Theory of Structures

Notes by-

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4/12 TCS-D

[Arches]

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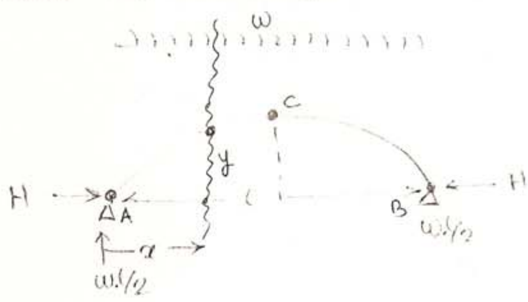
Types of arches:-

Shapes:- Parabolic (Imp), Circular, elliptical.

Imp. Show that the 3Hinge arch shown subjected to no BM anywhere on it.



Sol^n^:- $\Sigma F_y = 0$

$\therefore R_A + R_B = wl$

$R_A = R_B = wl/2$

$\Sigma M_C = 0$

$\therefore \frac{wl}{2} \times \frac{l}{2} = H \cdot h + \frac{wl}{2} \cdot \frac{l}{4}$

$\therefore H = \dfrac{\frac{wl^2}{4} - \frac{wl^2}{8}}{h}$

$$\boxed{H = \frac{wl^2}{8h}}$$

To show that BM at any section is zero,

$\Sigma M_{x\text{-}x} = 0$

$\Sigma M_x = \frac{wl}{2} x - Hy - w \cdot \frac{x^2}{2}$ & Eq^n^ of parabola is $\left\{ y = \frac{4hx(l-x)}{l^2} \right.$

$= \frac{wl}{2} x - \left(\frac{wl^2}{8h}\right)\left(\frac{4h}{l^2}\right) x(l-x) - \frac{wx^2}{2}$

$= \frac{wl}{2} x - \frac{wx}{2}(l-x) - \frac{wx^2}{2}$

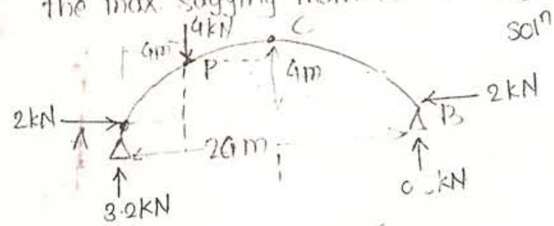
$= \frac{wl}{2} x - \frac{wxl}{2} - \frac{wx^2}{2} - \frac{wx^2}{2}$

$M_x = 0$

Note: The BM is zero for arch carrying udl on its entire span, irrespective of support levels; for 3H arch only.

Imp V.V. Imp

Find the thrust, radial shear, and bending moment at 'P'. Also find the max sagging moment & hogging moments on the arch.



Sol^n^ - $R_B \times 20 = 4 \times 4$

$\therefore R_B = 0.8\text{ kN}$

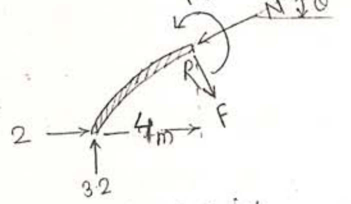
$R_A \times 20 = 4 \times 16$

$\therefore R_A = 3.2\text{ kN}$

$\Sigma M_C = 0$

$H \times 4 + 4 \times 6 = 3.2 \times 10$

$\boxed{H = 2\text{ kN}}$



Section at point 'P'

$\tan\theta = \frac{dy}{dx} = \frac{4h}{l^2}(l - 2x)$

$= \frac{4 \times 4}{(20)^2}(20 - 8)$

$\therefore \theta = 25.64°$

$\therefore \Sigma F_x = 0$

$2 + R\sin\theta = N\cos\theta \Rightarrow N\cos\theta - R\sin\theta = 2$

$3.2 = N\sin\theta + R\cos\theta \Rightarrow N\sin\theta + R\cos\theta = 3.2$

$\therefore R = 1.966 = 2.021$

$N = 3.22 = 3.186$

$\Sigma M_P = 0$

$\therefore M + 2(y_P) - 3.2(4) = 0$

$\therefore M + 2\left[\frac{4h}{l^2}\cdot x(l-x)\right] - 3.2\times 4 = 0$

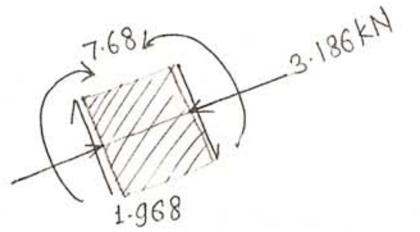
$\therefore M + 2\left[\frac{4\times 4}{20^2}\times 4(20-4)\right] - 3.2\times 4 = 0$

