Theory of Structures

Notes by-

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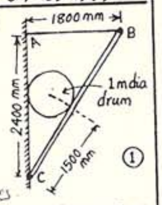
Kolhe PS

SOM Class test for SEM(A)

Answer all the five questions
8 + 11 + 12 + 14 + 5 = 50 marks

0930 hrs to 1130 hrs
04-09-1999

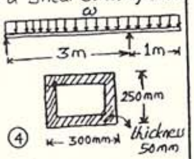


(1) A wall bracket is constructed as shown. All joints may be considered pin connected. Steel rod AB has a cross sectional area of $5\ mm^2$. Member BC is a rigid beam. If a 1 m dia frictionless drum weighing 5 kN is placed in the position shown, What will be the elongation of rod AB? $E = 200\ GN/m^2$

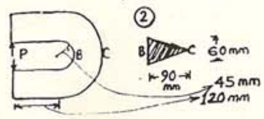
(2) The curved beam has a triangular cross section with the dimensions shown. If $P = 40\ kN$, determine the circumferential stresses at B & C

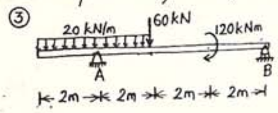
(3) Draw the SFD & BMD for the beam loaded as shown, highlighting all salient features

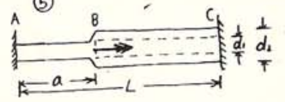
(4) The distributed load shown is supported by a box beam having the cross section shown. Find the maximum value of ω that will not exceed a flexural stress of 10 MPa or a shear stress of 1 MPa



(5) A bar ABC that is fixed at both ends is subjected to a torque at section B. The bar has a solid circular cross section from A to B of diameter d_1, and a hollow circular cross section from B to C of outer diameter d_2 and inner diameter d_1. Derive an expression for the ratio a/L such that the reactive torques at A and C are equal numerically







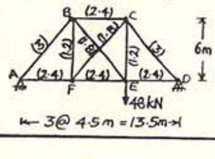
Kolhe PS

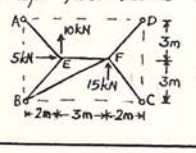
TOS-II Class test TE(C)

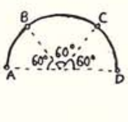
Answer all the five questions
(13) + (7) + (5) + (16) + (9) = 50 marks

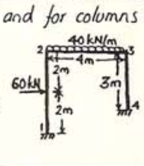
1130 hrs to 1330 hrs 25-08-99

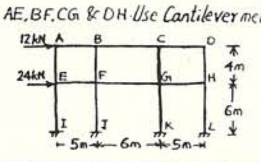
1. Find the forces in all the members of the truss shown. Number in paranthesis are areas in $10^{-3} m^2$. $E = 200\ GPa$
2. A space frame is supported at A, B, C and D in a horizontal plane through ball joints. Member EF is horizontal and 3 m above the base. Loads at E and F act in a horizontal plane. Find the forces in members EA, EB & EF
3. A semicircular beam ABCD is simply supported as shown at A and D. If the beam carries a udl of ω per unit length and has a radius r, find the support reactions at B & C.
4. Draw the BMD for the frame highlighting all the important features. EI is a constant.
5. Draw the BMD for the top floor beams AB, BC and CD and for columns AE, BF, CG & DH. Use Cantilever meth











Department of Civil Engineering, Amrutvahini College of Engineering, Sangamner 422 608.
Preliminary Examination in **THEORY OF STRUCTURES-II** for TE (C), 9.30 AM to 12.30 PM, Oct 11th 2002
Answer **any Three** questions from each Section I (Q.1through Q.5) and **any Three** from Section II(Q.6 through Q.10)

Q.1a) Write down the stress and strain tensors for Plane stress and Plane strain Problems. State the differential equations and compatibility equations in case of a 2D stress state. (8)

Q.1b) Strains measured by a rectangular rosette are ε_0=600μm/m(comp), ε_{45}=300μm/m(comp), and ε_{90}=500μm/m(tens). Locate the principal planes and find the principal stresses.Use sketches (8)

Q.2) For the beam*[Fig]*, draw BMD and sketch the elastic curve using Stiffness matrix method. (16)

[illegible]

Q.4) Solve Q.3) using Moment Distribution Method. (17)

Q.5a) Obtain the Shape factor for a circular cross section. (6)

