Theory of Structures

Notes by-

Pravin S Kolhe,

BE(Civil), Gold Medal, MTech (IIT-K)

Assistant Executive Engineer,

Water Resources Department,

www.pravinkolhe.com

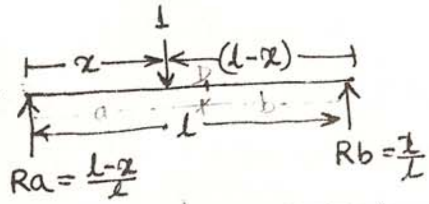
(F)

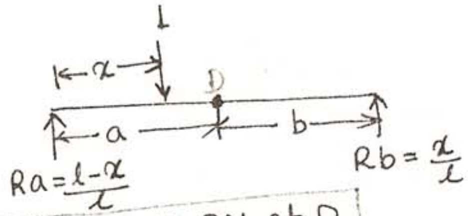
Moving Loads & ILD

Influence Line Diagram (ILD) :- A graphical representation of a function like reaction at support, shear force at section, bending moment at a section, for various position of unit load on the span of str.

Er. Pravin Kolhe
(B.E Civil)

I] Simply Supported Beam :-





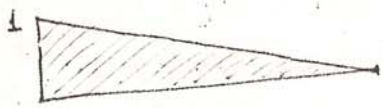
a] Influence line Diagram for Ra.

$\therefore R_a = \frac{(l-x)}{l}$

When $x=0$, $R_a = 1$

$x = l$, $R_a = 0$

(Ra)



b) ILD for Rb :-

$\therefore R_b = \frac{x}{l}$

When $x=0$, $R_b = 0$

$x = l$, $R_b = 1$

(Rb)

c] ILD for SF at a given section :- (D)

$\therefore SF @ D = \frac{l-x}{l} - 1$ (Load ON)

$= \frac{l-x}{l}$ (Load OFF)

$\therefore$ at $x = 0$, $SF_D = 0$ .. } Load (ON)

$x = a$, $SF_D = \frac{l-a}{l} - 1$ } $(0 \le x \le a)$ $\rightarrow 1 - \frac{a}{l} - 1 = -a/l$

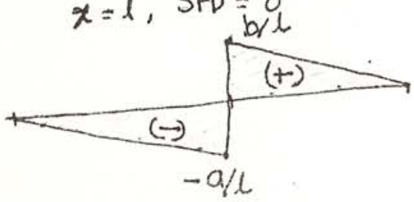
~~$x = l$, $SF_D = 1$~~

at ~~$x=0$, $SF_D = 1$~~ .. } load OFF

$x = a$, $SF_D = \frac{l-a}{l}$ } $(a \le x \le l)$

$x = l$, $SF_D = 0$

(VD)



d] ILD for BM at D

$BM_D = \left(\frac{l-x}{l}\right)a - 1(a-x)$ ... Load ON $[0 \le x \le a]$

at $x = 0$,

$BM_D = \left(\frac{l}{l}\right)a - 1(a) = 0$

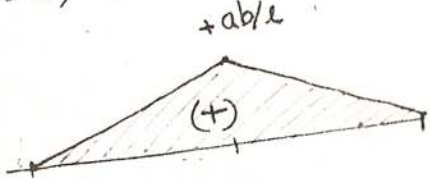
at $x = a$, $BM_D = \left(\frac{l-a}{l}\right)a - 0 = \left(\frac{l-a}{l}\right) \cdot a = \frac{ab}{l}$

$BM_D = \left(\frac{l-x}{l}\right) \cdot a$ . . . Load OFF $[a \le x \le l]$

at $x = a$, $BM_D = \left(\frac{l-a}{l}\right) \cdot a = \frac{ab}{l}$

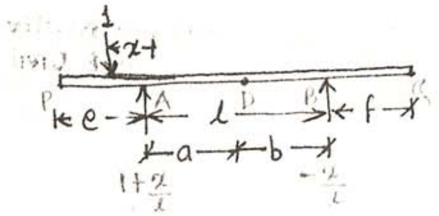
$x = l$, $BM_D = 0$

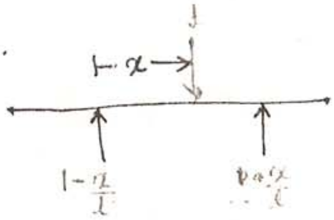
(MD)



* Once the ILD for choosen function is drawn, for maximise the function place the load ~~unit~~ (pt load or udl) such that it will give max. influence on the choosen function.

