Theory of Structures

Notes by-

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* (G)

* Criteria for maximum bending moment under a chosen wheel load :-

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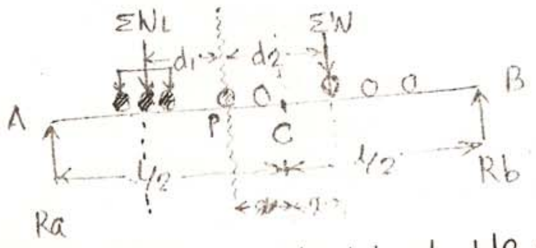


Let P be the chosen wheel load. We will ~~consider~~ evaluate that " For what position of wheel load system on the span, the BM under chosen load 'P' will be max."

Let ΣW = Resultant of all wheel loads, on the total span [AB]

ΣW_L = Resultant of all wheel loads that are on left part of 'P' on the beam. [i.e. betn AP]

d_1 = Dist. betn ΣW_L & chosen function P

d_2 = Dist. betn ΣW & chosen function P.

x = Dist. of ΣW from midspan 'C'.

$$\therefore \Sigma M@B = 0 \Rightarrow L\,R_a = \Sigma W\left(\frac{L}{2} - x\right)$$

$$\therefore R_a = \frac{\Sigma W}{L}\left(\frac{L}{2} - x\right)$$

$\therefore$ BM under chosen function $P = R_a \cdot \left(\frac{L}{2} + x - d_2\right) - \Sigma W_L \cdot d_1$

$$BM_P = \frac{\Sigma W}{L}\left(\frac{L}{2} - x\right)\cdot\left(\frac{L}{2} + x - d_2\right) - \Sigma W_L \cdot d_1$$

For BM_P to be max; $\frac{dM_P}{dx} = 0$; $BM_P = \frac{\Sigma W}{L}\left[\frac{L^2}{4} + \frac{xL}{2} - \frac{L \cdot d_2}{2} - \frac{xL}{2} - x^2 + x d_2\right] - \Sigma W_L d_1$

$$\therefore \frac{dM_P}{dx} = \frac{\Sigma W}{L}\left[0 + \frac{L}{2} - 0 - \frac{L}{2} - 2x + d_2\right] - 0 = 0$$

$$\Rightarrow \frac{\Sigma W}{L}\left[-2x + d_2\right] = 0$$

$$\Rightarrow \boxed{2x = d_2}$$

$$\Rightarrow \boxed{x = \frac{d_2}{2}}$$

$\therefore$ The chosen function (Load) & resultant of wheel load should be equidistant from the middle point of the girder.

i.e. BM under a chosen load of wheel load system will be max. when load system is so placed on the girder that the chosen load & resultant of all the wheel loads are equidistant from middle point of girder.

» Absolute Max BM

5) Find

Wheel loads :- The absolute max. BM will occur at a section near the centre of span.

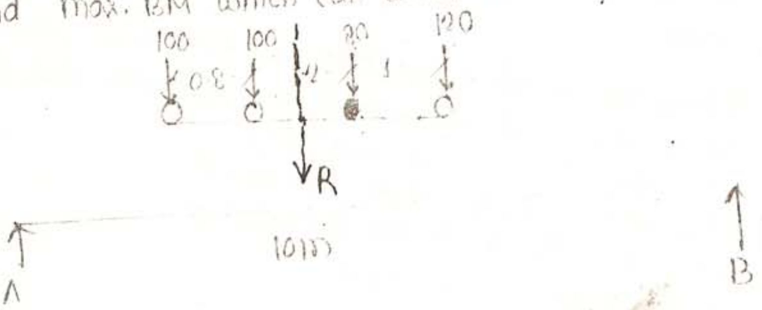
steps :- ① First by inspection, determine the load, which should be placed at midspan so that BM at midspan is max.

② Place the load system on the span such that resultant of all loads & load chosen are equidistant from midspan.

③ Determine the BM under the given i.e. chosen load.

Pro.) The load system moves from left to right on a girder of span 10m. Find max. BM which can occur under 80 kN load.

sol^n:-



The condition for BMmax under given (80 kN) load is," The wheel load system is so placed that resultant load & 80 kN load should be equidistant from midspan."