Q.5b) For the ultimate loads shown*[Fig]*, find the value of M_P required (uniform section throughout). Each of the four point loads is 168 kN and the udl on the left column is 14 kN/m. (10)

Q.6) A circular beam of radius r, curved in plan, is subjected to a udl w throughout and supported on 12 equispaced columns (θ=30°). Obtain an expression for the torsional moment at an intermediate point, defined by φ, on the beam. Find the angle φ_1 at which the torsional moment is maximum. What is the magnitude of this maximum torsional moment? (16)

Q.7a) Write notes on i) Berry functions ii) Perry's formula for intensity of 'column-end loading. (8)

Q.7b) During an experiment on a column, the deflection δ at the centre for axial load P are so recorded *[Table]*. Draw Southwell Plot and obtain P_{cr} and the initial curvature of the column. (?)

Q.8a) Find the forces in all the members of the truss shown*[Fig]*. Numbers in parenthesis are the member areas in 10^{-3} m². Take E= 200 GPa. (12)

Q.8b) Briefly explain the procedure of analysis in case there was an additional member AC. (4)

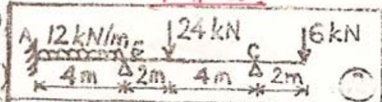
Q.9a) What are the various supports used in space trusses. Show the reactive components in these supports giving sketches. Explain in brief the method of analysis of a space truss. (5)

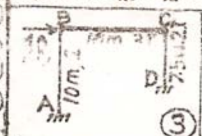
Q.9b) Analyze the beam*[Fig]* using flexibility method and obtain the moments at B and C. (12)

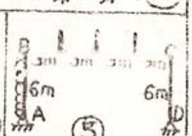
Q.10a) Why are the 'Portal' and 'Cantilever' methods also known as the 'method of proportionate shears' and 'method of proportionate stresses' respectively. (5)

Q.10b) Analyze the frame *[Fig]* using Portal method and draw the Axial force, Shear, and Moment diagrams for members AB, BC, CD, AE, BF, and CG. (12)

Rollers

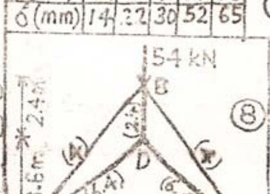


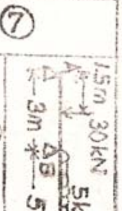


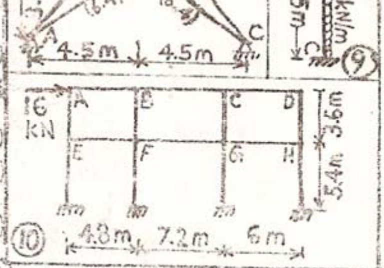


P(kN)	32	37	41	50	56
δ (mm)	14	22	30	52	65

(7)





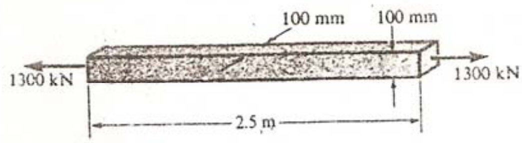


Eng. Grammer ⇒ Pal & Sari (MPSC)

Kolhe Pravin

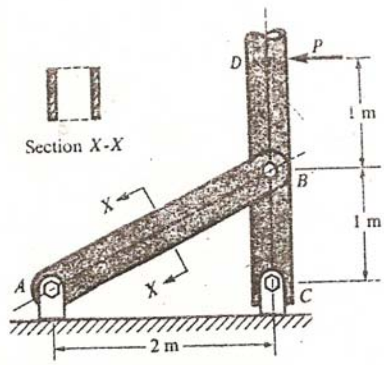
STRESSES AND STRAINS

1 A steel bar of length 2.5 m with a square cross section 100 mm on each side is subjected to an axial tensile force of 1300 kN (see figure). Assuming that $E = 200$ GPa and $\nu = 0.3$, find (a) the elongation of the bar, (b) the change in cross-sectional dimensions, and (c) the change in volume.