Case II : Simply Supported Beam with Overhang





i) ILD for Ra

Case I | Unit load is on PA | $[0 \le x \le e]$

$\Sigma M_B = 0 \Rightarrow 1\cdot(l+x) = Ra \cdot l$

$\therefore Ra = \left(\frac{l+x}{l}\right)$

& $Rb = 1 - \left(\frac{l+x}{l}\right) = \frac{l-l+x}{l} = -\frac{x}{l}$

Case II | Unit load is on AB | $[0 \le x \le l]$

$\Sigma M_B = 0 \Rightarrow Ra \cdot l = 1(l-x)$

$\left(\cancel{Ra = \frac{x}{l}}\right)$ $Ra = \frac{l-x}{l} = 1 - \frac{x}{l}$

& $Rb = \frac{x}{l} = \frac{x}{l}$

Case III | Unit load is on BQ | ~~$[l \le x \le (l+f)]$~~ $[0 \le x \le f]$

$\Sigma M_B = 0 \Rightarrow Ra \cdot l = -1 \cdot x$

$\therefore Ra = -x/l$

$Rb = 1 + \frac{x}{l}$

Case I at $x = e$; $Ra = 1 + \frac{e}{l}$; $Rb = -\frac{e}{l}$

$x = 0$, $Ra = 1$, $Rb = 0$

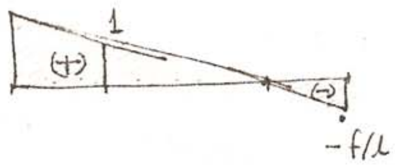
Case II, at $x = 0$, $Ra = 1$, $Rb = 0$

at $x = l$, $Ra = 0$, $Rb = 1$

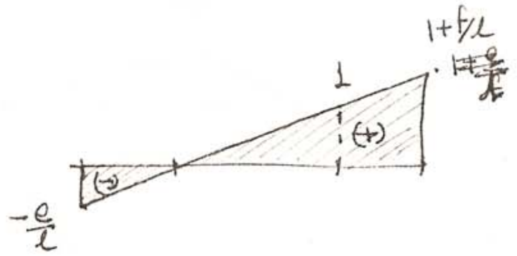
Case III, at $x = 0$, $Ra = 0$, $Rb = 1$

at $x = f$, $Ra = -f/l$, $Rb = 1 + \frac{f}{l}$.

Ra



Rb



b] ILD for SF at D

Case I) $V_D = \left(1 + \frac{x}{l}\right) - 1 = \frac{x}{l}$ $[0 \le x \le e]$

at $x = e$, $V_D = \frac{e}{l}$

$x = 0$, $V_D = 0$

Case II) $V_D = \left(1 - \frac{x}{l}\right) - 1 = -\frac{x}{l}$ $[0 \le x \le a$

at $x = 0$ $V_D = 0$

$x = a$, $V_D = -\frac{a}{l}$

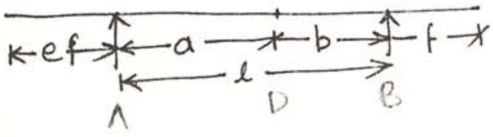
Case III) $V_D =$

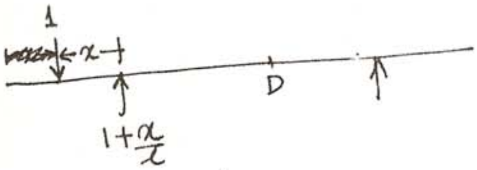
Case I) Load ON $[e \le x \le 0]$

$\therefore V_D = 1 + \frac{x}{l} - 1 = \frac{x}{l}$

Case II) Load OFF [

Simply supported beam with overhang.