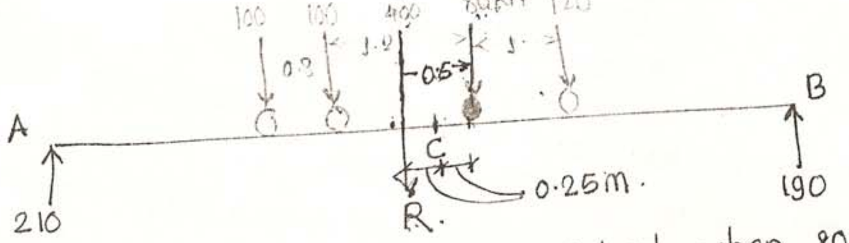
i.e. $d_2 = \cdot 2x$

∴ From given system of moving load ; Resultant will pass through a point 1.5 m from any end. (as Total span = 0.8 + 1.2 + 1 = 3m).

[& apply Varignon's Theorem — Extra Note]

∴ Distance bet^n resultant load & 80 kN load = 2 − 1.5 = 0.5 or
= 1.5 − 1 = 0.5m.

∴



∴ BM will be max; under 80 kN load, when 80 kN load is placed at 0.25m from right side of midspan.

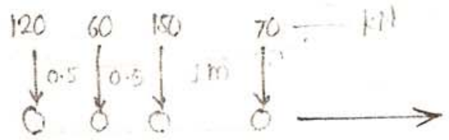
∴ $\Sigma M_B = 0 \Rightarrow 10\, R_a - 100\,(5 - 0.25 + 1.2 + 0.8) - 100\,(5 - 0.25 + 1.2) - 80 \times (5 - 0.25) - 120\,(5 - 0.25 - 1) = 0$ [or $10 R_a - 400(5 + 0.25$

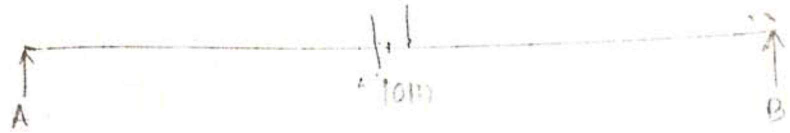
$\Rightarrow R_a = 210\text{ kN}$; $R_B = 400 - 210 = 190\text{ kN}$

∴ ΣM @ 80 kN load $= 210 \times (5 + 0.25) - 100 \times (0.8 + 1.2) - 100(1.2)$

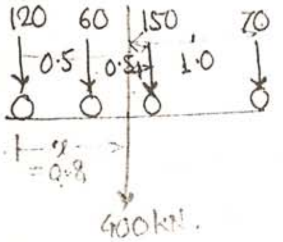
$= 782.5\text{ kNm}$

② Find absolute max. BM.





soln :-



$\Sigma W = 400 kN.$

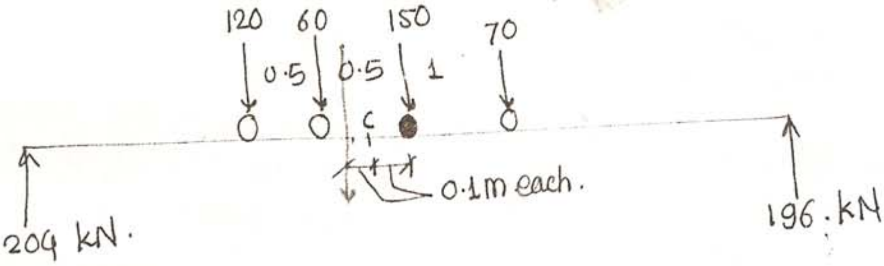
$\therefore \Sigma W \cdot x = 60 \times 0.5 + 150 \times 1 + 70 \times 2.0$

$\therefore x = 0.8$ from 120 kN load.

$\therefore$ ~~Dist.~~ The BM will be max. under 150 kN load.

$\therefore$ Dist. betn 150 kN load & ΣW = $1 - 0.8 = 0.2$ m.

$\therefore$ Place 150 kN load at 0.1 m right from midspan.



$\therefore \Sigma M_B = 0 \Rightarrow Ra \times 10 = 400 \times [5 + 0.1]$

$\therefore Ra = 204$ kN.