Ans: 1.62 mm, $\Delta = 0.02$ mm, $6500\ mm^3$ increase

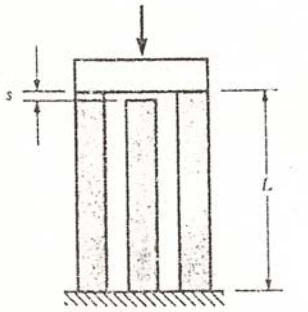
2 A frame is made of a 2 m long vertical pipe CD and a brace AB formed from two flat bars (see figure). The frame is supported by bolted connections at points A and C, which are 2 m apart. The brace is fastened to the pipe at point B, which is 1 m above point C, by a 20 mm diameter bolt. If a load $P = 12$ kN acts horizontally at D, determine the average shear stress τ_{aver} in the bolt at B.



Take moment @ C for CD to get the force at B

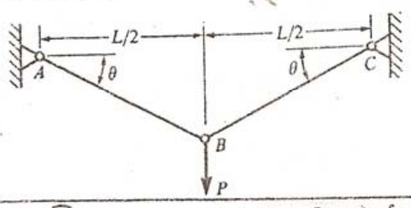
Ans: 43 MPa

3 A rigid steel plate is supported by three posts of high-strength concrete each having 200 mm × 200 mm square cross section and length $L = 2$ m (see figure). Before the load P is applied, the middle post is shorter than the others by an amount $s = 1.0$ mm. Determine the maximum permissible load P if the modulus of elasticity of the concrete is $E_c = 30$ GPa and the allowable stress in compression is $\sigma_{allow} = 18$ MPa.



Ans: 1.56 MN

4 Two bars AB and BC support a vertical load P (see figure). The distance L between supports remains constant, but the angle θ can be varied by changing the lengths of the bars. Assuming that both bars have the same cross-sectional areas and that they are fully stressed to the allowable stress in tension, determine the angle θ so that the structure has minimum volume. (Disregard the weights of the bars.)



Ans: 45°

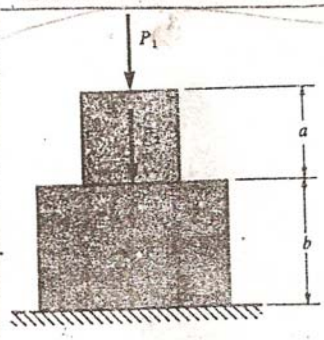
Get the tensions in terms of P. Minimize

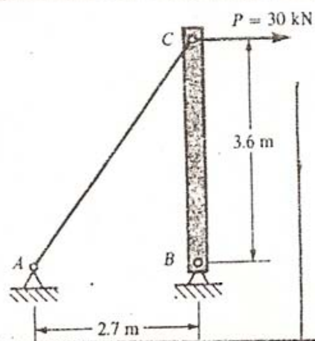
5 A concrete pedestal (see figure) of circular cross section has an upper part of diameter 0.5 m and height $a = 0.5$ m and a lower part of diameter 1.0 m and height $b = 1.2$ m. It is subjected to loads $P_1 = 7$ MN and $P_2 = 18$ MN. Assuming $E = 25$ GPa, calculate the deflection δ of the top of the pedestal.

Ans: 2.24mm

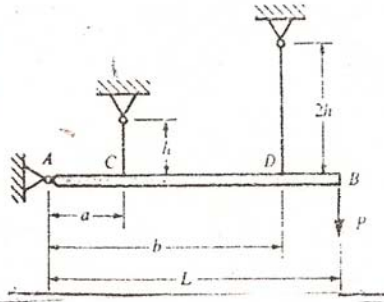
6 Calculate the horizontal and vertical deflections δ_h and δ_v, respectively, of the top of the wood post due to the horizontal load $P = 30$ kN (see figure). The post has cross-sectional area 32,000 mm² and modulus of elasticity $E_w = 10$ GPa. The post is supported by a steel rod AC of diameter 25 mm and modulus of elasticity $E_s = 210$ GPa.

Ans: $\delta_h = 4.24$ mm, $\delta_v = 0.45$ mm





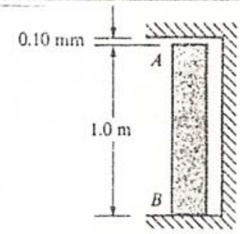
7 A rigid bar AB of length L is hinged to a wall at A and supported by two vertical wires attached at points C and D (see figure). The wires have the same cross-sectional areas and are made of the same material, but the wire at D is twice as long as the wire at C. Find the tensile forces T_c and T_d in the wires due to the vertical load P acting at end B.