$\therefore \boxed{M = +7.68\ kNm}$

To examine a function :- (curve)

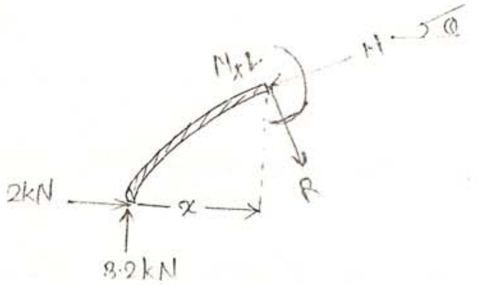
1) Put extreme limit in the analysied eqⁿ.

2) Check slope by Iˢᵗ derivative & IIⁿᵈ derivative



The element at 'p' (without considering 4kN)

We now go on to the BMD of arch



$M_x = 3.2x - 2y$ — $\frac{4h}{l^2}\cdot x(l-x)$ for $\tan\theta$ or $(l-2x)$ derivative

$M_x = 3.2x - \frac{2\times 4\times 4}{400}\cdot x(20-x)$

$M_x = 3.2x - \frac{x(20-x)}{12.5}$ at $x=0$, $M_4 = \underline{7.68 kNm}$

i.e. BM

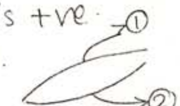
$\frac{dM}{dx} = 3.2 - \frac{1}{12.5}[20-2x]$

at $x=0$, $\frac{dM}{dx} = +1.6$ i.e. +ve slope

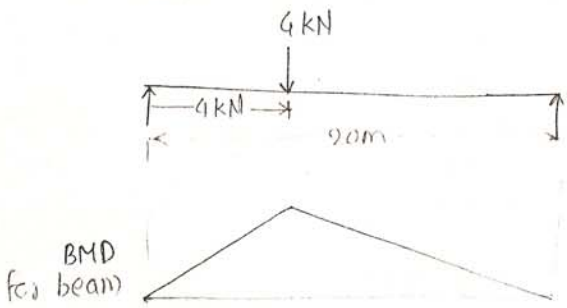
$\frac{d^2M}{dx^2} = \frac{2}{12.5}$ (+ve)

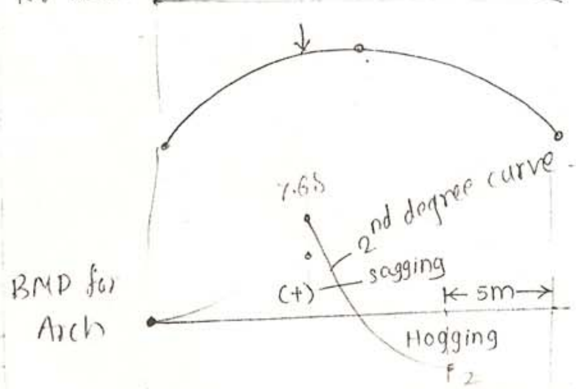
Means rate at which Iˢᵗ derivatative changes is +ve. ①

i.e.



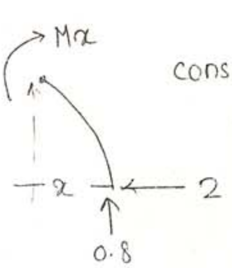
$\therefore$ 2ⁿᵈ curve is correct.