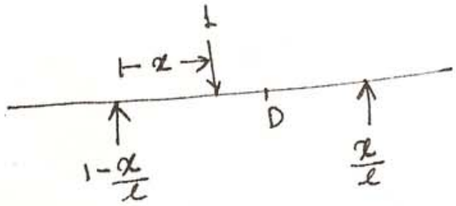


Load ON $[e \le x \le 0]$

$V_D = 1 + \frac{x}{l} - 1 = \frac{x}{l}$

at $x = e$, $V_D = e/l$

$x = 0$, $V_D = 0$

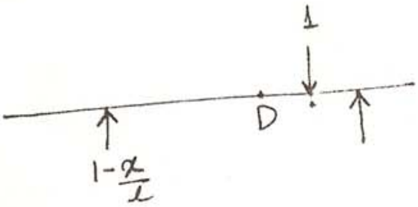


$[0 \le x \le a]$

$V_D = 1 - \frac{x}{l} - 1 = -\frac{x}{l}$

at $x = 0$, $V_D = 0$

$x = a$, $V_D = -\frac{a}{l}$

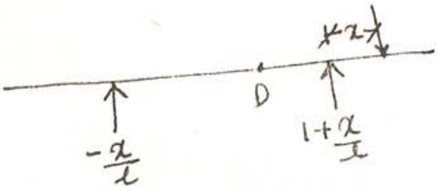


$[a \le x \le l]$

$V_D = 1 - \frac{x}{l}$

at $x = a$, $V_D = 1 - \frac{a}{l} = \frac{l-a}{l} = \frac{b}{l}$

$x = l$, $V_D = 1 - \frac{l}{l} = 0$.

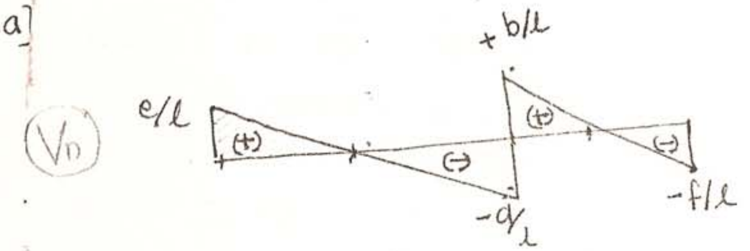


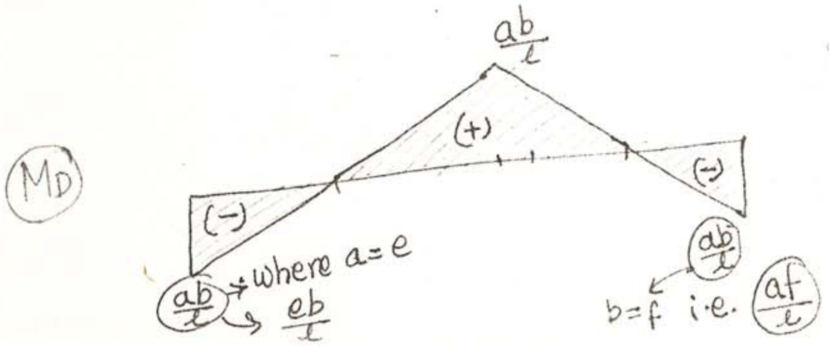
$[0 \le x \le f]$

$V_D = -\frac{x}{l}$

at $x = 0$, $V_D = 0$

$x = f$, $V_D = -f/l$.



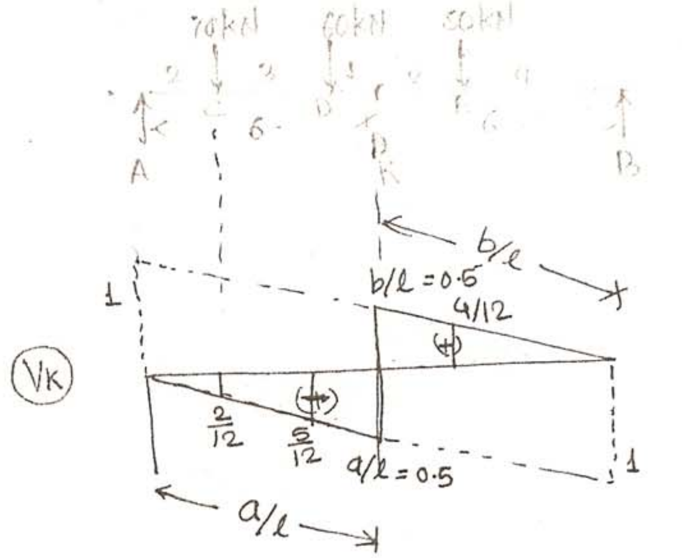


NOTE

i.e. Draw ILD for V_D without overhang & extend the IL's upto e/l & f/l in the "Progressive direction".

