Theory of Structures

Notes by-

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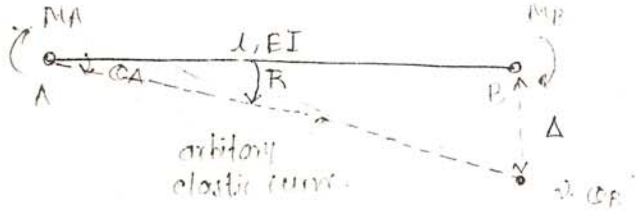
slope deflection eqⁿ :-

Sign conventions: All clockwise mmts & Rotations are positive. (↻ +)



$$\boxed{R = \frac{\Delta}{L}}$$

Generalised Fig.

MA & MB are the end moments of an unloaded beam element. When its 2 ends rotate by θ_A & θ_B and the element itself rotates by R the end moments & rotations are related through the eqⁿ,

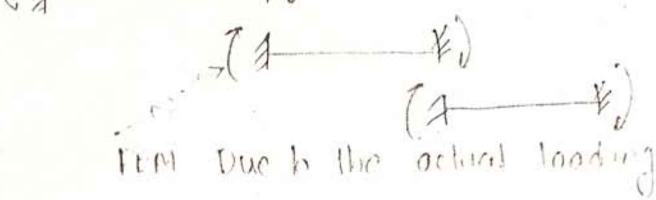
$$M_A = \frac{4EI\theta_A}{L} + \frac{2EI\theta_B}{L} - \frac{6EI\Delta}{L^2}$$

$$M_B = \frac{2EI\theta_A}{L} + \frac{4EI\theta_B}{L} - \frac{6EI\Delta}{L^2}$$

The slope deflection eqⁿ are derived for a loaded beam element by adding these end ~~rotations~~ moments to the fixed end moments/ locking mmt.

First lock all the three spans.

MA = Final mmt



FEM Due to the actual loading

$$M_A = (M_A)_0 + \frac{2EI}{L}\left[2\theta_A + \theta_B - 3R\right]$$

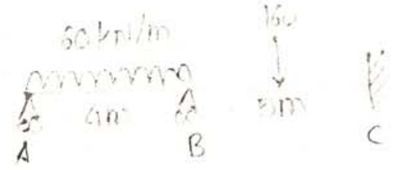
$$M_B = (M_B)_0 + \frac{2EI}{L}\left[\theta_A + 2\theta_B - 3R\right]$$

$R = \frac{\Delta}{L}$

R is positive if clockwise.

$$\boxed{\begin{aligned} M_{BA} + M_{BC} &= 0 \\ M_{CB} + M_{CD} &= 0 \\ M_{DC} &= 0 \end{aligned}}$$

Pro: Analyse and plot BMD.



$(\overline{M_{AB}})_0 = \frac{wl^2}{12} = \frac{60\times 4^2}{12} = -80$ kNm $M_{BA} = +80$ kNm

$(\overline{M_{BC}})_0 = wab^2/l^2 = -100$ $\overline{M_{CB}} = +100$ kNm

80 80 100 100

SD eq^n for AB,

$M_{AB} = -80 + \frac{2EI}{4}(2\theta A + \theta B) = -80 + \frac{4}{4}EI\theta A + \frac{2}{4}EI\theta B$

$M_{BA} = +80 + \frac{2EI}{4}(\theta A + 2\theta B) = 80 + 0.5EI\theta A + EI\theta B$

$M_{BC} = -100 + \frac{2EI}{5}(2\theta B + \theta C) = -100 + 0.8EI\theta B$ ($\theta C = 0$)

$M_{CB} = +100 + \frac{2EI}{5}(\theta B + 2\theta C) = +100 + 0.4EI\theta B$.

& $M_{BA} + M_{BC} = 0$

$\therefore -20 + \frac{2EI}{4}(\theta A + 2\theta B) + \frac{2EI}{5}(2\theta B) = 0$

$\therefore -20 + 0.5EI\theta A + EI\theta B + 0.8EI\theta B = 0$

$\therefore -20 + 0.5EI\theta A + 1.8EI\theta B = 0$.

$EI\theta A + 0.5EI\theta B = 80$

$0.5EI\theta A + 1.8EI\theta B = 20$

$\therefore \theta A = \frac{86.45}{EI}$

$\theta B = \frac{-12.9}{EI}$

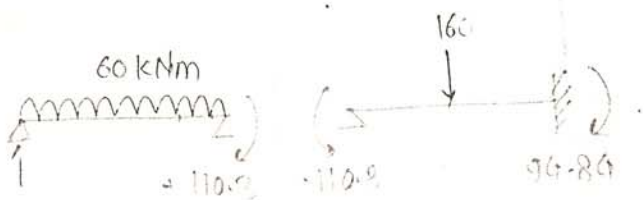
$\therefore M_{AB} = 0$

$M_{BA} = 110.32$

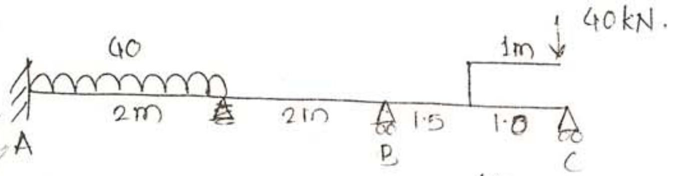
$M_{BC} = -110.32$

$M_{CB} = +99.84$.