$\therefore M$ sagging max $= 7.68$ kNm ... at $x = 16$

M Hogging max $= -2$ kNm at $x = 5$



considering Right part

$M_x = 0.8x - 2y$

$= 0.8x - \frac{4\times 4\times 2\, x(20-x)}{400}$

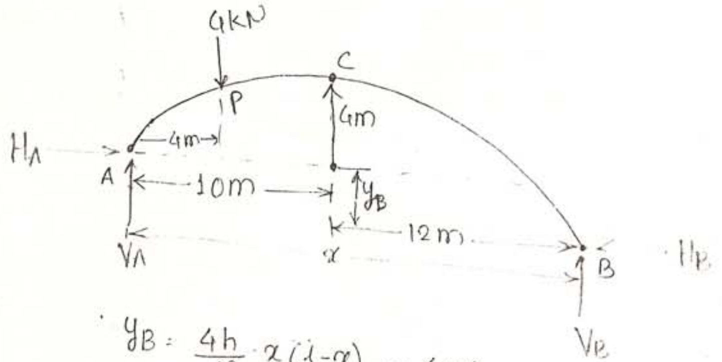
limits 0 to 16

$\therefore M_x = 0.8x - 0.08(20-x)\cdot x$

at $x=0$, $M_0 = 0$

$x = 10$, $M_{10} = 0$

$x = 16$, $M_{16} = 7.68$



$$y_B = \frac{4h}{l^2}\cdot x(l-x) = \frac{4\times 4}{20^2}\times 22(20-22) = -1.76 \text{ m}.$$

(assuming $h = 4$ m, $x = 22$ m)

$\Sigma M_B = 0$

$\therefore V_A(22) = H_A(1.76) + 4(18) \Rightarrow 22V_A - 1.76H_A = 72$

$\Sigma M_C = 0$

$\therefore H_A(4) + 4(6) = V_A(10) \Rightarrow 10V_A - 4H_A = +24$

$\therefore \boxed{V_A = 3.49 \text{ kN}}$

$\boxed{H_A = H_B = 2.727 \text{ kN}}$

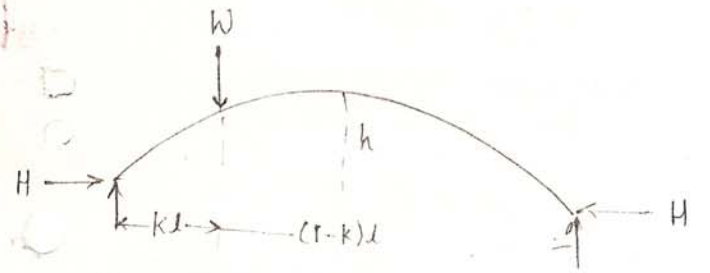
Then $\Sigma M_A = 0$

$\therefore 22V_B - 1.76H_B - 4\times 4 = 0$

$\therefore \boxed{V_B = 0.945 \text{ kN}}$

To find BM, consider section just left to 'P' & then upto 'B'.

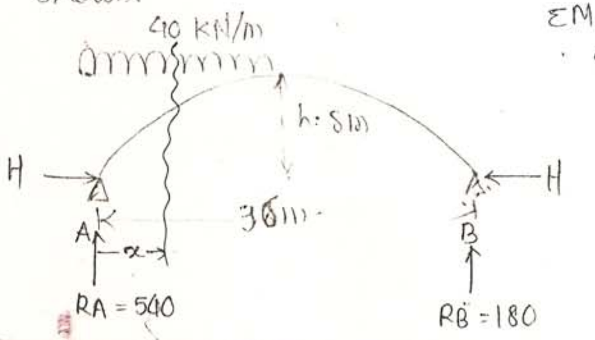
Two Hinge Arch



$$\boxed{H = \frac{5}{8}\frac{W.l}{h}(k - 2k^3 + k^4)}$$ V. Imp. or $$\boxed{H = \frac{5}{8}\frac{Wlk}{h}(1 - 2k^2 + k^3)}$$

If udl acts on arch

No: Determine the position & magnitude of the max BM for the arch shown.



$\Sigma M_A = 0$

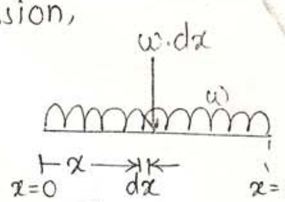
$36 R_B = 40\times 18\times 9$

$\therefore R_B = 180$ kN.

$R_A = 540$ kN

Horizontal reaction is obtained ~~by~~ from the basic expression,

i.e. $H = \frac{5}{8}\frac{\omega l}{h}k\,(1-2k^2+k^3)$



Integrating in the tributary length in the udl.

$\therefore dH = \frac{5}{8}\frac{\omega dx \cdot l}{h}(k-2k^3+k^4)$ where $\boxed{k = \frac{x}{l}}$ → Limits.