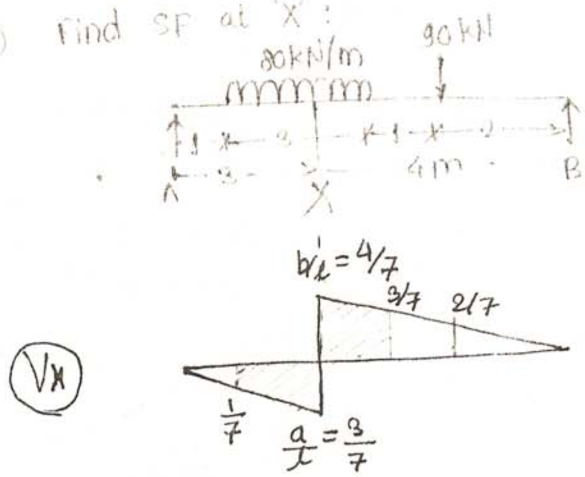
Same "funda" is applicable for M_D i.e. extend IL's till it reaches to extreme ends. & its m ordinate will be $\frac{eb}{l}$ & $\frac{fa}{l}$

① Find SF at section K for the loaded girder by mtd of ILD.



$$\therefore \text{SF at } K = -\frac{2}{12}\times 70 - \frac{5}{12}\times 60 + 50\left(\frac{4}{12}\right) = -20\,kN.$$

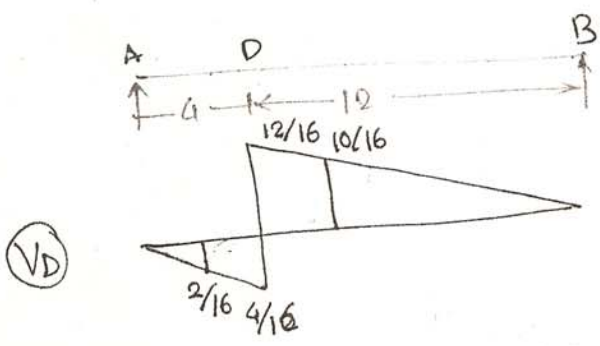
② Find SF at 'X':

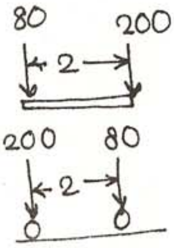


$$\therefore V_x = -\left[\frac{1}{7}+\frac{3}{7}\right]\times\frac{2}{2}\times 80 + \left[\frac{4}{7}+\frac{3}{7}\right]\times\frac{1}{2}\times 80 + \frac{2}{7}\times 90$$

$$= +20\,kN.$$

③ Two wheel loads 80, 200kN spaced 2m apart move on a girder of span 16m. Find +BM +SFmax & − SFmax at a section 4m from left end. Any wheel load can lead the other.





∴ for − SFmax ; 200 kN load is at section "JUST" left to D. (3)

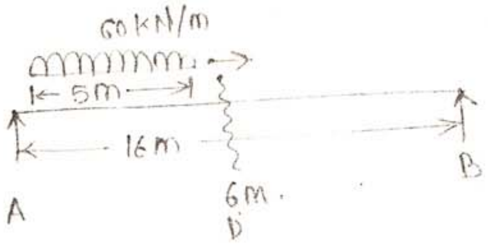
$$\therefore -SF_{max} = -\frac{2}{16}\times 80 - \frac{4}{16}\times 200 = -60\,kN$$

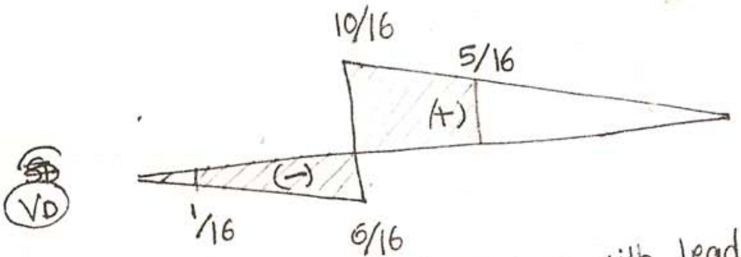
for + SFmax, 200 kN load is ~~at~~ placed JUST right to D.

$$\therefore +SF_{max} = +\frac{12}{16}\times 200 + \frac{10}{16}\times 80$$

$$= 200\,kN$$

(4) A udl of 60 kN/m of length 5 m moves on a girder of span 16 m. Find +SFmax & −SFmax at section 6 m from left end.