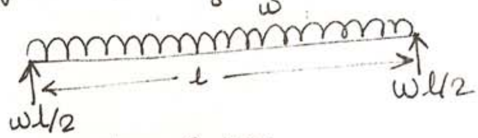
$\therefore$ BM at 150 kN load = absolute max. BM $196 \times [5 - 0.1] - 70(1)$

= ~~855.8 kN~~ 890 kN.

* Absolute max. BM for uniformly distributed load :-

a] udl greater (Longer) than span :-

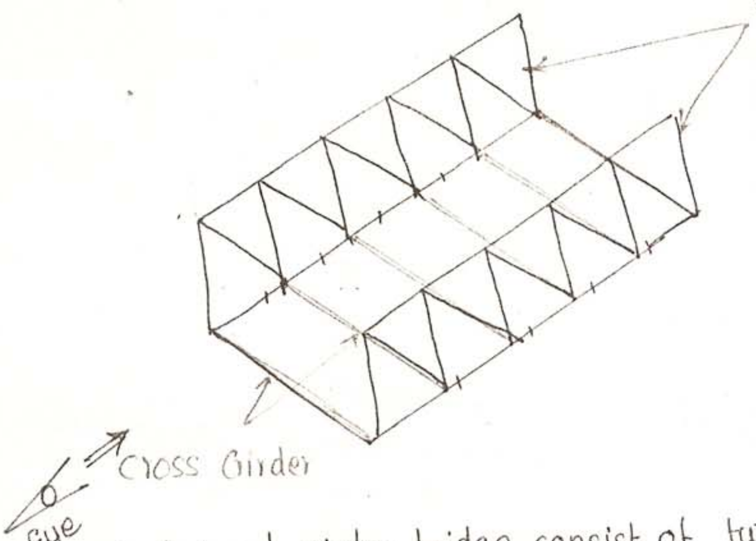
Absolute max. BM occurs when (at centre) when span is loaded & is given by = $\frac{wl^2}{8}$ (at midspan)

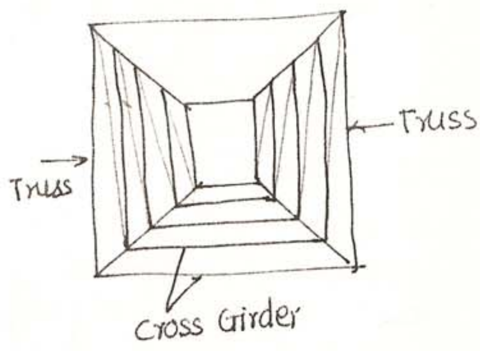


b] udl shorter than span :-

Absolute max. BM occurs at centre, when load is symmetrically placed on the span.

ILD for Bridge Truss Members





A trussed ~~girder~~ bridge consist of two longitudinal trusses cross-connected by cross girders. The LL recieved by cross girder is transfered to truss joints. The cross girders are generally supported by truss at panel points.

Classification of truss depending upon approaches - (for detail, see Bridge Engg. Notes)

a] Deck Type

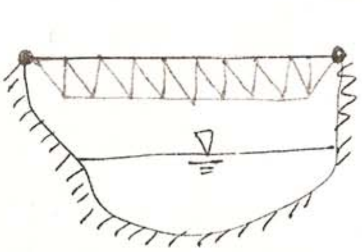
b] Through Type.

* Structural Difference

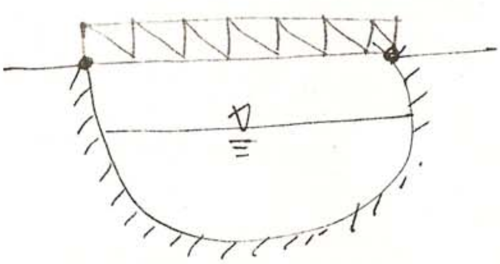
1] Deck Type bridge :- If cross girders are provided, connecting corresponding top chord joints of the truss, so that load is recieved at the top chord joints the truss is called as deck type truss.

2] Through Type Bridge :- If the cross girders are provided connecting corresponding bottom chord joints of the truss, so that load is recieved at the bottom chord joints, the truss is called a through type truss.