$\therefore \boxed{x = kl}$

$$dH = \int_0^{0.5} \frac{5}{8}\frac{w \cdot l \cdot dk \cdot l}{h}(k-2k^3+k^4)$$ It's better to replace dx by dk

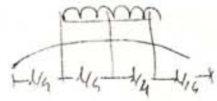
$\therefore dx = l \cdot dk.$

$$= \frac{5}{8}\frac{\omega l^2}{h}\int_0^{0.5}(k-2k^3+k^4)\,dk$$

$$= \frac{5\omega l^2}{8h}\left[\frac{k^2}{2} - 2\frac{k^4}{4} + \frac{k^5}{5}\right]_0^{0.5} = \frac{5}{8}\frac{\omega l^2}{h}\left[\frac{(0.5)^2}{2} - 2\frac{(0.5)^4}{4} + \frac{(0.5)^5}{5}\right]$$

$= \cancel{\frac{5\omega l^2}{8h}[0.1]}$ ✓ $\qquad \therefore \boxed{H = 405 \text{ kN}}$

$= \cancel{\frac{0.5}{8}\frac{\omega l^2}{h}}\ \boxed{H = \frac{0.5\,\omega l^2}{8h}}$



Limits $\frac{1}{2}$ to $\frac{3}{4}$

Consider a section at x from A.

$\therefore M_x = 540x - 40\frac{x^2}{2} - 405 \times y$ $\qquad (0 \le x \le 18)$

$y = \frac{4h}{l^2}\cdot x(l-x)$

$y = \frac{4 \times 8}{36^2} \times x(36-x)$

$\left.\begin{array}{l} M_0 = 0 \\ M_{18} = 0 \end{array}\right\}$ Speciality of Half loaded arch with udl with supports are at same level.

Examine the f^n:-

at $x=0$, $\frac{dM}{dx} = +ve$.

$\frac{dM}{dx} = 540 - 40x - 10(36-2x)$ $\Rightarrow_{+e}$ $540 - 40x - 360 + 20x = 0$

$\therefore \frac{dM}{dx}$ is zero at $\underline{x = 9}$

$\frac{d^2M}{dx^2} = -40 - 10(-2) = -20$

i.e. at turning point

i.e. curve starts with +ve slope. $\qquad \therefore \boxed{M_9 = 810 \text{ kNm}}$

for RHS :-

$M_x = 180x - 405 \times \frac{4 \times 8}{36^2} \times x(36-x)$ (9)

$= 180x - 10x(36-x)$

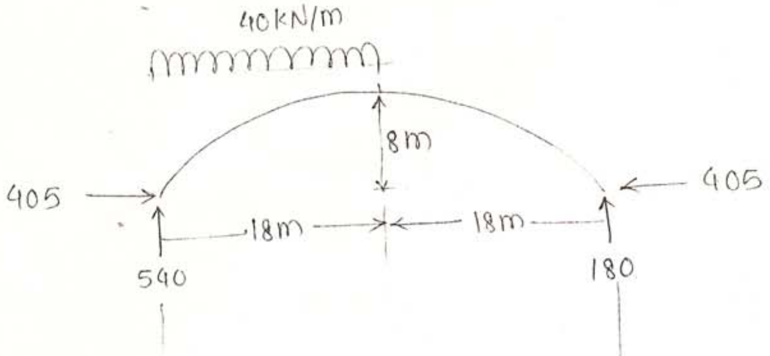
at $x=0$, $M_x = 0$

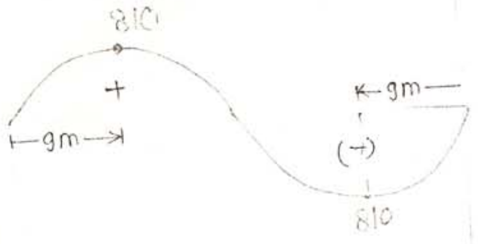
at $x=18$, $M_{18} = 0$

$\frac{dM_x}{dx} = 180 - 360 + 20x = -180 + 20x$

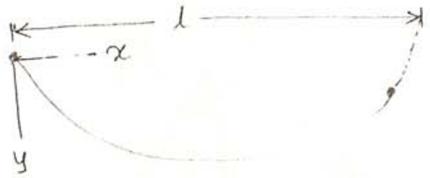
$\cancel{\frac{d^2M}{dx^2}} =$

Check $M_9 = \underline{-810 \text{ kNm}}$





[Suspension Cables]