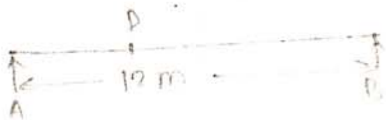


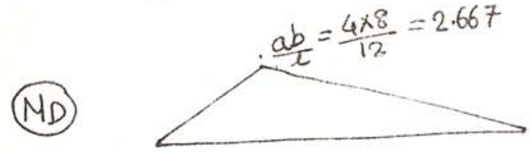
for −SFmax ; load is touching to D with leading ~~edg.~~ end.

$$\therefore -SF_{max} = -\left(\frac{1}{16}+\frac{6}{16}\right)\times\frac{5}{2}\times 60 = -65.625\,kN$$

$$+SF_{max} = \left(\frac{10}{16}+\frac{5}{16}\right)\times\frac{5}{2}\times 60 = +140.625\,kN.$$

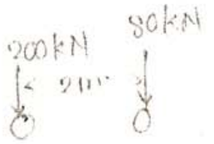
(5) A S.S. girder span 12 m, a 200 kN wheel load moves.
Find BMmax @ a sect^n 4 m from left end.



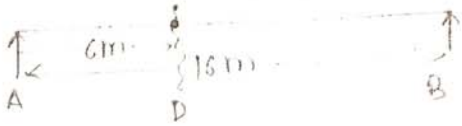


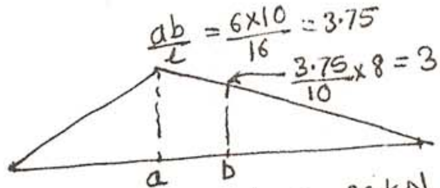
$$\therefore BM\ @\ D = 2.667\times 200 = 533.33\,kNm.$$

(5)



(any wheel load can lead the other)
find BMmax @ D





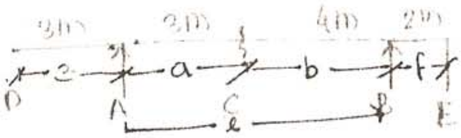
(M_D)

(I) ∴ Place 200kN at a, 80kN at b

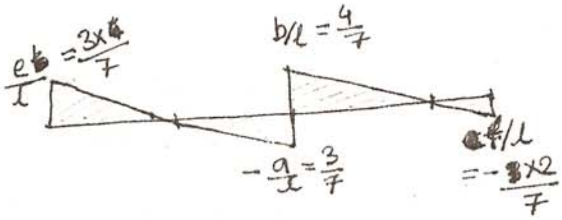
$\therefore BM = 3.75\times200 + 3\times80$

$\therefore BM_{max} = 990$ kNm.

(7) udl = 180kN/m occupy any position on girder.



Find +SF at C & −SF at C.



(V_C)

For +SFmax, udl on DA & CB.

$\therefore +SF_{max} = \frac{1}{2}\left[\frac{3\times1}{7}\times3\right]\times180 + \frac{1}{2}\left[\frac{4}{7}\times4\right]\times180$

$= \cancel{+66.857\ kN}.\ +142.857$ kN

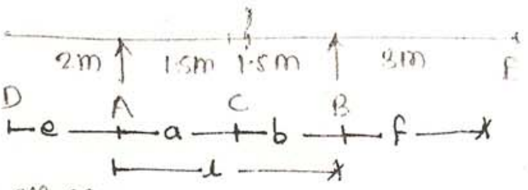
$-SF_{max} = -\frac{3}{7}\times0.5\times80^{\times3} - \frac{2}{7}\times0.5\times80\times2$

$= -74.286$ kN.