Ans: $T_c = 2PaL/(2a^2+b^2)$, $T_d = PbL/(2a^2+b^2)$

In Prob 8, consider the free expansion of the bar ($L\alpha\Delta T$) and the force exerted by the ceiling to reduce the expansion to a mere 0.1 mm.

8 A copper bar AB of length 1.0 m is placed in position at room temperature with a gap of 0.10 mm between end A and a rigid wall (see figure). Calculate the axial compressive stress σ in the bar if the temperature rises 40°C. (For copper, use $\alpha = 17 \times 10^{-6}$/°C and $E = 110$ GPa.)



$L\cdot\alpha\cdot\Delta t$ $L\alpha\Delta t$ P 0.1 mm

Ans: 63.8 MPa

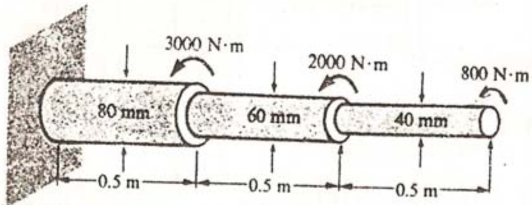
$\delta = \frac{PL}{AE} = \frac{\sigma L}{E} \Rightarrow \sigma = \frac{\delta L}{E}$

Kolhe Pravin

TORSION OF CIRCULAR SHAFTS

① A stepped shaft is subjected to torques as shown in the figure. The length of each section is 0.5 m and the diameters are 80 mm, 60 mm, and 40 mm. If the material has shear modulus of elasticity $G = 80$ GPa, what is the angle of twist ϕ (in degrees) at the free end?

Ans: 2.44°



Note Hz ⇒ cycles per s

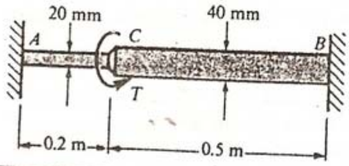
Power = Tω

② How much power P may be transmitted by a solid circular shaft of diameter 80 mm turning at 0.75 Hz if the shear stress is not to exceed 30 MPa?

Ans 14.2 kW

③ A stepped shaft of solid circular cross section (see figure) is held against rotation at the ends. If the allowable stress in shear is 55 MPa, what is the allowable torque T that may be applied to the shaft at C?

Ans 639 Nm



④ A hollow circular bar of steel ($G = 80$ GPa) is twisted by a torque T that produces a maximum shear strain $\gamma_{max} = 800 \times 10^{-6}$ rad. The bar has outside and inside radii of 75 and 60 mm, respectively. What is the maximum tensile stress σ_{max} in the shaft? What is the magnitude of the applied torque T?

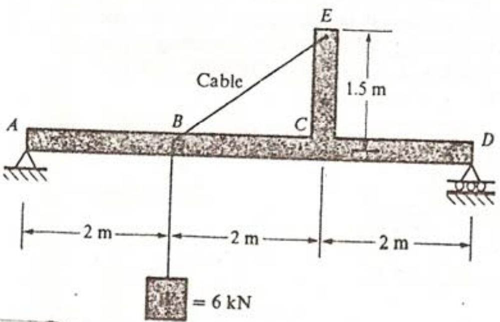
Ans 64 MPa 25 kNm

BENDING MOMENT AND SHEAR FORCE

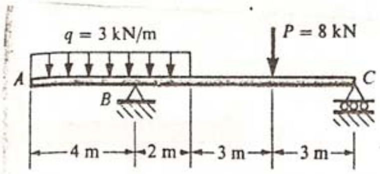
① The beam $ABCD$ is loaded by a force $W = 6$ kN by the arrangement shown in the figure. The cable passes over a small frictionless pulley at B and is attached at E to the vertical arm. Calculate the shear force V and bending moment M at section C, which is just to the left of the vertical arm.

