

Theory of Structures

Notes by-

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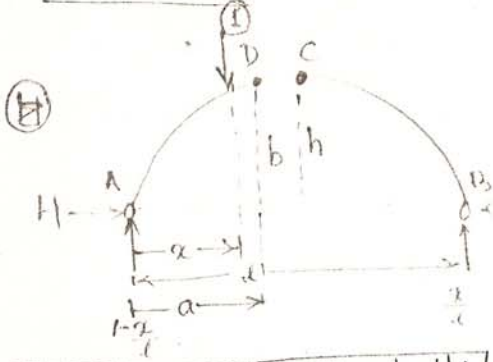
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ILD for Arches

3 Hinge Arch



ILD for Horizontal Thrust H:-

A, B, C - Hinges, h = Rise, l = span.

$V_a = 1 - \frac{x}{l}$; $V_b = \frac{x}{l}$

$\sum Mc = 0$

$\therefore H \cdot h = (1 - \frac{x}{l}) \times \frac{l}{2} - 1 \left(\frac{l}{2} - x \right)$ Load ON $(0 \leq x \leq \frac{l}{2})$

$= (1 - \frac{x}{l}) \cdot \frac{l}{2}$

Load OFF $(\frac{l}{2} \leq x \leq l)$

\therefore at $x=0$, $H=0$

$x = \frac{l}{2}$, $H = (1 - \frac{1}{2}) \frac{l}{2} - (\frac{l}{2} - \frac{l}{2})$
 $= \frac{l}{4}$
 $H = \frac{l}{4h}$

LOAD ON

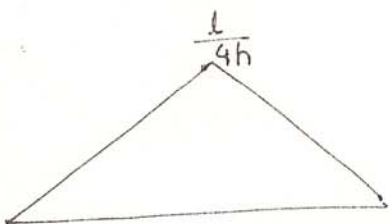
at $x = \frac{l}{2}$: $H = \frac{l}{4h}$

$x=l$, $H=0$

LOAD OFF

for simplicity,

$H=0$ to $\frac{l}{4h}$ at 'C' & c.



ILD for H

ILD for R_a



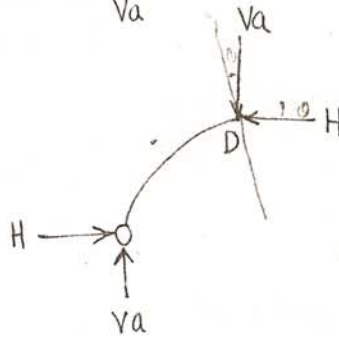
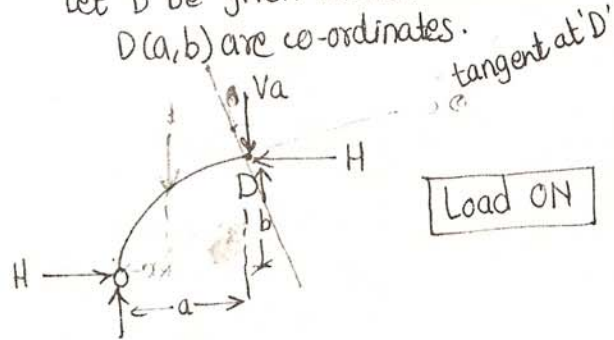
ILD for R_b



ILD for Normal Thrust

Let D be given section.

D(a, b) are co-ordinates.



Load ON

Load OFF

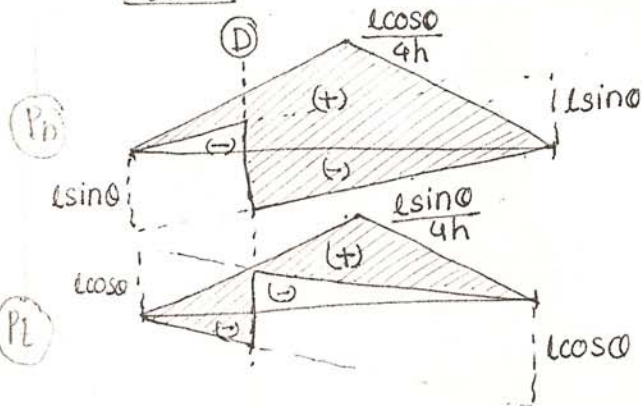
\therefore Load ON: Normal Thrust (P_n) = $H \cos \theta - V_b \sin \theta$