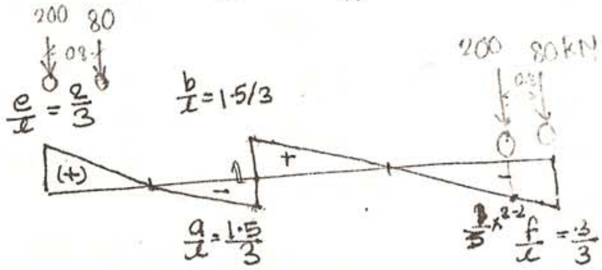
(8) 2 Wheel loads - 200kN & 80kN spaced 0.8 m apart moves on a girder.

Find :- +SFmax   +BMmax
     −SFmax at C   −BMmax

80kN load is leading.



(Vc)

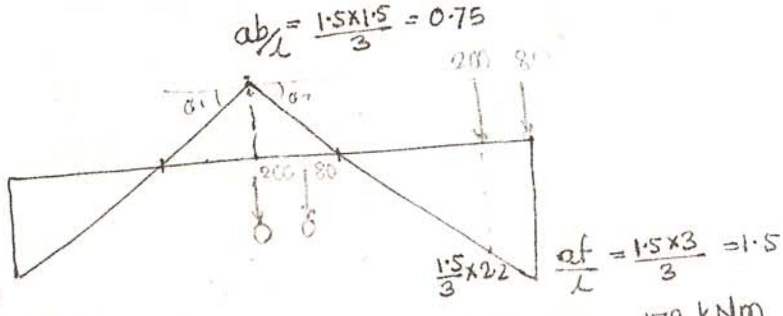


∴ for −SFmax : $-80\times\frac{3}{5}-200\times\frac{1}{3}\times2=-181.333$ kN

+SFmax :- $200\times\frac{2}{3}+80\times\frac{2/3}{2}\times1.2=165.333$ kN.

(9)

(Mc)



for +BMmax $=200\times0.75+80\times\frac{0.75}{1.5}\times0.7=178$ kNm

−BMmax $=-(30\times1.5+200\times1.5\times\frac{2.2}{3})=-340$ kNm

(9) Draw ILD for reaction at supports A, B, C & BM at B.
Find their max. values when travelling load of 60 kN/m may cover any part of span.

(RA)

$[0\le x\le6]$

$RA+RB+Rc=1$

$6RA-6Rc=1(6-x)\Rightarrow\Sigma M_B=0$

$\therefore RA-Rc=1-\frac{x}{6}$

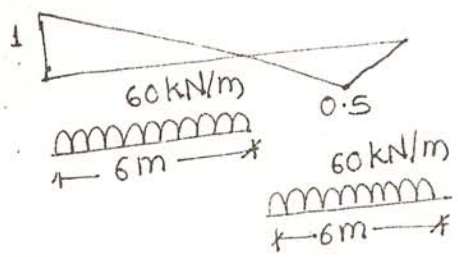
$\Sigma M_D=0 \therefore 9RA-3Rc+3RB=9-x$

∴ RA =

RB =

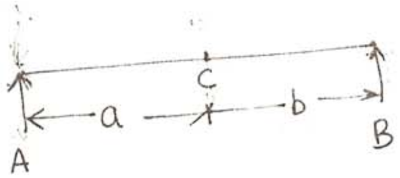
Rc =

(Ra)



* ~~Mt. Muler Breslau's Prin.~~
Miller
* Miller Breslau's principle

ILD can be drawn by First Principle (Basic) & by simplified method, suggested by Muler Breslau as -

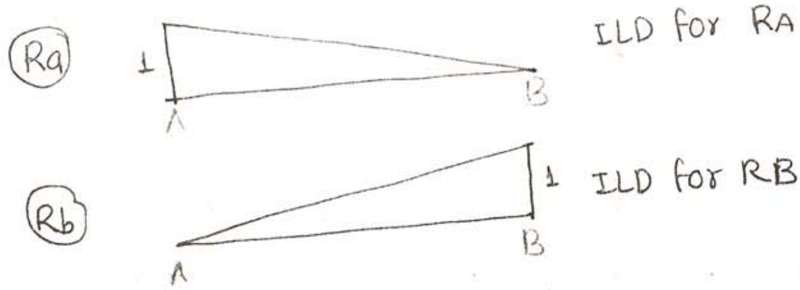


ILD for Reaction

Remove the constraint & give a unit displacement in the direction of constraint, without violating any static principle i.e. consistancy of statics complete ILD.

