN} \end{array}}$

$R_B = R_B' + R_B'' = 65.58 \text{ kN}$



Eq^n of BM:

$M = -17.96 + 23.28x - 10x^2/2$

$\therefore 5x^2 - 23.28x + 17.96 = 0$

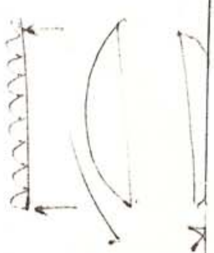
$\therefore x = 0.97$

$= 3.67$

$\therefore$ BMmax at $x = \frac{0.97 + 3.67}{2}$

$= 2.32$

Neglect ...



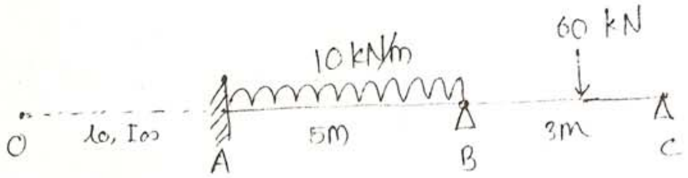
BMD should be drawn on Compression side.

Avoid superposition of BMD as they did not gives point of controflexure & position of BMmax.

Sinking of support:

26/8 (2)

In the previous problem if the support at B, sinks by 10 mm & EI = 8000 kNm². Set up the TME's.



$$M_A\left(\frac{l_1}{I_1}\right) + 2M_B\left(\frac{l_1}{I_1}+\frac{l_2}{I_2}\right) + M_C\left(\frac{l_2}{I_2}\right) = \frac{-6A_1\bar{a}_1}{l_1 I_1} - \frac{6A_2\bar{a}_2}{l_2 I_2} + \frac{6Eh_A}{l_1} + \frac{6Eh_C}{l_2}$$

If support is above from central, take +ve.

& in this case:

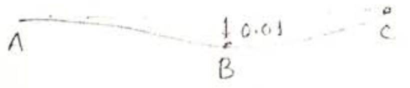
$\therefore h_B = -0.01$

Span OAB

$$\therefore 0 + 2M_A\left(0+\frac{5}{I}\right) + M_B\left(\frac{5}{I}\right) = 0 - \frac{6(104.2)\times 2.5}{5I}$$

$$\Rightarrow 10M_A + 5M_B = -312.5 - 0.012\,\boxed{EI = 8000}$$
$$= -312.5 - 96$$
$$= -408.5$$

Considering span ABC



A & C are above central support $\therefore$ h is +ve.

$$\therefore M_A\left(\frac{5}{I}\right) + 2M_B\left(\frac{5}{I}+\frac{3}{I}\right) + M_C\left(\frac{3}{I}\right) = -\frac{6(104.2)\times 2.5}{5I} - \frac{6(67.5)\times 1.5}{3I} + \frac{6E(0.01)}{5} + \frac{6E(0.01)}{3}$$

$$\Rightarrow 5M_A + 16M_B = -515 + 0.032EI$$
$$= -515 + 256$$
$$= -259.$$

$$\boxed{\begin{aligned} M_A &= -38.82\ \text{kNm} \\ M_B &= -4.05\ \text{kNm} \end{aligned}}$$

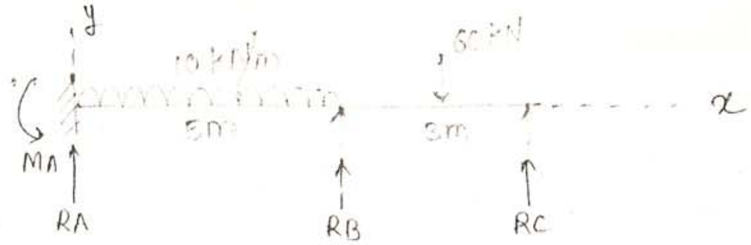
-102
-96 =

97 +
2
2.32

Tension side

Give point of contraflexure & position of BMmax.

Modified Double Integration Method:



$$EIy = 0 + 0 - MA\frac{x^2}{2!} + RA\frac{x^3}{3!} - 10\frac{x^4}{4!} - (-10)\frac{\langle x-5\rangle^4}{4!} + RB\frac{\langle x-5\rangle^3}{3!} - 60\frac{\langle x-6.5\rangle^3}{3!}$$

at $y = 0,\ x = 5$ — (a)
$y = 0,\ x = 8$ — (b)
$EI\frac{d^2y}{dx^2} = 0,\ x = 8$ — (c)

$\therefore$

$0 = -MA\cdot(12.5) + RA\,(28.83) - \cancel{2646}\ 260.4$ — (a)

$0 = -MA\,(32.0) + RA\,(85.33) + 33.75 + 4.5\,RB - \cancel{1.25}\ 33.75 - 1706.66$

$0 = -32.\,MA + 85.33\,RA - \cancel{1674.16}\ 1706. + RB\times 4.5$ — (b)

$0 = -MA + RA\,(8) - 10\times\frac{8^2}{2!} + \frac{10(3)^2}{2!} + RB\,(3) - 60(1.5)$

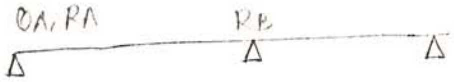
$0 = -MA + 8\,RA - 365 + 3\,RB$ — (c).

$-12.5\,MA + 28.83\,RA + 0\ RB = 260.4$ — (a)
$-32\,MA + 85.33\,RA + 4.5\,RB = 1706.66$ — (b)
$-MA + 8\,RA + 3\,RB = 365$ — (c)

$\therefore MA = -17.96$
$RA = 23.28$
$RB = 65.58$

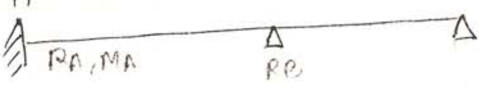
Unknowns:

$y=0,\ x=5$
$y=0,\ x=8$
$EI\frac{d^2y}{dx^2}=0,\ x=8$

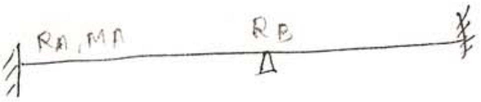


— If support sinks, take $y = 0.01$, at $x = 5$

$y=0,\ x=5$
$y=0,\ x=8$
$EI\frac{d^2y}{dx^2}=0,\ x=8$



$y=0,\ x=5$
$y=0,\ x=8$
$\frac{dy}{dx}=0,\ x=8.$



TMF	SD	MIP*
1	3	3
2	2	3
3	1	3

Independent on Unknowns. Depend on No. of spans.