Ans 1.6 kN 11.2 kNm

Draw the SFD & BMD also

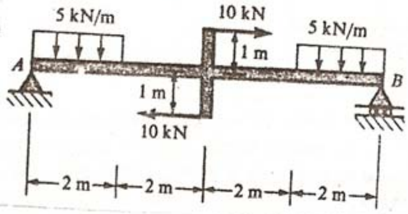


② Draw the SFD & BMD



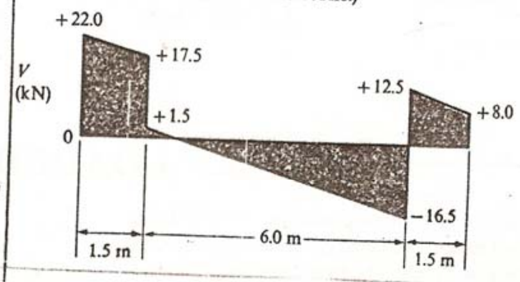
Ans Vmax = -12 kN Mmax = -24 kNm

③ Draw the SFD & BMD



Ans Mmax = 20 kNm

④ The shear-force diagram for a beam is shown in the figure. Assuming that no couples act as loads on the beam, draw the bending-moment diagram. (Note that the shear force has units of kilonewtons.)

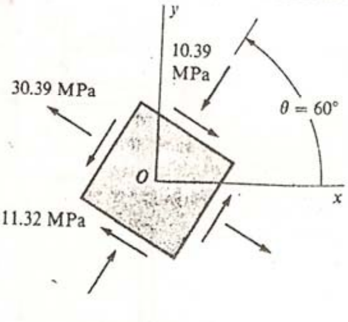


Ans Mmax = 30 kNm

TRANSFORMATION OF STRESSES

① An element in plane stress is rotated through a known angle θ (see figure). On the rotated element, the normal and shear stresses have the magnitudes and directions shown in the figure. Determine the normal and shear stresses on an element whose sides are parallel to the xy axes; that is, determine σ_x, σ_y, and τ_{xy}.

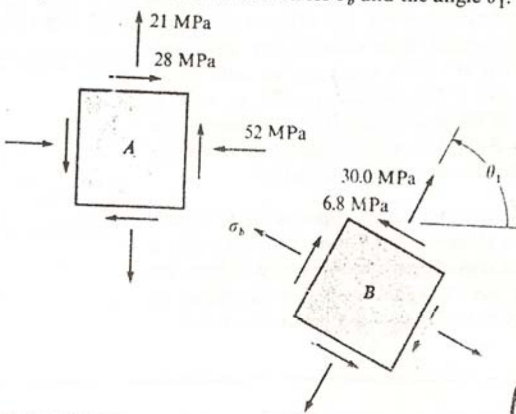
analytical or graphic



Ans: σ_x = 30 MPa, σ_y = -10 MPa, τ_{xy} = -12 MPa

② At a point in a structure subjected to plane stress, the stresses have the magnitudes and directions shown acting on element A in the first part of the figure. Element B, located at the same point in the structure, is rotated through an angle θ_1 of such magnitude that the stresses have the values shown in the second part of the figure. Calculate the normal stress σ_b and the angle θ_1.

Ans σ_b = -61 MPa θ_1 = 67°

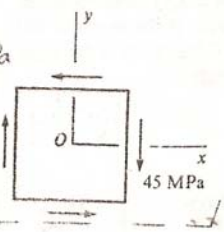


You may use analytical or graphical method.

Continued overleaf

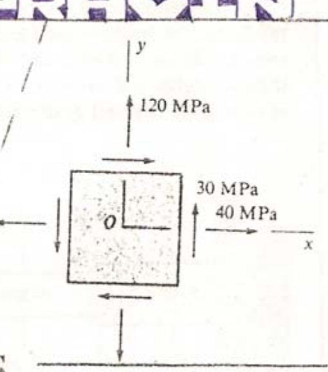
③ An element in pure shear is subjected to stresses τ_{xy} as shown in the figure. Using Mohr's circle, determine (a) the stresses acting on an element rotated through an angle $\theta = 75°$ from the x axis and (b) the principal stresses. Show the results on sketches of properly oriented elements.

Ans :
$\sigma_{x_1} = -22.5$ MPa
$\sigma_{y_1} = 22.5$ MPa
$\tau_{x_1y_1} = 39$ MPa
$\sigma_1 = 45$ MPa
$\sigma_2 = -45$ MPa



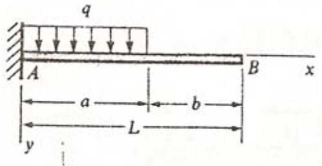
④ An element in plane stress is subjected to stresses σ_x, σ_y, and τ_{xy} as shown in the figure. Using Mohr's circle, determine the stresses acting on an element rotated through an angle $\theta = 20°$. Show the results on a sketch of a properly oriented element.