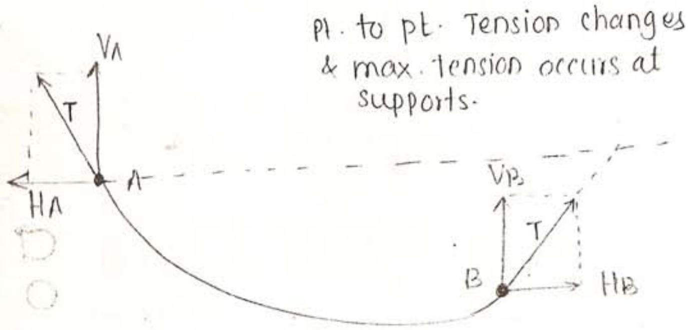
Shape is assumed to be parabolic

Eqⁿ of cable,

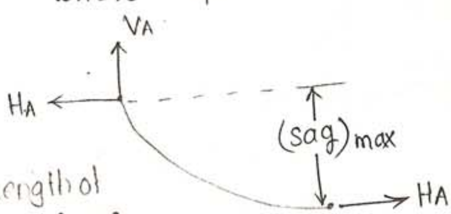
$y = \frac{4h}{l^2} x(l-x)$

* No BM
* No SF
* Only Tension.

Pt. to pt. Tension changes & max. tension occurs at supports.

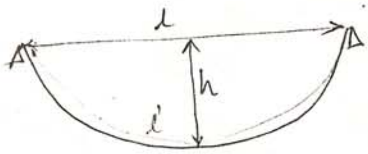


Max. sag occurs at the pt. where slope is zero.



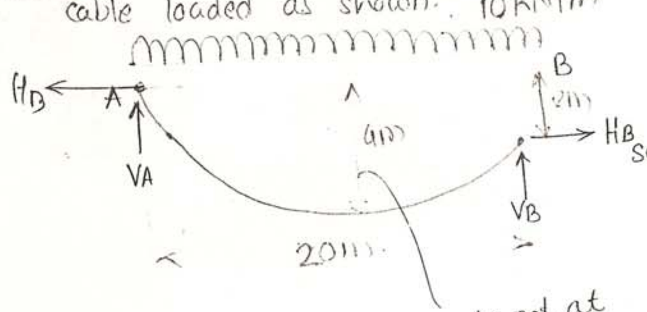
Length of the cable $= \int \sqrt{1+\left(\frac{dy}{dx}\right)^2} \cdot dx$. → length of any curve.

Using Binomial expansion we obtain an approx. expression for the length of the cable, supported at the centre.



$\boxed{\text{length of Curve}_{\text{Parabola}} = l' = l + \frac{8}{3}\frac{h^2}{l}}$

(Pro) Find the max. Tension for in the suspended cable loaded as shown.



4m is not at centre & this can be solved by Mechanics.

Solⁿ:- $\Sigma M_B = 0$

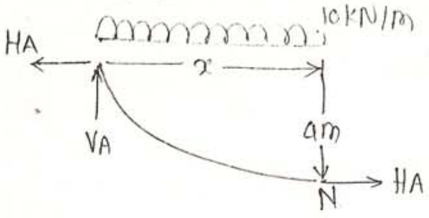
$\therefore 20V_A = 10 \times 20 \times 10 + 2H_A$

$20V_A - 2H_A = 2000$

$\Sigma M_A = 0$

$20V_B = 10 \times 20 \times 10 + 2H_B$

$20V_B - 2H_B = 2000$



$\Sigma M_N = 0$

$H_A(4) + 10\frac{x^2}{2} = V_A \cdot x$

$\Sigma f_y = 0$

$\boxed{V_A = 10x}$

$\therefore H_A(4) + 10\frac{x^2}{2} = 10x^2$

$\therefore H_A = \frac{10x^2 - 10\frac{x^2}{2}}{4} = \boxed{\frac{5x^2}{4} = H_A} = H_B$

$T_{max} = 172\ kN\ \sqrt{H_A^2 + V_A^2}$ $\quad \therefore$ solving for 'x', $x = 11.72\ m$

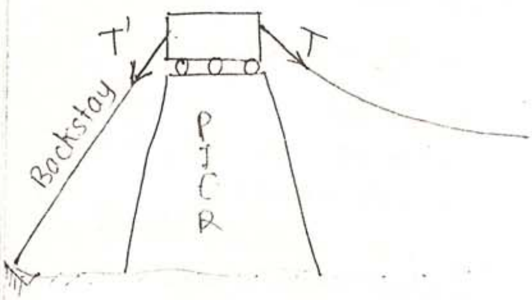
occurs at 'A' . i.e. Higher slope, or max. slope.