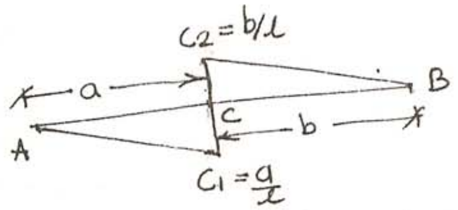
i.e. for drawing ILD for Reaction at 'A', remove support & shift the beam by unit displacement in the direction of reaction (i.e. upward.).

Similarly for drawing ILD for Rb, lift the beam by unit displacement at B in the direction of reaction & join with A.



ILD for shear force at a section:-

Due to shear there is sliding of two faces; this is constrain at that point. Therefore remove the constraint & shear that point in such a way that one goes down & second goes up & total displacement is unity.



$AC_1 \parallel^{el} C_2B \quad C_1C_2 = 1.$

i.e. To find ordinate, use principle of similar triangle.

i.e. $\frac{CC_1}{a} = \frac{CC_2}{b}$ But $CC_1 + CC_2 = 1$

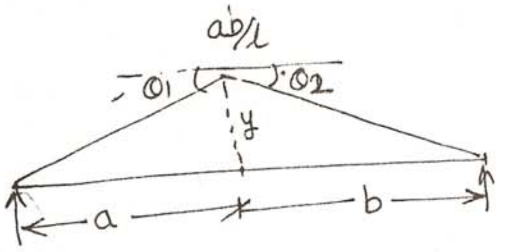
$\therefore \frac{CC_1}{a} = \frac{1 - CC_1}{b} \Rightarrow \frac{CC_1}{a} + \frac{CC_1}{b} = \frac{1}{b} \Rightarrow CC_1\left(\frac{a+b}{ab}\right) = \frac{1}{b}$

$\Rightarrow CC_1\left(\frac{l}{ab}\right) = \frac{1}{b} \Rightarrow CC_1 = \frac{ab}{l \cdot b} = \frac{a}{l}$

ILD for Bending moment at a section :- Generally sagging.

Due to bending, there will be sagging or hogging of a beam. This is constraint. Therefore we remove the constraint & allowing sag ~~or hog~~, so that at the given section lift the beam such that effective rotation is unity. (i.e. $\theta_1+\theta_2=1$)

Er. Pravin Kolhe
(B.E Civil)



$\theta_1 = \frac{y}{a}$

$\theta_2 = \frac{y}{b}$

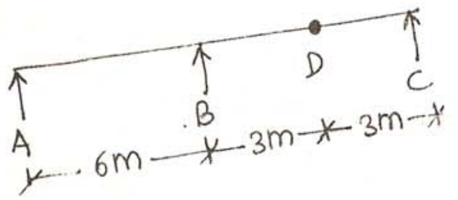
$\because \theta_1+\theta_2 = 1$

$\Rightarrow \frac{y}{a}+\frac{y}{b} = 1 \Rightarrow y\left(\frac{a+b}{ab}\right) = 1$

$\Rightarrow y = \frac{ab}{a+b} = \frac{ab}{l}.$

Miller Muters Breslau's Principle The IL for a given function of a statically determinate beam may be obtained by removing the constraint offered (i.e. support, sliding, sagging} for reaction, shear & moment resp.) & introducing a ~~linear & support~~ effective linear (for Reaction & shear) or angular (for BM) displacement at the given section & in the direction of the constraint.

for eg:-



(RA)

Hinge is provided so rotation is possible.
∴ We did not violate consistancy of str.

(RB)

(RC)

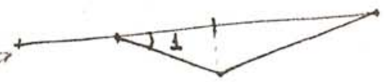
(RA)

constraint is ↑ ∴ Lift upward at 'A' by 1.
It should meet at B.

(RB)

constraint is ↑ at B ∴ Lift upward by 1
But at A, it is fixed, so it will remains same. up to hinge.
at hinge rotation is allowed.

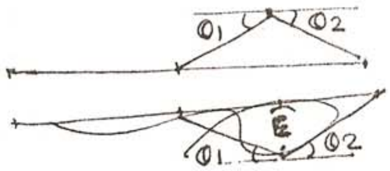
(MD)

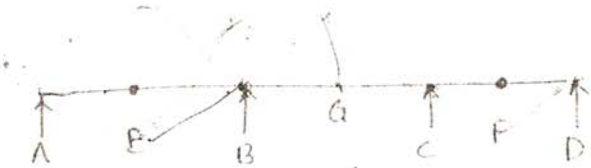


end is fixed
no rotation

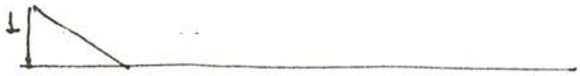
Constanib at D is hogging.
∴ Remove constraint & allow rotation such that total rotation in '1'.