Ans: $\sigma_{x_1} = 68.6$ MPa, $\tau_{x_1y_1} = 48.7$ MPa
$\sigma_{y_1} = 91.4$ MPa

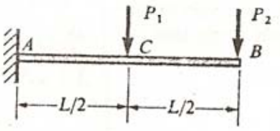


SLOPE AND DEFLECTION OF BEAMS

① Determine the equations of the deflection curve for a cantilever beam AB carrying a uniform load of intensity q over part of the span (see figure).

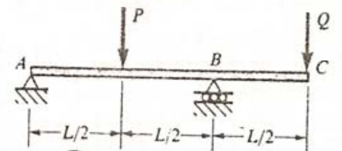


② A cantilever beam AB supports two concentrated loads P_1 and P_2 as shown in the figure. Calculate the deflections δ_b and δ_c at points B and C, respectively. Assume $P_1 = 10$ kN, $P_2 = 5$ kN, $L = 2.6$ m, $E = 200$ GPa, and $I = 20.1 \times 10^6$ mm^4.



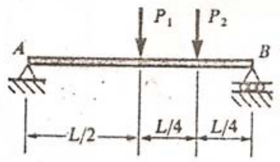
Ans:
$\delta_B = 11.8$ mm
$\delta_c = 4.1$ mm

③ A beam with an overhang supports loads P and Q as shown in the figure. Determine the ratio P/Q that will make the deflection at C equal to zero.



Ans:
4

④ A simple beam AB supports two concentrated loads P_1 and P_2 as shown in the figure. Calculate the maximum deflection δ_{max} of the beam, assuming $P_1 = 100$ kN, $P_2 = 200$ kN, $L = 10$ m, $E = 200$ GPa, and $I = 1.20 \times 10^9$ mm^4.



Ans:
20.72 mm

AXIALLY LOADED COLUMNS

① A pinned-end strut of aluminum ($E = 73$ GPa) with length $L = 2$ m is constructed of circular tubing with outside diameter $d = 50$ mm. The strut must resist an axial load $P = 14$ kN with a factor of safety $n = 2$ with respect to buckling. Determine the required thickness t of the tube.

Ans
4.05 mm

② Three pinned-end columns of the same material have the same length and the same cross-sectional area. The columns are free to buckle in any direction. The columns have cross sections as follows: (1) equilateral triangle, (2) square, and (3) circle. Determine the ratios $P_1 : P_2 : P_3$ of the critical loads for these columns.

Ans:
1.209 : 1.047 : 1

③ The horizontal bar shown in the figure is supported by columns AB and CD. Each column is pinned at the top to the horizontal bar, but support A is fixed and support D is pinned. Both columns are solid steel bars ($E = 200$ GPa) of square cross section with width equal to 15 mm. (a) If the distance $a = 0.4$ m, what is the critical value of the load Q? (b) If the distance a can be varied between 0 and 1 m, what is the maximum value of Q_{cr}? What is the corresponding value of a?

Ans
(a) 14.5 kN
(b) 22.8 kN, 0.253 m

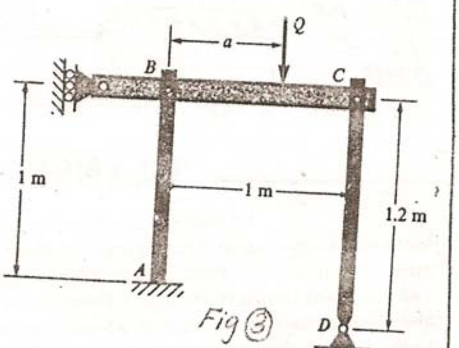


Fig ③

④ A horizontal bar AB is supported by a pinned-end column CD as shown in the figure. The column is a steel bar ($E = 200$ GPa) of square cross section (50 mm on a side). Calculate the allowable load Q if the factor of safety with respect to buckling of the column is $n = 3$.