$H_A = 172\ kN = H_B$

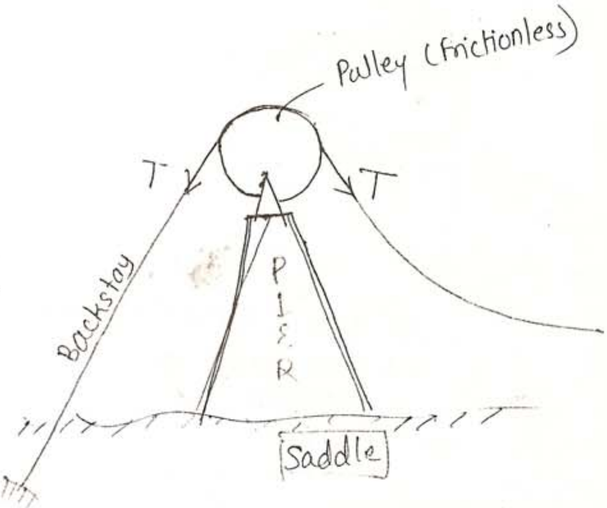
$V_A = 117.2$

$\boxed{T_{max} = 208\ kN}$.

* Anchorages:



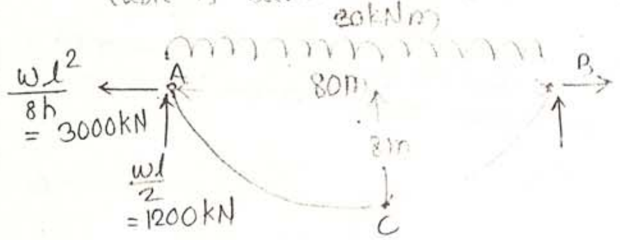
[saddle]



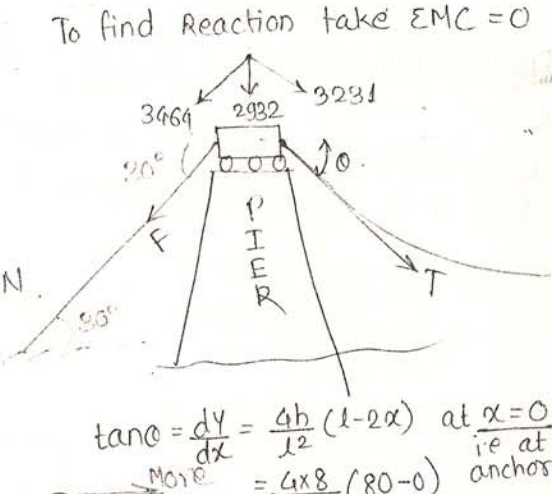
[Saddle]

"MPSC"

Pro:- Find the tension in the backstay & the pre. on the pier, if the cable is saddle & the backstay is at 30° with the horizontal.



To find Reaction take $\Sigma M_C = 0$



$\therefore T = \sqrt{1200^2 + 3000^2} = 3231\ kN$

$\therefore$ Eq^m condition,

$F\cos 30 = T\cos(21.8)$

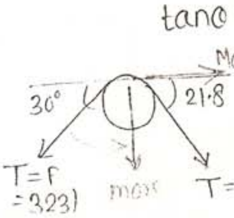
$\therefore \underline{F} = 3231 \frac{\cos(21.8)}{\cos 30}$

$\boxed{F = 3464\ kN}$

$\tan\theta = \frac{dy}{dx} = \frac{4h}{l^2}(l - 2x)$ at $x = 0$ i.e at anchorage

$= \frac{4 \times 8}{80^2}(80 - 0)$

$\boxed{\theta = 21.8°}$



Net pressure on pier $= \Sigma f_y = 0$

$\therefore F\sin 30 + T\sin(21.8) = 3464\sin 30 + 3231\sin(21.8)$

$= \underline{2932\ kN}$

$\therefore$ "saddle" the cable is running over a frictionless pulley & then $\therefore$ same angle, then Horizontal force on pier $= 3231(\cos 21.8 - \cos 30) = 201.8$