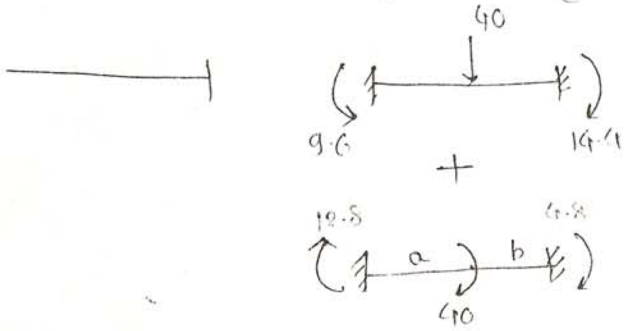
BMD (Nature).



MPSC Pro



Moment on a Beam



$\overline{M_{BC_1}} = \frac{40 \times 1.5 \times 1^2}{2.5^2} = 9.6$

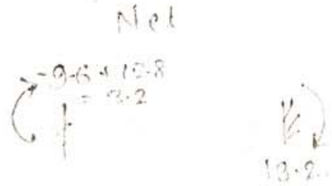
$\overline{M_{CB_2}} = \frac{40 \times 1.5^2 \times 1}{2.5^2} = 14.4$

$\overline{M_{BC_2}} = \frac{40 \times 1}{2.5^2}[3-1] = 12.8$

$\overline{M_{CB_2}} = \frac{40 \times 1.5}{2.5^2}[2-1.5] = 4.8$

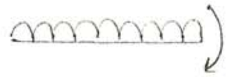
$\therefore \overline{M_{BC}} =$

Net



Extra case of overhang:

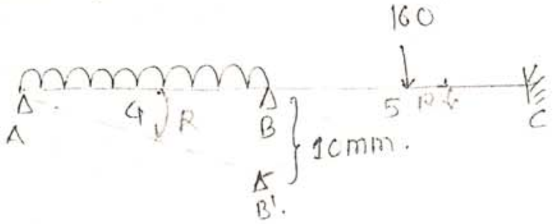
$\boxed{M_{AB} + M_{AD} = 0}$



Reactive mnt. $M_{AD} = \frac{wl^2}{2}$

$= \frac{60 \times 1.5^2}{2} = 67.5$

In case of Yielding of Support:- In previous prob. B yields by 10 mm.



$\therefore M_{AB} = -80 + \frac{2EI}{4}\left[2\theta_A + \theta_B - \left(+\frac{3 \times 0.01}{4}\right)\right]$

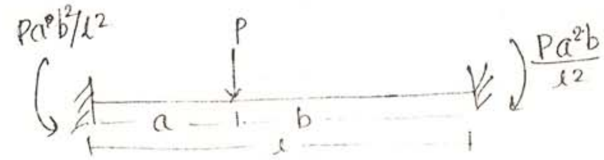
$M_{BA} = +80 + \frac{2EI}{4}\left[2\theta_A + 2\theta_B - \left(\frac{3 \times 0.01}{4}\right)\right]$

$M_{BC} = -100 + \frac{2EI}{5}\left[2\theta_B - 3 \times \left(\frac{-0.01}{5}\right)\right]$

$M_{CB} = +100 + \frac{2EI}{5}\left[\theta_B - 3 \times \left(\frac{-0.01}{5}\right)\right]$

as R rotating clockwise
i.e. AB rotating clockwise
$\therefore$ +ve.

eg:-



Find FEM for Triangular load.

$w' = w \cdot \frac{x}{l}$

Elementary mmt at A = $\frac{P \cdot a^2 b^2}{l^2}$

$\frac{\left(\frac{w \cdot x}{l}\right) dx \cdot x (l-x)^2 \cdot x}{l^2}$

$\therefore M_A = \frac{w}{l^3} \int_0^{l} x^2 (l-x)^2 \, dx$

$\therefore M_A = \frac{w}{l^3} \int_0^{l^2} x^2 (l^2 - 2lx + x^2) \, dx$

$= \frac{w}{l^3} \left[l^2 \times \frac{x^3}{3} - 2l \cdot \frac{x^4}{4} + \frac{x^5}{5} \right]_0^l = \frac{w}{l^3} \left[\frac{l^5}{3} - \frac{l^5}{2} + \frac{l^5}{5} \right]$

$$\boxed{M_A = \frac{w \cdot l^2}{30}}$$

$M_B = \frac{w}{l^3} \int_0^l x^3 (l-x)$

$$\boxed{M_B = \frac{w \cdot l^2}{20}}$$