ILD for forces ~~chosen~~ function in the members of any one truss may be drawn by assuming that the moving load on the bridge is 2 units along the centre line of the bridge & that a load of 1 unit is transmitted to each longitudinal truss.



Deck Type Bridge.

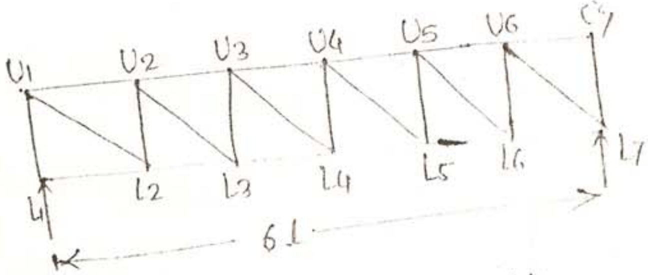


Through type Bridge.

③

* **Pratt Truss:-** The pratt truss consist of top & bottom chord members connected by verticals & diagonals so that the diagonal members are in tension & vertical members are in compression under the action of DL (only).

for eg:-



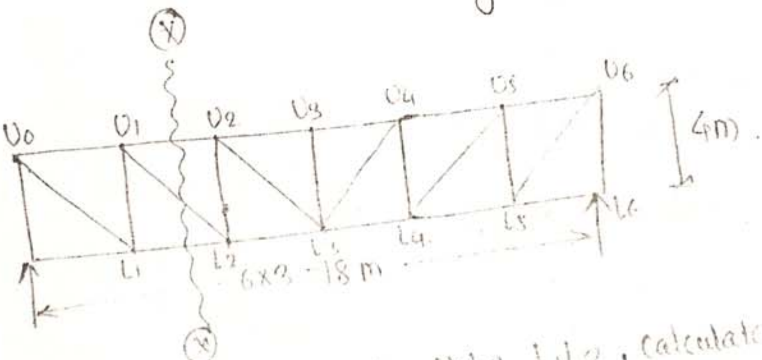
P. George Sir.

Here choosen function is "Axial Load."

Method of section : Cut the "RIGID BODY" in two parts such that, the reqd. member should be external force on each of the two bodies.

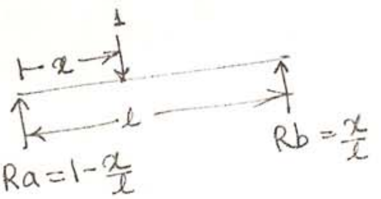
i.e.

Pro:



construct ILD for members U_1U_2, U_1L_2, L_1L_2. Calculate max force in U_1L_2 if due to - 1) DL = 5 kN/m
2) Moving load = 30 kN/m Greater than span
3) Moving load = 20 kN.

Reactions:-

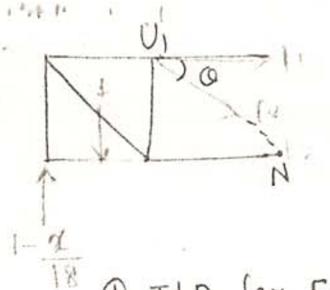


$\Sigma M @ B = 0$

$\therefore l \cdot Ra = 1(l - x)$

$\therefore Ra = \frac{l - x}{l} = 1 - \frac{x}{l}$

$Rb = \frac{x}{l}$



Load ON $[0 \le x \le 3]$
Load OFF $[\cancel{3}6 \le x \le 18]$

V. Imp

① ILD for F_1 (U_1U_2) — U_1U_2

$$\Sigma M_N = 0 \Rightarrow \left(1-\frac{x}{18}\right)\times 6 - 1(6-x) + F_1 \times 4 = 0$$

$$\therefore F_1 = \frac{1}{4}(6-x) - \frac{6}{4}\left(1-\frac{x}{18}\right) \quad \dots \quad \text{Load ON } [0 \le x \le 3]$$

$$F_1 = -\frac{6}{4}\left(1-\frac{x}{18}\right) \quad \dots \quad \text{Load OFF } [6 \le x \le 18]$$