Ans: 12.7 kN

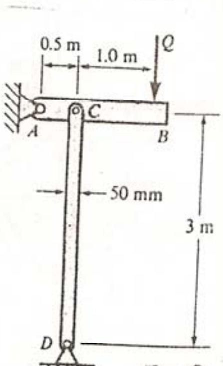
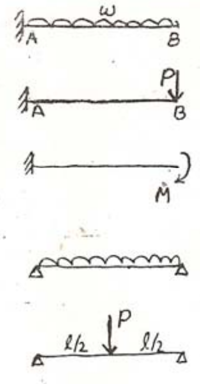


Fig ④

SLOPES & DEFLECTIONS



$\theta_B = wl^3/6EI$
$\delta_B = wl^4/8EI$

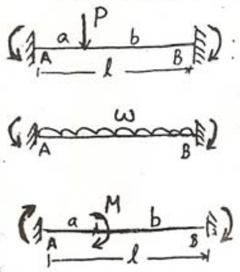
$\theta_B = Pl^2/2EI$
$\delta_B = Pl^3/3EI$

$\theta_B = M_o l/EI$
$\delta_B = M_o l^2/2EI$

$\theta_{ends} = wl^3/24EI$
$\delta_{max} = 5wl^4/384EI$

$\theta_{ends} = Pl^2/16EI$
$\delta_{max} = Pl^3/48EI$

FIXED-END MOMENTS



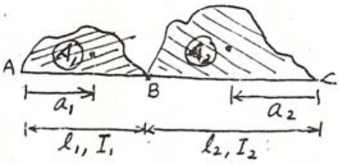
$M_A = Pab^2/l^2$
$M_B = Pa^2b/l^2$

$M_A = wl^2/12$
$M_B = wl^2/12$

$M_A = \frac{Mb}{l^2}(2a-b)$
$M_B = \frac{Ma}{l^2}(2b-a)$

THREE MOMENT EQUATION

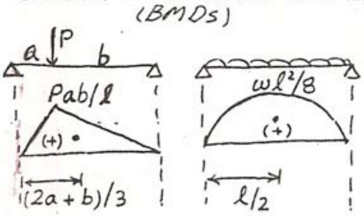
[Clapeyron (1857) & Mohr (1860)]

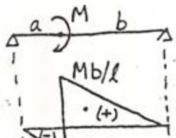


$M_A(l_1/I_1) + 2M_B(l_1/I_1 + l_2/I_2) + M_C(l_2/I_2) = -\frac{6A_1a_1}{l_1I_1} - \frac{6A_2a_2}{l_2I_2} + \frac{6Eh_A}{l_1} + \frac{6Eh_C}{l_2}$

h_A and h_C are positive, if they are above B. [in case of yielding]

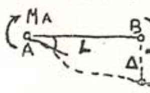
SIMPLY SUPPORTED BEAMS (BMDs)





Other type of distributed loads to be handled using the basics.

SLOPE DEFLECTION equations



$M_0 \rightarrow$ locking moment & $R = \Delta/L$

$M_A = M_0 + \frac{2EI}{L}(2\theta_A + \theta_B - 3R)$

R is positive, if clockwise.

STIFFNESS MATRIX METHOD (in a nutshell)

[A] expresses joint forces {P} in terms of independent element forces {F}

[S] expresses {F} in terms of end rotation matrix {θ}

[B] or $[A]^T$ expresses {θ} in terms of joint displacements {X}

$[S][A]^T$ expresses the generalized member end forces in terms of {X}

$[A][S][A]^T$ or [K] which is called the Global stiffness matrix expresses {P} in terms of {X}

The final end moments $\{F^*\}$, if $\{F^o\}$ are the fixed end moments are

$$\{F^*\} = \{F^o\} + [S][A]^T\{X\}$$

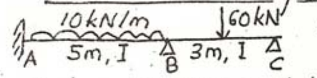
Note: $\{X\} = [K]^{-1}\{P\}$

The sequence of calculations can be summarized as

i) Obtain [A] and [S]
ii) Find $[S][A]^T$
iii) Find $[A][S][A]^T$ which is [K]
iv) Obtain $\{X\} = [K]^{-1}\{P\}$ where {P} is the unbalanced moments & forces at joints
v) Obtain $[S][A]^T\{X\}$
vi) Add the fixed end moments $\{F^o\}$ to $[S][A]^T\{X\}$ to get $\{F^*\} = \{F^o\} + [S][A]^T\{X\}$

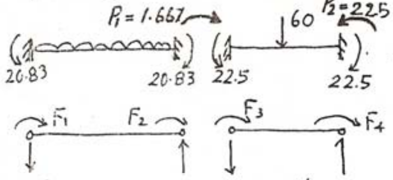
Note: [K] will be symmetrical and its order will be same as the number of unknown joint displacements

Demonstrative Example



Analyze, if 60kN is central.