Load OFF: $P_n = H \cos \theta + V_a \sin \theta$

Radial shear

Load ON: $P_r = H \sin \theta + V_b \cos \theta$

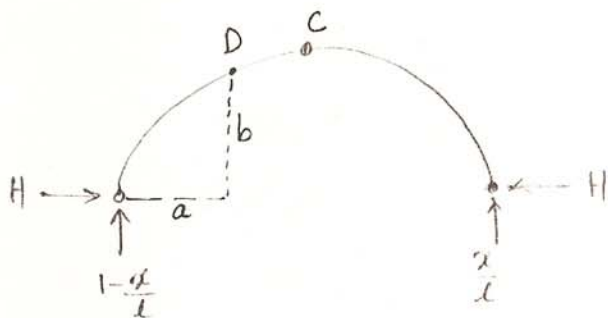
Load OFF: $P_r = H \sin \theta - V_a \cos \theta$



$\frac{l(l-a)}{2}$

ILD for BM at given section

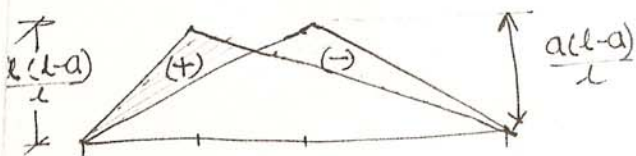
(5)



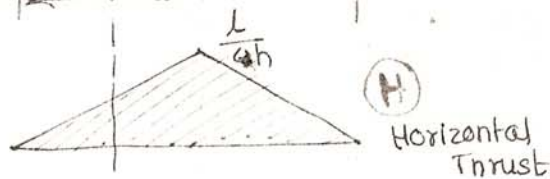
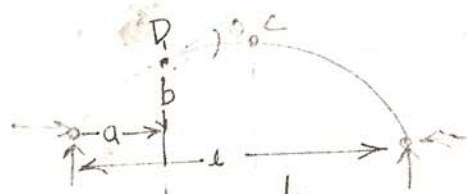
\therefore BMD = Beam moment - $H \cdot b$

But Beam moment at D = $(1 - \frac{a}{l}) \times a$
 $= (\frac{l-a}{l}) \times a$ (i.e. $\frac{ab}{l}$) & $H = \frac{4qh}{4h}$; $b = \frac{qh \cdot a(l-a)}{2l}$

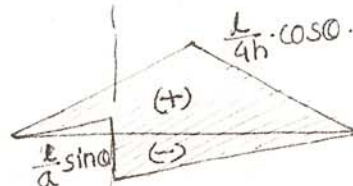
& $H \cdot b = \frac{4}{4h} \left[\frac{qh}{2} \cdot a(l-a) \right] = \frac{a}{2}(l-a)$



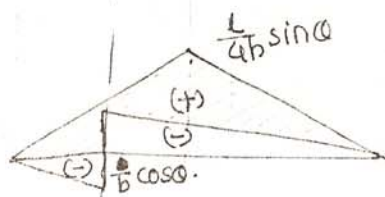
(MD)



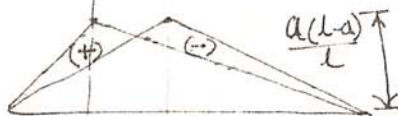
Horizontal Thrust



Normal Thrust



Radial shear



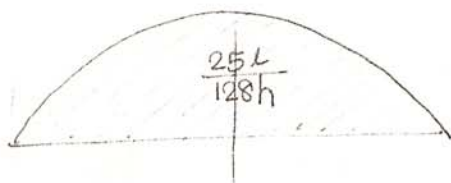
(MD)