② ILD for F_2 (U_1L_2) — U_1L_2

$$\Sigma F_y = 0 \Rightarrow \left(1-\frac{x}{18}\right) - 1 = F_2 \sin\theta \qquad \text{Load ON } [0 \le x \le 3]$$

$$\therefore F_2 = \frac{1}{\sin\theta}\left(\frac{3}{4}\right)\left(1-\frac{x}{18}\right) - \frac{1}{\sin\theta}\left(\frac{3}{4}\right) \quad ; \sin\theta = \frac{4}{3}$$

Load OFF $[6 \le x \le 18]$

$$F_2 = \frac{1}{\sin\theta}\left(1-\frac{x}{18}\right)$$

③ ILD for F_3 (L_1L_2) — L_1L_2

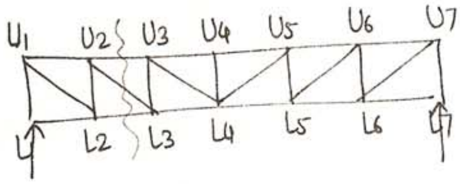
$$\Sigma M_{U_1} = 0 \Rightarrow \left(1-\frac{x}{18}\right)\times 3 - 1(3-x) = 4F_3$$

$$\therefore F_3 = \frac{3}{4}\left(1-\frac{x}{18}\right) - \frac{1}{4}(3-x) \qquad \text{Load ON } (0 \le x \le 3)$$

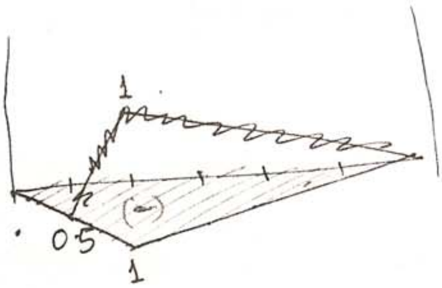
$$F_3 = \frac{3}{4}\left(1-\frac{x}{18}\right) \qquad \text{Load OFF } (6 \le x \le 18)$$

$\therefore$ ILD ordinates:-

U_1U_2	U_1L_2	L_1L_2
[Load ON]		
at $x=0$, $F_1 = \cancel{-0.833}\ 0$	$x=0$, $F_2 = \cancel{0.75}\ 0$	$x=0$, $F_3 = 0$
$x=3$, $F_1 = -0.5$	$x=3$, $F_2 = -0.125$	$x=3$, $F_3 = 0.625$
Load OFF	$x=6$, $F_2 = 0.5$	$x=6$, $F_3 = 0.5$
	$x=18$, $F_2 = 0$	$x=18$, $F_3 = 0$
$x=6$, $F_1 = -1$		
$x=18$, $F_1 = 0$		



U1U2



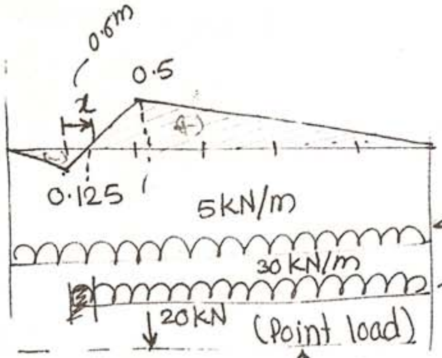
To calculate max. force in U1L2,

$\therefore \quad \frac{0.125}{x} = \frac{0.5}{3-x}$

$\therefore 0.125x + 0.5x = 0.375$

$\therefore x = 0.6\,m$

U1L2

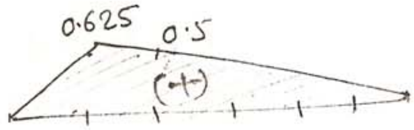


← DL (Total span)

← Moving load — only +ve side (Greater than span)

↑ at max. ordinate.

L1L2



$\therefore$ Max. force in U1L2 :-

$\left[-\frac{1}{2}\times0.125\times3.6+\frac{1}{2}\times0.5\times14.4\right]\times5+\left[\frac{1}{2}\times0.5\times14.4\times30\right]+0.5\times20$

$= 134.875$ kNm

Typical Pro Pg-765 / Pro-405 } Ramamurtham

Warren Girder (Truss without vertical members)