First find the fixed-end moms.



Joints B and C, where the unknown displacements are located, are balanced by

$\{P\} = \begin{Bmatrix} 1.667 \\ -22.5 \end{Bmatrix}$ Note: $P_1 = F_2 + F_3$, $-P_2 = F_4$

This implies $\begin{Bmatrix} 1.667 \\ -22.5 \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix}$

$\{P\} = [A]\{F\}$

But $\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} = \begin{bmatrix} 4EI/5 & 2EI/5 & 0 & 0 \\ 2EI/5 & 4E/5 & 0 & 0 \\ 0 & 0 & 4EI/3 & 2EI/3 \\ 0 & 0 & 2EI/3 & 4EI/3 \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{Bmatrix}$

$\{F\} = [S]\{\theta\}$

Now $\begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}$

$\{\theta\} = [A]^T\{X\}$

{Note that $\theta_1 = 0$, $\theta_2 = X_1$, $\theta_3 = X_1$, $\theta_4 = X_2$}

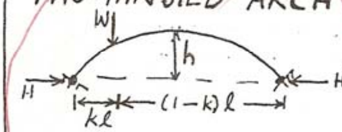
Thus $\{P\} = \underbrace{[A][S][A]^T}_{[K]}\{X\}$

{[K] is obtained as $[K] = EI\begin{bmatrix} 2.133 & 0.667 \\ 0.667 & 1.333 \end{bmatrix}$}

$\{X\} = [K]^{-1}\{P\} = \frac{1}{24EI}\begin{bmatrix} 1.333 & -0.667 \\ -0.667 & 2.133 \end{bmatrix}\begin{Bmatrix} 1.667 \\ -22.5 \end{Bmatrix} = \frac{1}{EI}\begin{Bmatrix} 7.176 \\ -20.46 \end{Bmatrix}$

$[S][A]^T\{X\} = \begin{Bmatrix} 2.87 \\ 5.74 \\ -4.07 \\ -22.5 \end{Bmatrix}$ & $\{F^*\} = \begin{Bmatrix} -20.83 \\ +20.83 \\ -22.50 \\ +22.50 \end{Bmatrix} + \begin{Bmatrix} 2.87 \\ 5.74 \\ -4.07 \\ -22.5 \end{Bmatrix} = \begin{Bmatrix} -17.96 \\ +26.57 \\ -26.57 \\ 0 \end{Bmatrix}$

TWO-HINGED ARCH

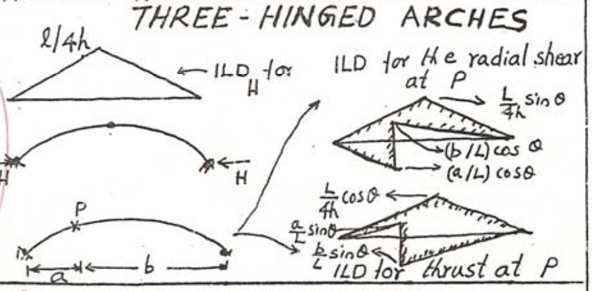


$H = \frac{5}{8}\frac{Wl}{h}k(1-2k^2+k^3)$

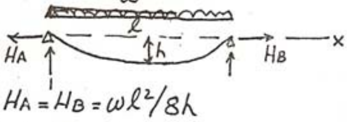
or

$= \frac{5}{8}\frac{Wl}{h}k(1-k)(1+k-k^2)$

THREE-HINGED ARCHES



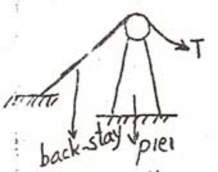
SUSPENSION CABLES & THEIR ANCHORAGES

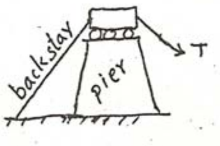


$H_A = H_B = wl^2/8h$

$y = \frac{4h}{l^2}x(l-x)$

Length of cable $\approx l + \frac{8}{3}\frac{h^2}{l}$





In case of saddled cable pier will not be subjected to horizontal thrust.

Due to a temperature change of ΔT, the change in dip is given by $dh = \frac{3}{16}\frac{l^2}{h}\Delta T\alpha$

Kolhe Pravin