V. Imp

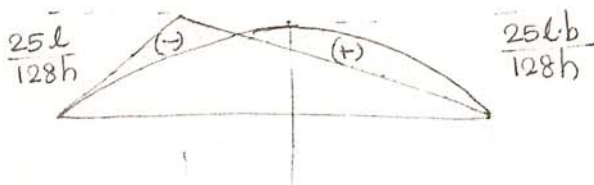
ILD for 2Hinge parabolic arch



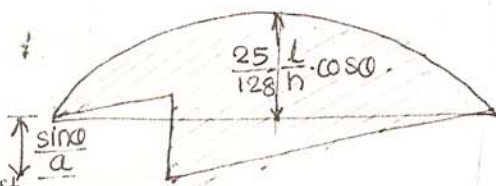
(H)
Horizontal Thrust



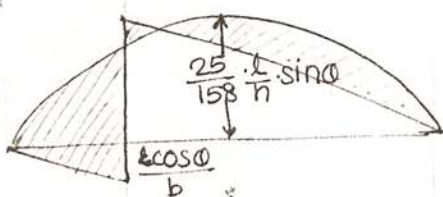
(M_D)
BM at D



(P_n)
Normal Thrust



(P_r)
Radial shear



Reaction

mmt
CBy

Sf at

mmt c
Sec

ILD for statically Indeterminate structures:-

ILD for propped cantilever



If $n =$ Given interval of span :- say 0.5 m, 0.25m etc. Say 0.25m

ordinate for

$$R_b = \frac{n^2(3-n)}{2}$$

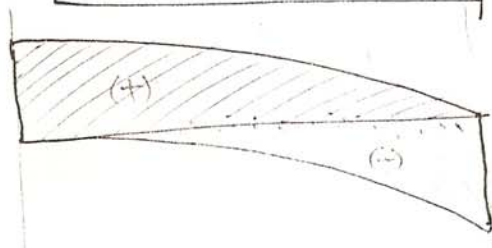
$$R_a = \frac{2-3n^2+n^3}{2}$$

min at A \rightarrow $M_d = -n(1-n)(2-n) \cdot \frac{l}{2}$

$$M_d = \frac{ba^2(3l-x)}{2l^3}$$

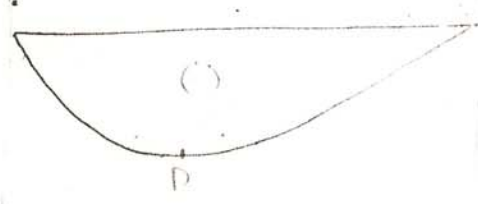
using this equations, find ordinates at 0.25, 0.5, 0.75, 1.0, 1.25, ... so on upto l. & Plot ILD by join all ordinates.

R_a
Reaction at A

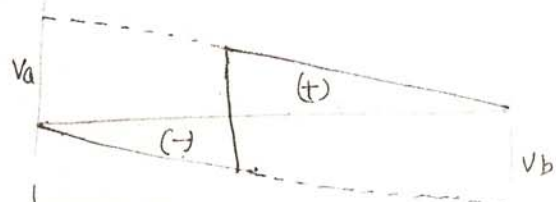


R_b
Reaction at B

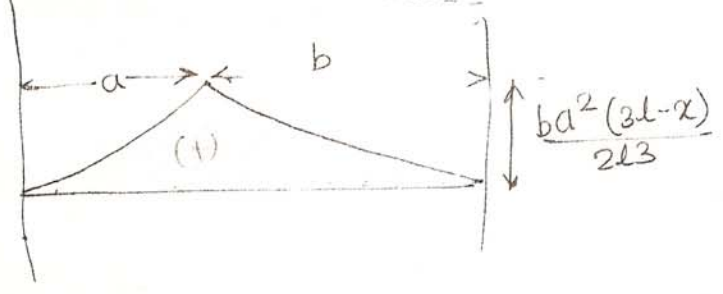
M_d
min at A
(By formula)



V_D
SF at D



M_d
min at given section

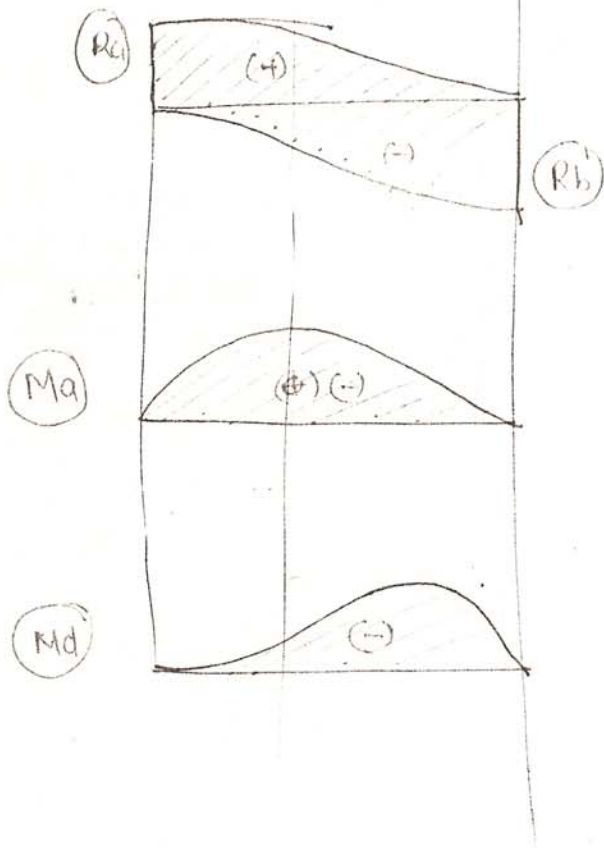


ILD for fixed beam



$R_a = \frac{1}{l} (1-n)^2 (1+2n)$
$R_b = \frac{1}{l} n^2 (3-2n)$
$M_a = n(1-n^2)$
$M_b = n^2(1-n)$

ILD for Two span continuous beam



See J.P. Greengard's Notes