(ME)



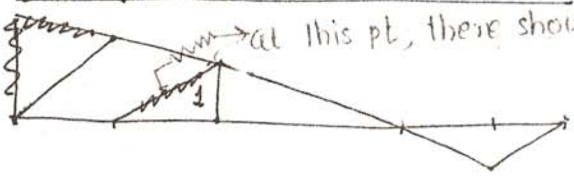


(Ra)



at this pt, there should not change of slope.

(Rb)

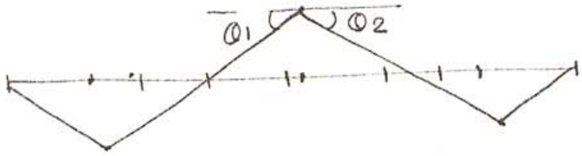


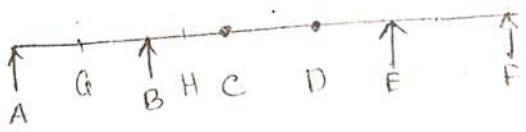
at hinge there

(Mb)

(Vg)

(MG)





(Ra)

Here change in direct^n.
∴ wrong

(Rb)

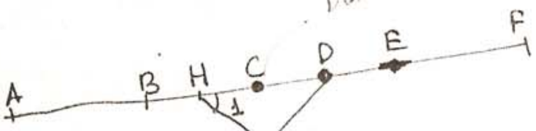
(VG)

(VH)

(MG)

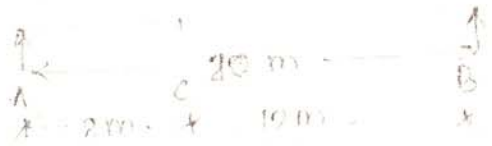
Due to hinge this portion will act as cantiver.
∴ BM - Resisting mmt which does not allows hogging
∴ Go downward direct

(MH)



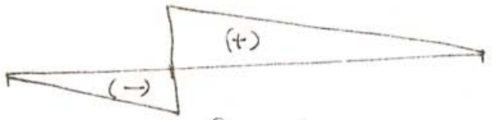
i.e. If higt hinge is provided on either side of beam then 2CD BM will be Downward (-ve) side.

MPSC Pro:- Find max. shear & BM at C when 50 kN/m load greater (longer) than span passes over the beam



$\frac{b}{l} = \frac{12}{20} = 0.6$

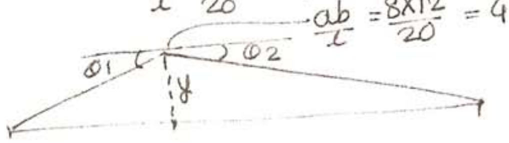
(Vc)



$\frac{a}{l} = \frac{8}{20} = 0.4$

$\frac{ab}{l} = \frac{8 \times 12}{20} = 4.8$

(Mc)



$\therefore$ $+$ Max SF $= 0.5 \times 0.6 \times 12 \times 50 = +180$ kNm.

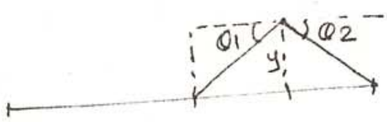
$-$ Max SF $= -0.5 \times 0.4 \times 8 \times 50 = -80$ kNm.

$+$ Max BM $= \frac{1}{2} \times 4.8 \times 20 \times 50 = +2400$ kNm

$-$ Max BM $= 0$.

Pro:

(Mc)



$\tan \theta_1 = \frac{y}{l/4} = \theta_1$

$\tan \theta_2 = y/l/4 = \theta_2$

$\therefore \frac{y}{l/4} + \frac{y}{l/4} = 1$

$\therefore y\left(\frac{l/4 + l/4}{l^2/4}\right) = 1$

$y = \frac{l^2/4}{l/2} = \frac{l}{8}$